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Confining quark condensate model of the nucleon

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We obtain a mean-field solution for the nucleon as a quark-meson soliton obtained from the action of the global color-symmetry model of QCD. All dynamics is generated from an effective interaction of quark currents. At the quark-meson level there are two novel features: (1) absolute confinement is produced from the space-time structure of the dynamical self-energy in the vacuum quark propagator; and (2) the related scalar meson field is an extended $\bar{q}q$ composite that couples nonlocally to quarks. The influence of these features upon the nucleon mass contributions and other nucleon properties is presented.

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I. INTRODUCTION

Because of the importance of chiral symmetry in low-energy modeling of QCD, the linear sigma model of Gell-Mann and Levy has formed the basis for many quark-meson models of baryons, usually at the mean-field level [1,2]. The local effective meson fields (σ,π) simulate the Goldstone $\bar{q}q$ collective modes from the flavor SU(2) sector of QCD. Given the evident success of chiral soliton models of the linear sigma variety for describing many nucleon properties, two questions immediately arise. First, are there important corrections to the dynamics due to the spatially extended nature of the meson-like $\bar{q}q$ modes? Second, how should quantum fluctuation corrections to the mean-field treatment be viewed if the meson fields already represent $\bar{q}q$ fluctuations? To deal with these issues, it is necessary to begin with something like a Nambu–Jona-Lasinio (NJL) [3] model where the $\bar{q}q$ meson modes can be derived through bosonization techniques [4]. However, the standard NJL four-quark contact interaction produces point meson fields locally coupled to the quarks, and the effect of meson size cannot be addressed. The derived meson parameters and quark self-energy are divergent vacuum quark loop integrals, and an additional parameter in the form of a cutoff must be introduced.

Extended meson fields can be produced through bosonization of a quark action if the current-current interaction is mediated by an effective gluon propagator with finite range [5,6]. It is then necessary to work with a nonlocal quark-meson action. However, an advantage is that the nonlocality from gluon dressing of the quarks produces convergent loop integrals for the derived meson parameters. In this work we present self-consistent numerical results for the nucleon as a mean-field solution of such a nonlocal linear sigma model in the form derived previously [7]. We refer the reader to Ref. [7] for further background and motivation for this type of approach. The dynamically generated quark self-energy Σ arises from the vacuum condensate constructed from bilocal combinations of $\bar{q}$ and q fields. The Dirac scalar component of Σ plays the dual roles of the internal form factor for the finite-size Goldstone $\bar{q}q$ mode in the (σ,π) channel and the vertex for the coupling of that mode to quarks. This economy is guaranteed by chiral symmetry and the axial Ward identity. We explore the consequences of an absolutely confining ansatz for the timelike behavior of $\Sigma(p)$ so that the vacuum quark propagator has no mass-shell pole. This is one of the proposed realizations of confinement in QCD [9,10], but has not before been employed in solutions of a quark-meson soliton because of the intrinsic nonlocality of the mechanism.

Soliton models often implement dynamical confinement in terms of a color dielectric function mediated by an auxiliary local scalar field attributed to a gluon condensate [11]. In contrast, the dynamical confinement in the present approach has its origin in the space-time structure of the quark condensate $\langle\bar{q}(x)q(y)\rangle$ and the associated scalar fluctuation field is the chiral partner of the $\bar{q}q$ pion. No other field need be introduced to obtain a self-confining chiral soliton. One of our principal results is that the valence quark wave functions and nucleon mass produced from the simplest application of this mechanism are quite acceptable.

The underlying model for the present work is taken to be the global color-symmetry model (GCM) [6] of QCD based on a finite-range current-current interaction. This has the hidden chiral-symmetry property of NJL-type models, but can also accommodate dynamical confinement. A quark-meson soliton model arising from the bosonization of the GCM was put forward some time ago [5]. However, no previous attempt has been made to obtain numerical solutions that retain the intrinsic nonlocalities. We recently explored [7] the formal development of such a generalized soliton model by identifying the meson loop expansion that produces the mean-field approximation as the lowest term. There arguments are made for the mechanism whereby a confining vacuum quark propagator without a mass shell combines with a constant classical scalar meson field to create a constituent mass shell and thus well-defined quark eigenstates and energies. In a subsequent work [12], we confirmed this mechanism through numerical results for the confining solutions of the corresponding Dirac equation that contains a nonlocal coupling to a finite-range scalar field of Gaussian form. The fully self-consistent, nonlinear case in which the meson field is obtained from the valence quark source is the subject of the present work.

In this numerical work, we ignore the pion degree of freedom so that, with good quark isospin, we may develop solution methods that handle the intrinsic nonlocalities in the simplest possible setting. The case which is solved is that of three $1S_{1/2}$ quarks in the lowest-energy baryon state (average of the nucleon and delta), interacting with a $\bar{q}q$ composite, static, scalar mean field generated self-consistently by the quarks. The quark self-energy in the absence of the scalar field is modeled to provide confinement, and the associated single parameter of the model characterizes the strength of the effective gluon propagator. In Sec. II a brief review of this generalized soliton model and of the employed confinement mechanism is given along with the equations of motion to be solved. Detailed derivations occur elsewhere [5,7,12,13] and will not be repeated. The numerical methods employed in obtaining the solutions are described in Sec. III. In Sec. IV the results are presented along with a discussion of the effects of the confinement and the related nonlocal quark-meson vertex. In Sec. V we summarize our findings.

