Near-barrier fusion of ^{11}Li with heavy spherical and deformed targets

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The cross sections for the fusion of ^{11}Li with ^{208}Pb and ^{238}U are calculated at near-barrier energies. The coupling of the entrance channel to the soft giant dipole resonance in ^{11}Li is taken into account together with the coupling to the breakup channel ^{9}Li$+2n$. The deformation of ^{238}U is also considered. The cross section is found to exhibit important structure around the barrier.

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Recently, the low-energy fusion of radioactive beams, such as ^{11}Li, with heavy target nuclei has been discussed [1–4]. The principal motivation is twofold: (i) the enhancement of the fusion cross section σ_f that arises from the existence of the halo neutrons can be used to further understand these exotic nuclei, and (ii) the potential production of superheavy cold compound nuclei, with reasonably measurable cross sections.

In the calculations made so far, two features of the halo are taken into account: the lowering of the static Coulomb barrier and the coupling of the entrance channel to the low-lying soft giant dipole resonance (the pygmy resonance). Both of these effects lead to an enhanced fusion cross section.

The existence of the pygmy resonance at about 1.2 MeV has recently been firmly established though the study of the double-charge-exchange reaction ^{11}B$(\pi^-,\pi^+)^{11}$Li [5]. One anticipates, on general ground, that this state has a large width due to the very low binding energy of the dineutron (~ 0.2 MeV). Thus, it is of great importance in any fusion calculation to consider the finite lifetime of the pygmy resonance. This leads to an enhancement of σ_f reduced with respect to that already reported. The purpose of this work is to consider the above effect by coupling the pygmy resonance to the breakup channel in the calculation of the fusion cross section.

In a coupled-channel description of a heavy-ion reaction, the fusion cross section can be calculated from the total reaction cross section as [6]

$$\sigma_f=\sigma_R-\sigma_D \ , \tag{1}$$

where σ_D is the direct reaction cross section and σ_R is given by

$$\sigma_R=\frac{k}{E}\langle \Psi_{\mathbf{k}}^{(+)}|-\mathrm{Im}V|\Psi_{\mathbf{k}}^{(+)}\rangle \ , \tag{2}$$

where $\langle \mathbf{r}|\psi_{\mathbf{k}}^{(+)}\rangle$ is the wave function that describes the elastic scattering and V is the optical potential that generates $\langle \mathbf{r}|\Psi_{\mathbf{k}}^{(+)}\rangle$. It can be shown [6] that the cross section σ_f [Eq. (1)] can be written in the form

$$\sigma_f=\frac{k}{E}\sum_i \langle \Psi_{\mathbf{k}_i}^{(+)}|-\mathrm{Im}\mathring{V}_i|\Psi_{\mathbf{k}_i}^{(+)}\rangle \ , \tag{3}$$

where $\mathring{V}_i$ is the bare optical potential in channel i (no channel coupling) and $|\Psi_{\mathbf{k}_i}^{(+)}\rangle$ is the exact scattering wave function in that channel. Equation (3) has been used by several authors to calculate σ_f using coupled-channels codes [7]. Other models based on this equation, but with the further assumption of infinite absorption once the barrier is penetrated, have also been developed [8]. Here we generalize the second class of models by incorporating the effect of the breakup channel (included in $|\Psi_{\mathbf{k}_i}^{(+)}\rangle$).

To be more specific, we deal here with a case involving the coupling of the elastic channel to a resonant state in the projectile. If this resonant state is approximated by an excited state whose width is very small, then σ_f can be written as (ignoring the excitation energy of the state) [8]

$$\sigma_f=\tfrac{1}{2}[\mathring{\sigma}_f(+F)+\mathring{\sigma}_f(-F)] \ , \tag{4}$$

where $\mathring{\sigma}_f$ is the one-channel fusion cross section and F is the channel coupling potential evaluated at the barrier radius. The $+$ $(-)$ sign indicates addition (subtraction) to (from) the barrier height.

