β_4 systematics in rare-earth and actinide nuclei: sdg interacting boson model description

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The observed variation of hexadecupole deformation parameter β_4 with mass number A in rare-earth and actinide nuclei is studied in the *sdg* interacting boson model (IBM) using single *j*-shell Otsuka-Arima-Iachello mapped and IBM-2 to IBM-1 projected hexadecupole transition operator together with $SU_{sdg}(3)$ and $SU_{sdg}(5)$ coherent states. The $SU_{sdg}(3)$ limit is found to provide a good description of data.

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Coulomb excitation, electron scattering, Coulomb/ nuclear interference, etc., generated data on hexadecupole deformation parameter β_4 all across the rare-earth region and the latest compilation of these data are due to Janecke [1]. Similarly Bemis et al. [2], employing Coulomb excitation with ⁴He ions, and Zumbro et al. [3,4], analyzing muonic K, L, and M x rays, produced β_4 data for some of the actinide nuclei. An important feature of the β_4 data in rare-earth and actinide regions is the change in the sign of β_4 after some value of the mass number A. Bertsch's [5] polar cap model gives a qualitative understanding of the variation in β_4 with A. The Nilsson-Strutinsky renormalization method [6] and its variants [7,8] and the more microscopic Hartree-Fock + BCS method [9] are so far employed for the description of β_4 data.

Recently considerable progress was made in establishing that the sdg interacting boson model (sdgIBM or simply gIBM) is a viable and powerful tool for the analysis and understanding of E4 data (i.e., select E4 matrix elements [10], E4 strength distributions [11,12], hexadecupole vibrational bands [13], etc.) which has started accumulating in the past few years. Therefore it is important and essential to understand the β_4 systematics using sdgIBM. As demonstrated in [14], the sdIBM fails to describe the β_4 systematics in rare-earth nuclei. In Refs. [15,11] it is shown that the inclusion of g bosons together with the OAI (Otsuka, Arima, and Iachello) [16] mapped hexadecupole (E4) transition operator can in principle explain the change in sign of β_4 with A. However, as the formalism in [15,11] is strictly confined to sdgIBM-1, so far the experimental data are not analyzed. To this end one should start with proton-neutron sdgIBM (pnsdgIBM or sdgIBM-2). Therefore OAI mapping of the E4 transition operator in p, n spaces is carried out separately and the resulting IBM-2 E4 operator is projected onto IBM-1 space. The E4 operator thus obtained is used in the $SU_{sdg}(3)$ and $SU_{sdg}(5)$ limits of sdgIBM [17] to analyze β_4 data in rare-earth and actinide nuclei.

In sdgIBM-2 quadrupole $(\lambda=2)$ and hexadecupole $(\lambda=4)$ transition operators T^{λ}_{μ} are written as

$$T^{\lambda}_{\mu}(\pi\nu) = \sum_{\rho=\pi,\nu} e^{(\lambda)}_{\rho} Q^{\lambda}_{\mu;\rho} ,$$

$$Q^{\lambda}_{\mu;\rho} = \sum_{l,l'=0,2,4} q^{(\lambda)}_{l,l';\rho} (b^{\dagger}_{l;\rho} \tilde{b}_{l';\rho})^{\lambda}_{\mu} ,$$
 (1)

where π (v) represents proton (neutron) bosons. In (1)

 $q_{l,l';\pi}^{(\lambda)}(q_{l,l';\nu}^{(\lambda)})$ are effective charges that define the onebody transition operators $Q_{\pi}^{(\lambda)}(Q_{\nu}^{(\lambda)})$ and $e_{\pi}^{(\lambda)}(e_{\nu}^{(\lambda)})$ are the overall effective charges, respectively, in proton (neutron) boson space. Using the OAI correspondence [16,18],

$$\begin{aligned} |(j_{\rho})^{2N_{\rho}}, v_{\rho} = 0, J_{\rho} = 0 \rangle \leftrightarrow |n_{s;\rho} = N_{\rho}, L_{\rho} = 0 \rangle , \\ |(j_{\rho})^{2N_{\rho}}, v_{\rho} = 2, J_{\rho} = 2 \rangle \leftrightarrow |n_{s;\rho} = N_{\rho} - 1, n_{d;\rho} = 1, L_{\rho} = 2 \rangle , \\ |(j_{\rho})^{2N_{\rho}}, v_{\rho} = 2, J_{\rho} = 4 \rangle \leftrightarrow |n_{s;\rho} = N_{\rho} - 1, n_{g;\rho} = 1, L_{\rho} = 4 \rangle , \\ \rho = \pi \text{ or } \nu , \qquad (2) \end{aligned}$$