II. GCM AND QUARK-MESON MODELS

A. Chiral model

The underlying action is taken to be the GCM [6] given in Euclidean space as

$$S[\bar{q},q]=\int d^4x\, d^4y \left[\bar{q}(x)(\gamma\cdot\partial+m)\delta(x-y)q(y) + \frac{g^2}{2}j^a_\mu(x)D_{\mu\nu}(x-y)j^a_\nu(y)\right], \quad (1)$$

where the quark current is $j^a_\mu(x)=\bar{q}(x)(\lambda^a/2)\gamma_\mu q(x)$. In the limit of zero-current-quark mass m, the GCM has $SU(2)_L\otimes SU(2)_R$ chiral symmetry in the two-flavor version considered here. The GCM implements only a global SU(3) color symmetry. With the assumption $D_{\mu\nu}(x-y)=\delta_{\mu\nu}D(x-y)$ and the effective gluon two-point function $D(x-y)$ treated as a parameter function constrained only by asymptotic freedom in the Euclidean domain, this action has provided a successful modeling of meson properties and dynamics [5,6]. Here we wish to see whether the model admits acceptable valence quark states for baryons. For this initial exploration the function $D(x-y)$ will be parametrized to give a simple confining and explicit form for the dynamical vacuum self-energy of quarks. Chiral symmetry dictates that the Goldstone meson modes will have to couple with a vertex largely fixed by the self-energy dynamics.

The meson modes of the model are produced through the bosonization procedure [5,14] in which the quartic term in quark fields is exactly reformulated as a functional integration over auxiliary bilocal Bose fields $\mathcal{B}^\theta(x,y)$ having the transformation properties of $\bar{q}(y)\Lambda^\theta q(x)$. Here Λ^θ are various direct product combinations of color, flavor, and spin matrices resulting from the Fierz reordering of the current-current term of the action in Eq. (1). Fluctuations in these fields will be interpreted as effective meson fields. For the fluctuations we will ignore the color-octet sector and deal only with color-singlet effective meson fields. At this level the Fierz-reordered form of (1) is essentially a nonlocal version of the NJL model. The limit $D(x-y)\propto\delta(x-y)$ recovers the local NJL model. With the quartic quark terms replaced by Bose field integrations, the remaining bilinear quark field term can be handled by Grassmann integration in the standard way. To obtain a mean-field model for a baryon, one can [7] use a canonical transformation to introduce chemical potentials μ to fix the baryon number and flavor. Then integration over Grassmann fields with the appropriate adjustment of boundary condition produces the grand partition function Z given by

$$Z=N\int D\mathcal{B}^\theta \exp\{-S[\mu,\mathcal{B}^\theta]\}\ , \quad (2)$$

where the bosonized action is

$$S[\mu,\mathcal{B}^\theta]=-\operatorname{Tr}\{\ln G^{-1}[\mu,\mathcal{B}^\theta] - \ln G^{-1}[\mu=0,\mathcal{B}^\theta]\}+S[\mathcal{B}^\theta]\ , \quad (3)$$

with the vacuum action given by

$$S[\mathcal{B}^\theta]=-\operatorname{Tr}\ln G^{-1}[\mu=0,\mathcal{B}^\theta] + \frac{1}{2}\int d^4x\, d^4y \frac{\mathcal{B}^\theta(x,y)\mathcal{B}^\theta(y,x)}{g^2D(x-y)}\ . \quad (4)$$

The separation in Eq. (3) isolates the valence quark contribution and requires that meson modes be produced from the vacuum action. The inverse propagator appearing in (3) is [7]

$$\begin{aligned}G^{-1}(\mu;x,y)&=e^{\mu x_4}G^{-1}(x,y)e^{-\mu y_4}\\ &=(\gamma\cdot\partial+m-\gamma_4\mu)\delta(x-y)\\ &\quad+e^{\mu x_4}\Lambda^\theta\mathcal{B}^\theta(x,y)e^{-\mu y_4}\ .\end{aligned} \quad (5)$$

The quarks are Yukawa coupled to the auxiliary Bose field variables of integration with bare vertices Λ^θ. Besides the familiar shift of the time derivative, the additional μ dependence in (5) is due to the nonlocality of the Bose fields. With appropriate boundary conditions, the μ dependence of G will serve to shift the pole structure in the momentum component conjugate to x_4-y_4 so that valence and vacuum configurations are treated together in the usual way. The saddle-point or classical vacuum configuration $\mathcal{B}^\theta_0$, defined by $\delta S/\delta\mathcal{B}^\theta_0=0$, produces a translation invariant quark self-energy $\Sigma(x-y)$

$=\Lambda^\theta \mathcal{B}_0^\theta(x-y)$, which in momentum space satisfies

$$\Sigma(p)=i\gamma\cdot p[A(p^2)-1]+B(p^2)$$
$$=g^2\int\frac{d^4q}{(2\pi)^4}D(p-q)\frac{\lambda^a}{2}\gamma_\mu \times\frac{1}{i\gamma\cdot q+m+\Sigma(q)}\frac{\lambda^a}{2}\gamma_\mu\ , \qquad (6)$$

an equation of Schwinger-Dyson form, where $D(p)$ is the Fourier transform of $D(x-y)$.