To include the breakup channel coupling effect, in Eq. (4), namely, the nonzero width of the excited state, it is convenient first to express the cross section as a sum of partial-wave contributions

$$\mathring{\sigma}_f=\frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)T_l^f \ ,$$
$$T_l^f=\left\{1+\exp\left[\frac{2\pi}{\hbar\omega}\left[V_B+\frac{\hbar^2 l(l+1)}{2\mu R_B^2}-E_{\mathrm{c.m.}}\right]\right]\right\}^{-1} . \tag{5}$$

In the above, R_B and V_B are the Coulomb barrier radius and height, respectively. When incorporating the breakup channel coupling effect, the partial fusion probability T_l^f has to be multiplied by the breakup survival probability, $(1-T_l^{\mathrm{bu}})$. Thus,

$$\tilde{\sigma}_f^b = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)(1-T_l^{\rm bu})T_l^f \ . \tag{6}$$

And therefore we finally have

$$\sigma_f = \frac{1}{2}\frac{\pi}{k^2}\left[\sum_{l=0}^{\infty}(2l+1)(1-T_l^{\rm bu})T_l^f(+F) + \sum_{l=0}^{\infty}(2l+1)(1-T_l^{\rm bu})T_l^f(-F)\right] = \tfrac{1}{2}[\tilde{\sigma}_f^b(+F)+\tilde{\sigma}_f^b(-F)] \ . \tag{7}$$

Here the Coulomb breakup does not contribute since it is significant only at l larger than those for which fusion is relevant. The nuclear breakup transmission factor $T_l^{\rm bu}$ has been recently calculated for several radioactive systems [9,10]. The major conclusion of these studies is that the dynamic polarization potential which enters in the evaluation of $T_l^{\rm bu}$ via

$$T_l^{\rm bu} = 1 - \exp\left[-2\int_{\rho_0}^{\infty} \frac{{\rm Im}\, V_{\rm pol}/E_{\rm c.m.}}{\sqrt{1-2\eta/\rho - l(l+1)/\rho^2}}\, d\rho\right] \tag{8}$$

is very sensitive to the binding energy of the breakup cluster. In Eq. (8), η is the Sommerfeld parameter and ρ_0 is the distance of closest approach, multiplied by the wave number k, obtained from $1-2\eta/\rho_0 - l(l+1)/\rho_0^2=0$. A closed-form expression for $T_l^{\rm bu}$ was derived in Ref. [10] and it reads

$$T_l^{\rm bu} = 1 - \exp\left[-\frac{4\mathcal{F}_0^2}{E^2}|S_l^{(1)}| I_l^2(\eta,s)\right] , \tag{9}$$

where $\mathcal{F}_0$ is a coupling strength factor, which was found to be 4.859 MeV for ^{11}Li+^{208}Pb. $|S_l^{(1)}|$ is the modulus of the elastic S matrix in the breakup channel and $I_l(\eta,s)$ is a Coulomb radial integral evaluated and discussed in Ref. [10]. The sensitivity of $V_{\rm pol}$ and $T_l^{\rm bu}$ to the binding energy of the dineutron in ^{11}Li resides in the l dependence of $I_l(\eta,s)$.

In the following, we use the fusion calculation of Takigawa and Sagawa [4] as a background for the study of the effect of the coupling to the breakup channel. We took the height of the Coulomb barrier ($V_B=26$ MeV), its radius ($R_B=11.1$ fm), and curvature ($\hbar\omega=3$ MeV) from Fig. 1 of Ref. [4] and used these in our Hill-Wheeler transmission coefficients of Eq. (5). The strength F was adjusted to reproduce the values of σ_f of Ref. [4]. We found $F\sim+3.0$ MeV. The breakup effect was then investigated, through the modified fusion cross section, Eq. (7), with the help of Eq. (9) and taking $|S_l^{(1)}|=[1-T_l^f(E_{\rm c.m.}-0.2)]^{1/2}$. The result of our calculation is shown in Fig. 1. It is clear that the inclusion of the breakup coupling, and thus the lifetime of the pygmy resonance, reduces the enhancement of σ_f by as much as a factor of 100 at energies slightly below the barrier. More important is the fact that the breakup of the projectile renders the fusion cross section *lower* than the one-dimensional calculation at energies extending from above the barrier ($26<E_{\rm c.m.}<45$ MeV) to slightly below the barrier ($24<E_{\rm c.m.}<26$ MeV). At energies less than 24 MeV, the enhancement sets in. The increase of the enhancement with increasing $E_{\rm c.m.}^{-1}$ is, however, much slower than the case without breakup, Eq. (4). Only at energies $E_{\rm c.m.}\le 10$ MeV, does the breakup effect subside completely, letting the pygmy resonance act as a complete vibrational enhancer.