where N_{π} (N_{ν}) is proton (neutron) boson number, $j_{\rho} = (2\Omega_{\rho} - 1)/2$ with $2\Omega_{\pi} (2\Omega_{\nu})$ being the shell degeneracy for protons (neutrons), and equating the matrix elements of multipole operators in fermion $[r_{\rho}^{\lambda}Y_{\mu}^{\lambda}(\theta_{\rho},\phi_{\rho})]$ and boson $(Q_{\mu;\rho}^{\lambda})$ spaces one obtains the effective charges $q_{l,l';\rho}^{(\lambda)}$. Note that (j_{π},j_{ν}) takes values $(\frac{31}{2},\frac{43}{2})$ and $(\frac{43}{2},\frac{57}{2})$ for rare earths and actinides, respectively. Now carrying out IBM-2 to IBM-1 projection [19] by assuming that the low-lying levels belong to F spin $F = F_{max} = (N_{\pi} + N_{\nu})/2$ and using the simple result that $\langle FF_z | e_{\pi}(b_{\pi}^{\dagger}\tilde{b}_{\pi}) + e_{\nu}(b_{\nu}^{\dagger}\tilde{b}_{\nu}) | FF_z \rangle = [(e_{\pi}N_{\pi} + e_{\nu}N_{\nu}) / N] \langle FF | b^{\dagger}\tilde{b} | FF \rangle;$ $F_z = (N_{\pi} - N_{\nu})/2$ (IBM-1 states correspond to $F = F_z = N/2; N = N_{\pi} + N_{\nu}$) which follows from the Wigner-Eckart theorem in F-spin space, the OAI mapped and IBM-2 to IBM-1 projected transition operator T_{μ}^{λ} is

$$\begin{split} T^{\lambda}_{\mu} &= \sum_{l,l'} e^{(\lambda)}_{l,l'} (b^{\dagger}_{l} \tilde{b}_{l'})^{\lambda}_{\mu} ;\\ e^{(\lambda)}_{l,0} &= e^{(\lambda)}_{0,l} \\ &= e^{(\lambda)}_{\pi} \sqrt{2(\Omega_{\pi} - N_{\pi})} N_{\pi} / N + e^{(\lambda)}_{\nu} \sqrt{2(\Omega_{\nu} - N_{\nu})} N_{\nu} / N ,\\ e^{(\lambda)}_{l,l'} &= e^{(\lambda)}_{l',l} \\ &= (\mp) \sum_{\rho = \pi, \nu} (e^{(\lambda)}_{\rho} q^{(\lambda)}_{l,l';\rho} N_{\rho} / N), \quad (l,l') \neq 0 ,\\ q^{(\lambda)}_{l,l';\rho} &= \frac{\sqrt{\Omega_{\rho}(\Omega_{\rho} - 1)}}{(\Omega_{\rho} - 2)} (\Omega_{\rho} - 2N_{\rho}) \\ &\qquad \times \sqrt{4(2l+1)(2l'+1)} \begin{cases} l & l' & \lambda \\ j_{\rho} & j_{\rho} & j_{\rho} \end{cases} \\ k \sqrt{4(2l+1)\Omega_{\rho}(\Omega_{\rho} - 1)} ,\\ q^{(\lambda)}_{l,l';\rho} &= q^{(\lambda)}_{l,l';\rho} e^{(\lambda)}_{\rho} / e^{(\lambda)}_{\rho} . \end{split}$$
(3)

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The two free parameters $e_{\rho}^{(\lambda)}$ (or equivalently $\mathbf{e}_{\rho}^{(\lambda)}$) are defined in (1) and the OAI mapped expressions for the effective charges $q_{l,l';\rho}^{(\lambda)}$ appearing in (1) are defined by the last equation in (3). The (-) sign for $e_{l,l'}^{(\lambda)}$ in (3) is for particle bosons (fermion number $N_f \leq \Omega; N = N_f/2$) and the (+) sign is for hole bosons [fermion number $N_f \geq \Omega; N = (2\Omega - N_f)/2$].

In sdgIBM, the N-boson coherent state [17] is written as

$$|N;\alpha\rangle = \left(\left[\sum_{\substack{l=0,2,4\\-l\leq\mu\leq l}} \alpha_{l\mu} b_{l\mu}^{\dagger} \right]^{N} / [N!\alpha^{2N}]^{1/2} \right] |0\rangle \qquad (4)$$

where $\alpha^2 = 1 + \alpha_2 \cdot \alpha_2 + \alpha_4 \cdot \alpha_4$, $\alpha_{l,-\mu} = (-1)^{l+\mu} \alpha_{l,\mu}$, $b_0^{\dagger} = s_0^{\dagger}$, $b_{2\mu}^{\dagger} = d_{\mu}^{\dagger}$, $b_{4\mu}^{\dagger} = g_{\mu}^{\dagger}$. Using the coherent state $|N;\alpha\rangle$, the intrinsic quadrupole and hexadecupole moments M_2 and M_4 are given by

$$M_{2} = \langle N; \alpha | T_{0}^{2} | N; \alpha \rangle$$

$$= \frac{N}{\alpha^{2}} \sum_{l,l'} (\alpha_{l} \alpha_{l'})_{0}^{2} e_{ll'}^{(2)} ,$$

$$M_{4} = \langle N; \alpha | T_{0}^{4} | N; \alpha \rangle$$

$$= \frac{N}{\alpha^{2}} \sum_{l,l'} (\alpha_{l} \alpha_{l'})_{0}^{4} e_{ll'}^{(4)} .$$
(5)