The propagating Bose fields are identified as the fluctuations $\hat{\mathcal{B}}^\theta(x,y)=\mathcal{B}^\theta(x,y)-\mathcal{B}_0^\theta(x-y)$. We retain only color-singlet $(\sigma,\boldsymbol{\pi})$ propagating modes in the expansion of $S[\mathcal{B}^\theta]$ about the saddle-point configuration [5,7]. Quantum loop effects from all the auxiliary Bose modes can presumably give the quark-gluon vertex structure that is missing from (6). We do not do this, but look for classical solutions in the selected $\hat{\mathcal{B}}^\theta$ induced by the valence quark source within the baryon. At this level in the $m\to 0$ limit, which we employ from this point forward, dynamical chiral-symmetry breaking in the QCD vacuum is attributed to $B(p^2)\neq 0$. The same amplitude $B(p^2)$ is identifiable, through the Ward identity for the axial-vector vertex [8], as the on-mass-shell vertex (internal meson form factor) for quark coupling to the massless pion and its scalar partner. This result can also be obtained directly by considering the eigenfunctions of the inverse propagator for the $\hat{\mathcal{B}}^\theta$ fields [15], which are solutions of the ladder Bethe-Salpeter equation consistent with the self-energy Σ. These eigenfunctions define the off-mass-shell vertices, and the pion vertex reduces to the scalar portion B of the quark self-energy when the on-mass-shell condition is invoked. The color-singlet scalar-isoscalar and pseudoscalar-isovector fluctuations that are retained for the chiral model can be written as [5,6]

$$\Lambda^\theta\hat{\mathcal{B}}^\theta(x,y)=\frac{B(r)}{f_\pi}\chi(R)e^{i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}(R)/f_\pi}\ , \qquad (7)$$

where $r=x-y$ and $R=(x+y)/2$, and we have approximated the off-mass-shell vertex with the on-shell form factor B.

A chirally symmetry derivative expansion of the vacuum action (4) to leading order in derivatives of the fields $\sigma=\chi\cos[\phi/f_\pi]$ and $\boldsymbol{\pi}=\hat{\boldsymbol{\phi}}\chi\sin[\phi/f_\pi]$ can finally be written as

$$S[\sigma,\boldsymbol{\pi}]-S[f_\pi,0] =\int d^4R\,\{\tfrac{1}{2}[(\partial_\mu\sigma)^2+(\partial_\mu\boldsymbol{\pi})^2]+U(\chi^2(R))\}\ , \qquad (8)$$

where $U(\chi^2)$ is the effective potential (meson self-interactions) given by

$$U(\chi^2)=-12\int\frac{d^4q}{(2\pi)^4}\left\{\ln\left[\frac{q^2A^2(q^2)+B^2(q^2)(\chi/f_\pi)^2}{q^2A^2(q^2)+B^2(q^2)}\right] -\frac{B^2(q^2)[(\chi/f_\pi)^2-1]}{q^2A^2(q^2)+B^2(q^2)}\right\}\ , \qquad (9)$$

and where f_π is the pion decay constant given by

$$f_\pi^2=12\int\frac{d^4q}{(2\pi)^4}\left\{\frac{B^2A^2}{[q^2A^2+B^2]^2} -\frac{\frac{1}{2}q^2[(B')^2+BB'']+BB'}{q^2A^2+B^2}\right\}\ . \qquad (10)$$

Here the argument of A and B is q^2 and primes denote differentiation with respect that argument. The logarithm term in (9) is the sum of single quark vacuum loops with all possible insertions of chiral meson fields to zeroth order in their derivatives. The expression for f_π^2 comes as usual from the quark loop with two insertations. All integrals are finite because of the natural regulation provided by the amplitude $B(p^2)$. The potential $U(\chi^2)$ has turning points at $\chi^2=0$ and at the degenerate vacuum configuration $\chi^2=f_\pi^2$ corresponding to a local maximum and absolute minima, respectively. The obtained Mexican hat structure is displayed later. The meson masses can be obtained from the second derivative of the potential $U(\chi^2)$ with respect to the corresponding field at the absolute minimum. The pion mass is zero in the exact chiral limit of zero-current-quark mass, while the mass of the scalar is finite and is given by

$$m_\chi^2=\frac{48}{f_\pi^2}\int\frac{d^4q}{(2\pi)^4}\frac{B^4(q^2)}{[q^2A^2(q^2)+B^2(q^2)]^2}\ . \qquad (11)$$

Numerical values obtained for the pion decay constant f_π and the scalar field mass m_χ are given in Sec. IV.

B. Scalar model

In this initial numerical work, we focus upon treatment of the nonlocalities from the quark self-energy and quark-meson coupling. We therefore truncate to $\boldsymbol{\phi}=0$, thus giving up explicit chiral symmetry for the convenience of good quark isospin. This amounts to keeping only radial fluctuations away from the chiral circle at the vacuum point $\boldsymbol{\phi}=0$, where χ is the radial field. When the constant vacuum value of the action is discarded, the complete action for the scalar soliton model can be written

$$S[\mu,\chi]=-\mathrm{Tr}[\ln G^{-1}(\mu,\chi)-\ln G^{-1}(0,\chi)] +\int d^4R\,[\tfrac{1}{2}(\partial_\mu\chi)^2+U(\chi^2)]\ . \qquad (12)$$

The chemical potential dependence of the fermion Tr ln term ensures that a meson source from valence quarks will be generated. The inverse quark propagator occurring in (12) is, for $\mu=0$,

$$G^{-1}(x,y)=\gamma\cdot\partial_x A(x-y)+f_\pi^{-1}B(x-y)\chi\left[\frac{x+y}{2}\right]\ . \qquad (13)$$

If the chemical potential μ in the action $S[\mu,\chi]$ is set to zero, the saddle-point configuration will be $\chi=f_\pi$. With a finite chemical potential, there will be a classical field expectation value of χ that reflects the spatial source distribution of the valence quarks.