Figure 2 exhibits more clearly the above features through the behavior of the enhancement factor, ε, defined as the ratio of the fusion cross section to the one-dimensional barrier penetration cross section. The breakup effect is contained in the interval $10<E_{\rm c.m.}<45$ MeV. Further, there is a sharp dip at the barrier. This dip is easily understood. At energies above the barrier, the nuclear breakup process inhibits fusion. This inhibition becomes less effective as the energy approaches the barrier,

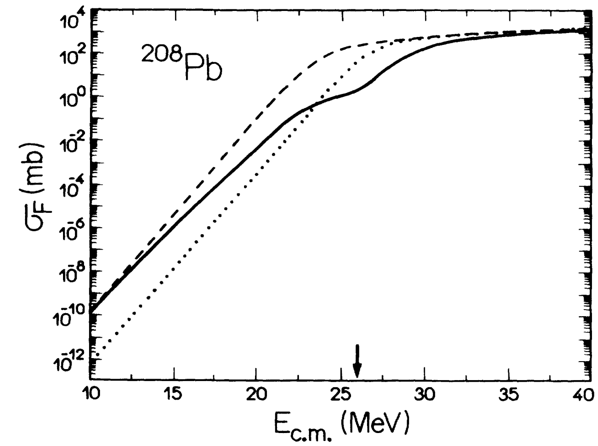

FIG. 1. Excitation function for the fusion cross section of ^{11}Li+^{208}Pb. The dotted curve is the one-dimensional Hill-Wheeler cross section [Eq. (5)], the dashed curve is the pygmy resonance enhanced cross section [Eq. (4)], and the full curve represents the result with inclusion of the breakup coupling [Eq. (7)] (see text for details).

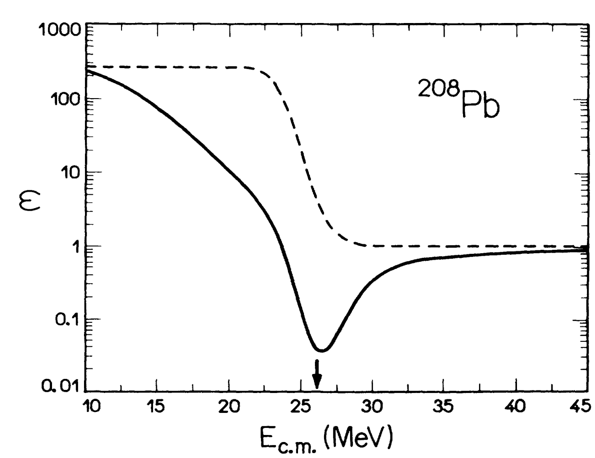

FIG. 2. The enhancement factor ε vs $E_{\rm c.m.}$ for ^{11}Li+Pb. The dashed curve is Eq. (6)/ Eq. (5) while the full curve represents Eq. (7)/Eq. (4).

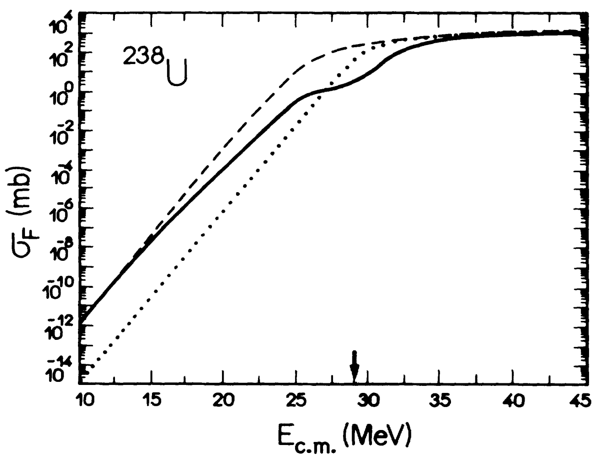

FIG. 3. Same as Fig. 1 for $^{11}Li+^{238}U$ (dotted curve). The dashed curve is the pygmy resonance and target rotation enhanced cross section [Eq. (10)]. The full curve includes the effect of the ^{11}Li breakup on Eq. (10).