In [20,17] $\alpha_{\lambda\mu}$ are parametrized in terms of the quadrupole and hexadecupole deformation parameters β_2^c, β_4^c and the asymmetry angle γ^c (*c* denotes coherent state),

$$\begin{aligned} \alpha_{20} &= \beta_2^c \cos \gamma^c ,\\ \alpha_{21} &= \alpha_{2-1} = 0 ,\\ \alpha_{22} &= \alpha_{2-2} = \frac{\beta_2^c \sin \gamma^c}{\sqrt{2}} ,\\ \alpha_{40} &= \frac{1}{6} \beta_4^c (5 \cos^2 \gamma^c + 1) ,\\ \alpha_{41} &= \alpha_{4-1} = 0 , \end{aligned}$$
(6)
$$\alpha_{42} &= \alpha_{4-2} = \frac{1}{6} \sqrt{\frac{15}{2}} \beta_4^c \sin 2 \gamma^c ,\\ \alpha_{43} &= \alpha_{4-3} = 0 ,\\ \alpha_{44} &= \alpha_{4-4} = \frac{1}{6} \sqrt{\frac{35}{2}} \beta_4^c \sin^2 \gamma^c ,\\ \beta_2^c &\geq 0 , -\infty \leq \beta_4^c \leq +\infty , 0^c \leq \gamma^c \leq 60^c . \end{aligned}$$

In the SU_{sdg} (3) and SU_{sdg} (5) limits, which are appropriate for deformed nuclei [17,21], the equilibrium shape parameters $(\beta_2^0, \beta_4^0, \gamma^0)$ are given by

$$\begin{aligned} & \mathbf{SU}_{sdg}(3): \ \beta_{2}^{0} = \sqrt{\frac{20}{7}}, \ \beta_{4}^{0} = \sqrt{\frac{8}{7}}, \ \gamma^{0} = 0^{\circ}, \\ & \mathbf{SU}_{sdg}(5): \ \beta_{2}^{0} = \sqrt{\frac{10}{7}}, \ \beta_{4}^{0} = \sqrt{\frac{18}{7}}, \ \gamma^{0} = 60^{\circ}. \end{aligned}$$
(7)

The coherent state matrix elements M_2 and M_4 are related to the geometric model deformation parameters (β_2, β_4) for axially symmetric shapes [as is the case with

$$SU_{sdg}(3)$$
 and $SU_{sdg}(5)$ limits [17]] by the relation [22]

$$\beta_2 = \left[\frac{4\pi}{3ZeR_0^2}\right] M_2 ,$$

$$\beta_4 = \left[\frac{4\pi}{3ZeR_0^4}\right] M_4 - \frac{9}{7\sqrt{\pi}} \beta_2^2 .$$
(8)

In the geometric model analysis [22] that gives rise to (8), the surface diffuseness corrections are neglected. Note that $R_0 = r_0 A^{1/3}$ and r_0 is chosen to be 1.2 fm.

The mapped E4 operator T^4_{μ} with $\mathbf{e}^{(4)}_{\pi}, \mathbf{e}^{(4)}_{\nu}$ as the two free parameters (3) is used to calculate the coherent state matrix element M_4 via (5) and (6) in the $SU_{sdg}(3)$ and $SU_{sdg}(5)$ limits by using the equilibrium shape parameters $(\beta^0_2, \beta^0_4, \gamma^0)$ given in (7). The M_4 's thus obtained and the experimental or theoretical [using (3) and (5)] β_2 values when used in (8) give β_4 values. The calculated β_4 's are fitted to experimental data and the two free parameters



FIG. 1. Hexadecupole deformation β_4 vs mass number for rare-earth nuclei. (a) The $SU_{sdg}(3)$ and $SU_{sdg}(5)$ predictions are compared with data. (b) The theoretical results due to Nilsson *et al.* [6] are compared with data. In the inset to (b), the polar cap model prediction for the variation in M_4 with A is also shown. The experimental data are taken from Janecke [1].