At the level of a mean-field approximation, only a static $\chi(R)$ field appears and the baryon energy functional can be identified as the zeroth term in the meson loop expansion of the effective action. For the present model the energy functional corresponding to a fixed set of quark occupation numbers n_j is [7]

$$E[n,\chi]=E_q[n,\chi]+E_m[\chi] , \tag{14}$$

where the valence quark contribution is

$$E_q[n,\chi]\left[-\int dx_4\right] = \mathrm{Tr}\{\ln G^{-1}[\mu,\chi]-\ln G^{-1}[0,\chi]\}-\mu_j n_j , \tag{15}$$

and the scalar field contribution is

$$E_m[\chi]=\int d^3x[\tfrac{1}{2}(\nabla\chi)^2+U(\chi^2)] . \tag{16}$$

The field χ satisfies the equation of motion $\delta E/\delta\chi=0$. The chemical potentials μ_j are now functionals of the field χ and the particle numbers due to the constraint $n_j=\partial\,\mathrm{Tr}\ln G^{-1}/\partial\mu_j$.

With static meson fields, $G^{-1}(x,y)$ depends on time only through the variable $\tau=x_4-y_4$. The time-translation invariance of $G^{-1}(x,y)$ allows stationary eigenstates of the form $u_j(\mathbf{x})e^{i\omega x_4}$, which satisfy

$$\int d^3y\, G^{-1}(\omega;\mathbf{x},\mathbf{y})u_j(\mathbf{y})=i\gamma_4\lambda_j(\omega)u_j(\mathbf{x}) . \tag{17}$$

The eigenvalues have the form $\lambda_j(\omega)=\omega-i\epsilon_j(\omega)$, where ϵ_j is the quark eigenenergy for state j. The ω dependence of $\epsilon_j(\omega)$ arises from the dynamical nature of the self-energy $\Sigma(\omega,\mathbf{x}-\mathbf{y})$. The index j labels the set of distinct states of the spectrum for a given value of ω.

The quark component (15) of the baryon energy functional reduces to the sum of positive eigenenergies bounded by the chemical potentials and satisfying $\lambda_j=0$, that is, $\epsilon_j=-i\omega_p$. The associated states satisfy a self-consistent Dirac equation, which, in momentum space, is

$$\int d^3k\, G^{-1}(i\epsilon_j;\mathbf{p},\mathbf{k})u_j(\mathbf{k})=0 , \tag{18}$$

that is,

$$[i\gamma\cdot pA(p^2)+B(p^2)]u_j(\mathbf{p}) +f_\pi^{-1}\int\frac{d^3k}{(2\pi)^{3/2}}B\left[\frac{p+k}{2}\right]\hat{\chi}(\mathbf{p}-\mathbf{k})u_j(\mathbf{k})=0 , \tag{19}$$

where the eigenvalue $p_4=k_4=i\epsilon_j$ enters in a nonlinear way. The spatially dependent part of the χ field has been separated out, that is, $\chi=f_\pi+\hat{\chi}$. Note that the meson vertex is energy dependent. A wave-function renormalization constant Z_j can be identified from the residue at the pole of the Green's function and is given by [7]

$$Z_j=-\int d^3p\, d^3k\, \bar{u}_j(\mathbf{p})\frac{\partial G^{-1}(i\epsilon_j;\mathbf{p},\mathbf{k})}{\partial\epsilon_j}u_j(\mathbf{k}) . \tag{20}$$

The net result for the energy functional of the baryon is

$$E[n,\chi]=3\epsilon_0[\chi]+\int d^3x[\tfrac{1}{2}(\nabla\chi)^2+U(\chi^2)] , \tag{21}$$

where ϵ_0 is the energy of the degenerate lowest S state. This is the standard result for a scalar soliton model except that here the potential U and the dynamical relation between ϵ_0 and χ are calculated from the nonlocal confining quark self-energy amplitudes.

The meson field equation $\delta E/\delta\chi=0$, after use of (21) and accounting for the self-consistent energy dependence of the $\bar{q}\chi q$ coupling, becomes

$$-\nabla^2\chi(\mathbf{z})+\frac{\delta U}{\delta\chi(\mathbf{z})}+Q_\chi(\mathbf{z})=0 , \tag{22}$$

where the meson source provided by valence quarks is [7]

$$Q_\chi(\mathbf{z})=\sum_j\frac{1}{f_\pi Z_j}\int d^3x\, d^3y\, \bar{u}_j(\mathbf{x})B(-\epsilon_j^2;\mathbf{x}-\mathbf{y}) \times\delta\left[\frac{\mathbf{x}+\mathbf{y}}{2}-\mathbf{z}\right]u_j(\mathbf{y}) . \tag{23}$$

In the limit of point coupling where the amplitude B above becomes $\bar{B}\delta(\mathbf{x}-\mathbf{y})$, the source reduces to the local form of conventional soliton models. The frequency dependence of the dynamical quark self-energy is responsible for the wave-function renormalization Z_j. Departures of Z_j from unity are produced when the self-energy amplitude $A(x-y)$ departs from $\delta(x-y)$ and when $B(x-y)$ is not static. Equations (19) and (22) are the equations of motion we solve for the scalar model.

C. Absolute confinement

The issue of how confinement is realized in QCD is unresolved. One of the goals of this work is to investigate the effects of a novel confinement ansatz on the self-consistent solutions of the soliton equations of motion. We take up the proposal that confinement is realized through the absence of a mass-shell pole in the vacuum propagator for dressed quarks [9,10]. Such a behavior has been noted in studies of model Schwinger-Dyson equations if the effective gluon propagator has sufficient infrared strength [16]. So confinement in the present model is defined as the *inability of a quark to propagate in the vacuum* where $\chi=f_\pi$. In this situation the Dirac equation (19) reduces to

$$0=[i\gamma\cdot pA(p^2)+B(p^2)]u_j(\mathbf{p}) . \tag{24}$$

If there is no solution to $p^2+M^2(p^2)=0$, where $M(p^2)=B(p^2)/A(p^2)$ is the dynamical mass function, then we say that the quark is confined. This implies that a quark is restricted to a region in which $\chi\neq f_\pi$. In the present model this region will have finite spatial extent due to the localized valence quark source. This confinement mechanism has no knowledge of a bag surface and does not presuppose a hadronic environment for its implementation. Rather, it is the presence of a hadronic environment that induces $\chi\neq f_\pi$ and creates a constituent quark mass shell through self-consistent solutions to (19) and (22). A simple illustration of constituent mass generation from a confining propagator of this type has been presented previously [7].