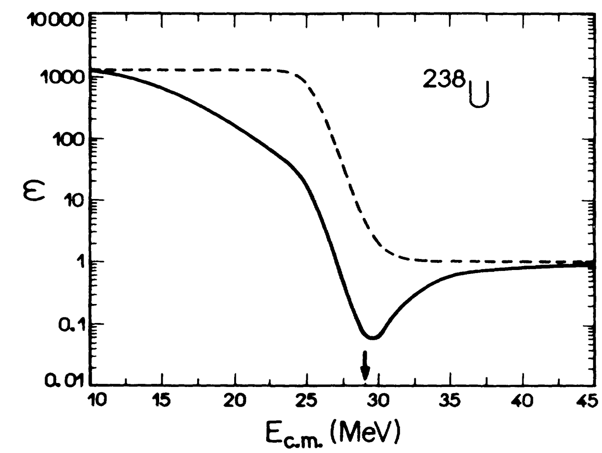

FIG. 4. The enhancement factor ε vs $E_{c.m.}$ for $^{11}Li+^{238}U$. The dashed curve is Eq. (10)/Eq. (4) while the full curve includes the breakup effect. See text for details.

which acts as a natural threshold. At sub-barriers energies, the increase in the distance of closest approach leads to a further reduction in the breakup effects. These striking features arising from the halo should be easy to verify experimentally. The saturation value of ε is 250 and it represents simply the value of $\frac{1}{2}\exp(2\pi/\hbar\omega F)$ [see Eq. (4)], which is attained at much lower energies (~ 10 MeV) than predicted by Takigawa and Sagawa [4].

We have repeated the above calculation for the deformed nucleus ^{238}U. Here the enhancement of σ_f arises from both the coupling to the pygmy resonance of the projectile and the coupling to the states of the target rotor. Taking only the 0^+ and 2^+ states of ^{238}U into consideration, the fusion formula reads [3] (the sudden limit is assumed)

$$\sigma_f=\tfrac{1}{2}(0.562\{\mathring{\sigma}_f[F+0.73\beta_2 f(R_B)]+\mathring{\sigma}_f[-F+0.73\beta_2 f(R_B)]\}+0.348\{\mathring{\sigma}_f[F-1.37\beta_2 f(R_B)]+\mathring{\sigma}_f[-F-1.37\beta_2 f(R_B)]\}) , \quad (10)$$

where $f(R_B)$ is the rotational coupling form factor given approximately by

$$f(R_B)=\frac{1}{\sqrt{4\pi}}V_B\frac{R_2}{R_B}\left[1-\frac{3}{5}\frac{R_2}{R_B}\right] . \quad (11)$$

In Eqs. (10) and (11), R_2 is the radius of ^{238}U (7.4 fm) and β_2 is the deformation parameter ($\beta_2 \simeq 0.27$), and we estimate V_B to be about 29 MeV, so that $f(R_B)=3.3$ MeV.

In Fig. 3, we present the result of our calculation of σ_f, according to Eq. (5) (one-dimensional barrier penetration model), Eq. (10) (pygmy resonance vibration and target rotation coupling model) and with the inclusion of the breakup survival probability in Eq. (10), obtained by replacing $\mathring{\sigma}_f$ by $\mathring{\sigma}_f^b$ (full curve). We find here a fusion behavior similar to that of the $^{11}Li+^{208}Pb$ system except that the enhancement is a factor of 11 larger. The corresponding enhancement factors are shown in Fig. 4, which shows very similar behavior to Fig. 2. The saturation value of ε is 1000, which is attained at about $E_{c.m.}=13$ MeV. Again one sees the sharp dip of ε at the barrier (2.9 MeV).

In conclusion, we have calculated the influence of the nonzero width of the pygmy resonance on the fusion of ^{11}Li with heavy spherical and deformed nuclei at close-to-barrier energies. This is accomplished by taking into account in the multidimensional fusion calculation, the effect of the breakup channel $^{11}Li \rightarrow ^{9}Li+2n$. The usual vibrational and vibrational and target rotational enhancement of the sub-barrier fusion cross section is appreciably reduced. Further, the enhancement factor ε is found to exhibit nontrivial structure around the barrier. This is clearly related to the halo neutrons in ^{11}Li and should be easily verified experimentally.

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