 $\mathbf{e}_{\pi}^{(4)}, \mathbf{e}_{\nu}^{(4)}$ are determined. In the present analysis experimental β_2 values are used, which is equivalent to comparing the calculated and experimental *E*4 matrix elements $(M_4 = \langle 4_{g.s.}^+ || T^4 || 0_{g.s.}^+ \rangle$; g.s. stands for ground state $K^{\pi} = 0^+$ band). The results for rare-earth and actinide nuclei are shown in Figs. 1(a) and 2(a), respectively.

For rare-earth nuclei the β_4 data are taken from Janecke's compilation [1] and the β_2 data are taken from the adopted values given by Raman *et al.* [23]. The effective charges $(\mathbf{e}_{\pi}^{(4)}, \mathbf{e}_{\nu}^{(4)})$ in *e* b² units are determined to be (0.0121, 0.0045) and (0.0396, -0.0181) in the SU_{sdg}(3) and $SU_{sdg}(5)$ limits, respectively. It is seen from the results in Fig. 1(a) that the $SU_{sdg}(3)$ limit provides a good description of experimental data. In Fig. 1(b) the results of Nilsson et al. [6] are shown. Here the potential energy surfaces are constructed as a function of quadrupole (β_2) and hexadecupole (β_4) deformation parameters including Coulomb and pairing energies and then minimizing the potential energy, the ground state deformation parameters are determined. In the inset to Fig. 1(b) Bertsch's polar cap model [5] prediction for M_4 vs A is shown. Where, with \mathcal{N} the sum of proton and neutron shell degeneracies (for rare earths $\mathcal{N}=76$), m(A) the number of valence nucleons [for rare earths m(A)= A - 132] and the Legendre polynomial $P_4(x)$ $=(35x^4-30x^2+3)/8$

$$M_4(A) \propto \int_{\mu(A)}^1 P_4(x) dx, \ \mu(A) = 1 - \frac{m(A)}{N}$$

It is to be noted that the variation of β_4 with A is similar to that of M_4 with A. It is seen from Figs. 1(a) and 1(b) that the SU_{sdg}(3) limit results are similar to those of Nilsson *et al.* [6].

For actinide nuclei the β_4 data are taken from Bemis et al. [2] and Zumbro et al. [3,4] and the β_2 data are taken from Ref. [2]. The effective charges $(\mathbf{e}_{\pi}^{(4)}, \mathbf{e}_{\nu}^{(4)})$, in $e b^2$ units, are determined to be (-0.078, 0.066) and (-0.0133, 0.026) in the SU_{sdg}(3) and SU_{sdg}(5) limits, respectively. Once again the results given in Fig. 2(a) show that SU_{sdg}(3) limit provides a good description of data. In Fig. 2(b) the Hartree-Fock + BCS results of Libert and Quentin [9] and the Nilsson-Strutinsky calculations of Brack et al. [8] are shown and the SU_{sdg}(3) results are somewhat better than the microscopic calculations.

By comparing the $SU_{sdg}(3)$ and $SU_{sdg}(5)$ results with experimental and other theoretical calculations shown in Figs. 1 and 2 the following conclusions can be drawn: (1) With two free parameters the data for rare earths and actinides are described very well. (2) $SU_{sdg}(3)$ limit provides a good description of data and it is much better than $SU_{sdg}(5)$ limit. This is in conformity with the fact that the $SU_{sdg}(3)$ limit gives a good description of various properties of rotational nuclei [21,24]. (3) In actinides $SU_{sdg}(3)$ limit shows change in sign of β_4 for Cm isotopes which is consistent with data; other theoretical calculations do not produce the change in sign. (4) The free parameters $e_{\pi}^{(4)}$ and $e_{\nu}^{(4)}$ should be determined by microscop-



FIG. 2. Hexadecupole deformation β_4 vs mass number for actinides. (a) The SU_{sdg}(3) and SU_{sdg}(5) predictions are compared with data. (b) The theoretical results due to Brack *et al.* [8] and Libert and Quentin [9] are compared with data. The experimental data are due to Bemis *et al.* [2] and Zumbro *et al.* [3,4].

ic theories involving multi-*j*-shell mappings (see, for example, Ref. [25]) and this is being pursued. The results of this exercise will be reported in a longer publication. (5) In the calculations experimental β_2 values are employed which amounts to comparing the calculated and experimentally determined M_4 's rather than the deduced [via Eq. (8)] β_4 values. It is useful to note that the experimental M_4 's are 4-8 times the single particle unit for rare earths and 6-10 times for actinides (except in a few cases).

The results given in this paper confirm that sdgIBM provides a good framework for describing E4 properties of nuclei. The present formalism is being extended to study the recently deduced $B(IS4:4^+_{\gamma} \rightarrow 0^+_{g.s.})$ systematics in rare-earth nuclei [26] and the results will be reported in a longer publication.

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