The confinement condition $p^2+M^2(p^2)\neq 0$ implies as a minimum that $M^2(p^2)$ for $p^2<0$ is linear with slope less than or equal to -1 and is nonzero at $p^2=0$. A simple realization of this is produced from use of the model gluon propagator [16]

$$D(p)=(2\pi)^4 \tfrac{3}{16}\alpha^2\delta^{(4)}(p) , \tag{25}$$

in the Schwinger-Dyson equation (6). The resulting self-energy amplitudes are

$$A(p^2)=\begin{cases} 2, & p^2\le\alpha^2/4 \\ \frac{1}{2}[1+(1+2\alpha^2/p^2)^{1/2}], & p^2\ge\alpha^2/4 , \end{cases}$$
$$B(p^2)=\begin{cases} (\alpha^2-4p^2)^{1/2}, & p^2\le\alpha^2/4 \\ 0, & p^2\ge\alpha^2/4 . \end{cases} \tag{26}$$

For $p^2<0$ this gives $M^2(p^2)=\alpha^2/4-p^2$. Several studies have obtained useful results with this simple confining dynamical mass function [5,12,17]. We use the parametrization (26) for the present numerical work. The resulting quark propagator is asymptotically free for large spacelike momenta, but not for large timelike momenta. There are arguments [18] for timelike asymptotic freedom and also arguments [10] that such behavior is not implied by perturbative analysis of timelike production processes. The present calculation, however, is not affected by the deep timelike region. Loop evaluations such as that in Eq. (9) are performed purely in the Euclidean domain, and the constituent quark eigenvalue determination from Eq. (19) entails a very limited continuation into the timelike region. We have previously shown [12] that for these amplitudes A and B, the Dirac equation (19) has only discrete solutions in the presence of a finite-range $\hat{\chi}$ field. This follows because, in position space, the large-distance behavior of the states is not governed by vacuum solutions since none exist. No solutions with scattering boundary conditions are possible. This clearly implements one distinction between confined systems and energetically bound systems: *A confined system has no continuum.* Our previous numerical work [12] also confirmed that with such a confinement mechanism the quark eigenenergies necessarily tend to infinity as the strength of a finite-range $\hat{\chi}$ field approaches zero. Thus the constituent mass shell disappears as other valence quarks are moved away. This indicates that a finite-size soliton in this model will always exist and is the only type of solution possible.

III. METHOD OF CALCULATION

The coupled equations of motion to be solved are the coordinate-space equation for the scalar field (22), and the momentum-space Dirac equation (19). The Dirac equation is solved in momentum space because the dynamical nature of the nonlocality and comparison to a local limit (point coupling and constant mass) are most easily handled in that format. After projection onto S waves, the Dirac equation (19) is brought to the matrix form $\mathcal{H}(\epsilon_j)u_j=0$ by use of Gauss-Legendre quadrature. The lowest positive $S_{1/2}$ energy is found from the condition $\det(\mathcal{H})=0$ by stepping slowly away from $\epsilon_j=0$ in the positive direction until a root is bracketed. The root is then obtained to the desired accuracy using Brent's method [19]. The eigenfunctions u_j are then obtained by iteration.

The scalar field Klein-Gordon equation, on the other hand, is most easily handled in position space because of its nonlinear form. This implies that at each iteration in the solution of the coupled equations a Fourier transform must be performed twice. The source term Q_χ of Eq. (23) is calculated in momentum space from the quark states and is then Fourier transformed to position space. The scalar field solution $\chi(\mathbf{x})$ is Fourier transformed back to momentum space for use in the Dirac equation. The restriction to S wave allows use of a fast-Fourier-transform algorithm [19] designed strictly for one-dimensional problems. With an initial guess for a finite-range scalar field, the quark states are calculated, and the resulting source term is constructed. A new scalar field is generated, and a comparison with the previous field provides a better estimate. The pair of coupled equations of motion is iterated this way until convergence is achieved. The convergence criteria used is that the components of the baryon energy should be accurate to 10^{-4}.

The nonlinear scalar field equation is solved iteratively by the application of Newton's method to a functional. For example, given a functional $F[\chi]$ for which a solution χ_s defined by $F[\chi_s]=0$ is sought, one can proceed as follows. Expansion of $F[\chi]$ to first order about an initial solution χ_0 gives

$$F[\chi]=F[\chi_0]+\int d^4x \left.\frac{\delta F[\chi]}{\delta\chi(x)}\right|_{\chi_0}(\chi-\chi_0)(x)+\cdots . \tag{27}$$

The condition $F[\chi]=0$ implies

$$\int d^4x \left.\frac{\delta F[\chi]}{\delta\chi(x)}\right|_{\chi_0}\eta(x)\approx -F[\chi_0] . \tag{28}$$

This can be solved as a matrix equation for the vector $\eta(x)=\chi(x)-\chi_0(x)$, from which the new solution $\chi_i(x)=\eta_i(x)+\chi_{i-1}(x)$ is formed. Equation (28) is then iterated to the desired accuracy. For the present case the functional F is that given in Eq. (22), from which one can see that the integration in (28) is removed by the delta function generated from the functional differentiation. The starting solution is that obtained from the linearized Klein-Gordon equation formed by the substitution $\delta U/\delta\chi\rightarrow m_\chi^2\chi$.

IV. NUCLEON RESULTS

The nonlinear potential U as calculated from Eq. (9), with the amplitudes A and B of Eq. (26), is displayed in Fig. 1. The shape is very similar to the standard fourth-order polynomial form adopted in most chiral models, except that here U increases quadratically with χ for very large χ. The obtained Mexican hat structure is quite insensitive to the detailed form of the amplitudes A and B and is dictated by the underlying chiral symmetry. The results for the mean-field nucleon calculations are summarized in Tables I and II and in the graphs of Figs. 2–5. The soliton mass is estimated from the calculated energy E_s by approximate removal of the spurious center-of-

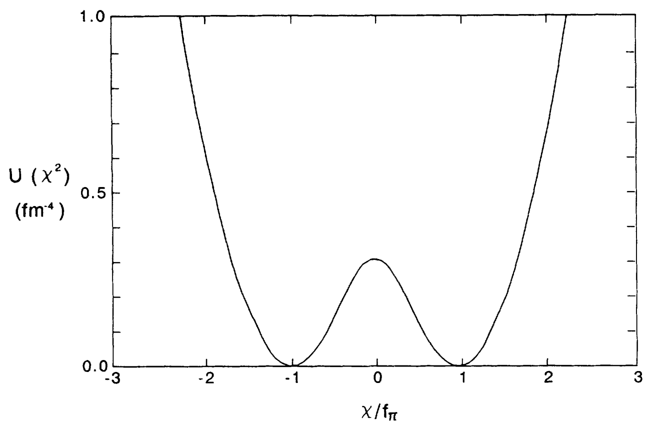

FIG. 1. Calculated vacuum effective potential of Eq. (9) is plotted as a function of the chiral-invariant scalar field χ for the case in which the self-energy amplitudes A and B are those of Eq. (26). The form shown here displays the Mexican hat behavior adopted in most chiral models.

mass component present in a mean-field model. That is, $M_s \approx [E_s^2 - 3\langle p^2 \rangle]^{1/2}$, where $\langle p^2 \rangle$ is the expectation value of the square of the quark momentum. The first two columns of Table I contain results from a pair of calculations that retain the confining dynamical quark mass and nonlocal quark-meson coupling. We refer to this as the DM-NLC case. The two versions shown correspond to different choices of the single parameter α for the gluon propagator strength; in the first column the pion decay constant f_π is fitted to its experimental value, while in the second column the soliton mass M_s is fitted to the average of the nucleon and delta masses. Since the pion, excluded from the present work, would lower the soliton mass by about 200 MeV, the mass obtained with the correct f_π value is encouraging. The second column illustrates that a modest decreased of α can lower the baryon mass by 300 MeV while increasing the quark rms radius by 25%. The increase in binding there is due

TABLE I. Mean-field calculations of the nucleon/delta. The mass M_s is calculated from the scalar field χ and quark eigenenergy ϵ. Results that include a confining dynamical mass and nonlocal coupling (DM-NLC) are compared with results that follow from the constant mass and local coupling limit (CM-LC). The single model parameter is α, a characterization of the infrared strength of the effective gluon propagator. The quantity Z is the quark wave-function renormalization constant.

	DM-NLC		CM-LC	Expt.
α (GeV)	1.04	0.86	1.04	
ϵ (MeV)	390	301	434	
χ potential energy (MeV)	119	116	119	
χ kinetic energy (MeV)	256	208	82	
m_χ (MeV)	465	374	465	
E_s (MeV)	1545	1227	1504	
M_s (MeV)	1359	1086	1406	1086
$R_{\rm rms}$ (fm)	0.79	1.0	1.0	0.83
f_π (MeV)	93	74	93	93
Z	1.8	1.8	2.0	

TABLE II. Mean-field calculations of the nucleon/delta. Results that include nonlocal quark-meson coupling (CM-NLC) are compared with results from the purely local reduction of the model (CM-LC). In both calculations the dynamical mass is frozen at a constant value and there is no confinement.

	CM-NLC	CM-LC	Expt.
α (GeV)	1.04	1.04	
ϵ (MeV)	395	434	
χ potential energy (MeV)	137	119	
χ kinetic energy (MeV)	108	82	
m_χ (MeV)	465	465	
E_s (MeV)	1430	1504	
M_s (MeV)	1328	1406	1086
$R_{\rm rms}$ (fm)	1.0	1.0	0.83
f_π (MeV)	93	93	93
Z	2.5	2.0	

mainly to the increased range of the distributed quark-meson vertex. Clearly, there are relevant processes which have been neglected in this work; however, the range between these two solutions is not large, indicating that this model with one parameter gives a reasonable description of the physics involved. Obviously, a major uncertainty in a model of this type is the effect of a more realistic gluon propagator.

For comparison we report the results of a calculation for the limit of local coupling and no confinement. For this limit the dynamical nature of the amplitudes A and B in the Dirac equation is suppressed by employing the constant $p^2=0$ values ($A=2$, $B=\alpha$) for all p^2. The original potential $U[\chi^2]$ and the value $f_\pi=93$ MeV are retained. The result is a local soliton model with a constant constituent quark mass $M_q=\alpha/2$ and a point coupling to the scalar field with a coupling constant $g=M_q/f_\pi$. We label this case as CM-LC. It is well known for such a local model that there is a critical value of the coupling constant g below which a stable soliton cannot form [1].

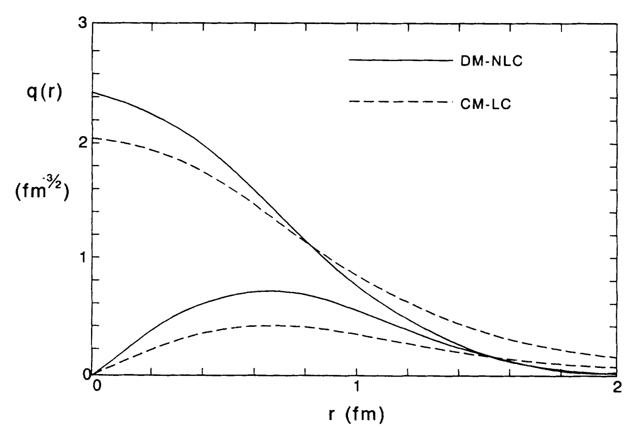

FIG. 2. Upper and lower components of the quark wave functions are plotted for two treatments of the dynamics. The solid lines follow from the full model containing a confining dynamical mass and nonlocal coupling. The dashed lines follow from a reduction to a constant (nondynamical) mass and local coupling. The differing asymptotic behavior is a result of the confinement.

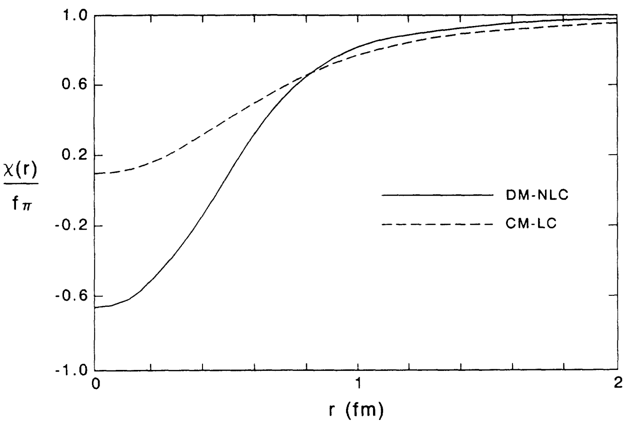

FIG. 3. Self-consistent scalar field χ is plotted for the cases of confining dynamical mass and nonlocal coupling (solid line) and constant mass and local coupling (dashed line).

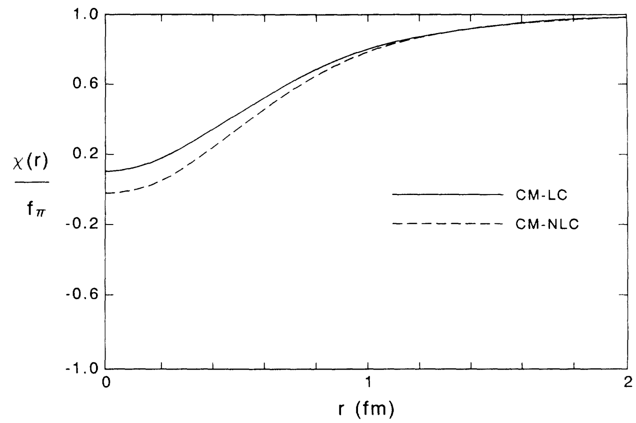

FIG. 5. Self-consistent scalar field χ is plotted for the cases of constant mass and local coupling (solid line) and constant mass and nonlocal coupling (dashed line).

In contrast, for a confining model such as the present DM-NLC calculation, any nonzero value of α ensures a stable soliton.

As can be seen from Table I, the combined effect of the removal of the confinement mechanism and nonlocal coupling is a slight increase in the baryon mass, but a 25% increase in the quark rms radius. This is the result of several mechanisms. In the confining case, the large distance falloff of the quark wave functions is guaranteed to be significantly faster than the exponential behavior of typical bound states associated with a constant quark mass parameter. This effect is illustrated in Fig. 2 where wave functions from the confining DM-NLC calculation are compared with those of the local CM-LC case. The quark source for the meson field is consequently more compact in the presence of confinement. The dynamical nature of the coupling vertex provided by the scalar self-energy amplitude $B(p^2)$ introduces an energy-dependent and finite-range spatially dependent coupling strength. For low three-space momenta of the quark states, the DM-NLC model has much stronger coupling than the local CM-LC model, and the reverse is true for high-momentum components. This also induces a more compact behavior for quark states. One may expect that the self-consistently determined meson field would be weaker and of smaller range in the confining nonlocal case. However, this is not so, and the scalar fields obtained in the two cases are displayed in Fig. 3. Although the ranges are similar, the confinement mechanism induces a deeper potential in agreement with a more compact quark source. The integrated potential energy of the meson field is largely unaffected, but the meson kinetic energy is greatly increased in the confined case.

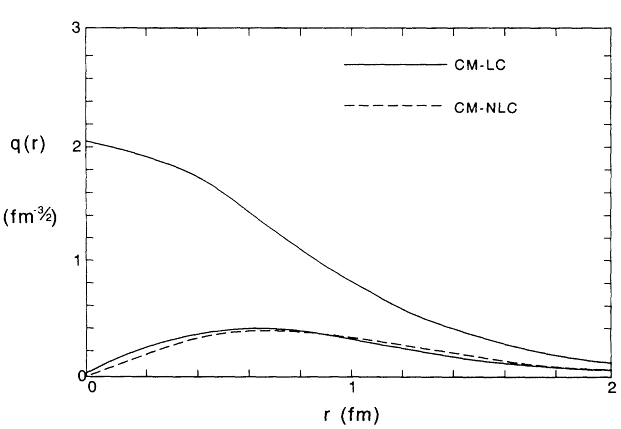

FIG. 4. Upper and lower components of the quark wave functions are plotted for the cases of constant mass and local coupling (solid lines) and constant mass and nonlocal coupling (dashed lines).

Although in the present model the dynamical quark self-energy and the nonlocal quark-meson coupling arise from a single mechanism as expressed through the amplitude $B(p^2)$, it is of interest to investigate the effect of the nonlocal coupling separately. We have performed a calculation in which the self-energy dynamics was suppressed as described above, but the distributed nature of the coupling vertex was retained. We label this calculation as CM-NLC. In Table II we compare the results to the earlier purely local limit. The principal effect of the nonlocal coupling is a lowering of the quark energy and a raising of the meson energy without change in the quark rms radius. This effect is largely due to the energy-dependent increase of the strength of the coupling vertex and accounts for essentially all of the reduction of the quark energy evident from Table I in the presence of confinement. On the other hand, it can be concluded that it is the confinement mechanism that produces the reduction in the quark rms radius. The effect of the nonlocal coupling on the wave functions and scalar field is quite small, as indicated by Figs. 4 and 5, respectively.

We consider now the nucleon axial coupling constant within this model. In general, it is of interest to explore the coupling to various fields, such as the electromagnetic field and isovector axial field, since the nonlocality of the quark-meson action (12) can make significant dynamical contributions. There will not be associated currents that are local combinations of the quark and meson fields. This is because the quarks there are dressed and the coupling to mesons respects the extended $\bar{q}q$ structure. However, the original action (1) of the GCM is unambiguous in requiring that the bare quarks couple to an external vector field A_μ (here taken to be Abelian) through the covariant derivative $\partial - i\lambda A$. Here λ is the quark charge operator $\frac{1}{2}(\tau_3+\frac{1}{3})$ for electromagnetic coupling or $\gamma_5\tau_3$ for axial coupling. The generating functional Z of Eq. (2) then acquires a dependence upon the external field, and after the saddle-point configuration for quark dressing is obtained, the current that couples to the external field is identifiable as

$$J_\mu(q) = \left.\frac{\delta \ln Z}{\delta A_\mu(q)}\right|_{A=0} = \frac{1}{(2\pi)^2}\,\mathrm{tr}\int d^4p\, d^4k\, G(k,p)\Gamma_\mu(p,k;q) \,. \quad (29)$$

Here G is the quark propagator corresponding to (19) and Γ_μ is the irreducible three-point vertex for the coupling of dressed quarks to the external field. In the present model this quantity can, in principle, be obtained from its defining integral equation that is the counterpart of the simplified Schwinger-Dyson equation (6). This is still quite difficult.

For the axial coupling constant, we use a simple minimal-substitution approximation for Γ_μ^5 that still permits us to explore the effects of the confinement mechanism and nonlocal coupling upon g_A. Since the present work omits the pion, only the valence quark contribution to axial coupling is required, and the self-energy function contains the relevant information. The Dirac scalar amplitude $B(p^2)$ is a chiral-symmetry-breaking term that does not contribute to the axial current. Minimal substitution into the relevant part of the quark inverse propagator then leads to

$$\Gamma_\mu^5(P,Q;q) \approx -\delta(Q-q)\frac{\partial}{\partial P_\mu} i\gamma\cdot P A(P^2)\gamma_5\tau_3 \,, \quad (30)$$

where $P=\frac{1}{2}(p+k)$ and $Q=p-k$. This approximation amounts to use of the $q=0$ value for all q and should be accurate for a coupling constant calculation. The dependence upon the momentum P is a consequence of the quark self-energy dressing and (30) satisfies the axial Ward identity with the induced pseudoscalar mode removed as is appropriate for the present pion-free calculation. The axial coupling constant is given by the valence quark contribution to $\int d^3x\, J_3^5(x)$, and it is easily seen to be independent of x_4. The result is

$$g_A = \sum_j{}' Z_j^{-1} \int d^3p\, \bar{u}_j(\mathbf{p}) \left\{ \frac{\partial}{\partial p_3} i\gamma\cdot p A(p^2) \right\} \gamma_5\tau_3 u_j(\mathbf{p}) \,, \quad (31)$$

where $p=(i\epsilon_j,\mathbf{p})$ and the spin-flavor summation over j is weighted by the occupation probabilities of the standard SU(4) valence quark model of the nucleon. After angular integration the dominant contribution can be expressed as

$$g_A = \frac{5}{3}\frac{1}{Z}\int dp\, p^2 A(-\epsilon^2+p^2)\left\{ g^2(p) - \frac{1}{3}f^2(p) \right\} \,, \quad (32)$$

TABLE III. Comparison of values calculated for the axial-vector coupling constant in three cases of differing dynamical content.

	DM-NLC	CM-LC	CM-NLC	Expt.
g_A	1.23	1.42	1.15	1.24

where $g(p)$ is the upper Dirac component and $-\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}f(p)$ is the lower component of the S-wave quark state. We have not displayed terms that involve derivatives of the amplitude A since they make a negligible contribution here. In the limit where A and Z are constants that cancel, (32) reduces to the standard result for a point-coupling model.

In Table III we display results from evaluation of (32) for the three different dynamical cases discussed earlier. Although the value of g_A obtained in the full dynamical model with confinement and nonlocal coupling (DM-NLC) is essentially identical to the empirical value, this cannot be taken seriously in the absence of a pion. In local models of this type [1], pionic effects can increase the value of some 70%. Whether this will also be the case in the present model is under investigation. The purpose of the g_A calculations here is to explore the effect of the confinement mechanism and nonlocal coupling on the valence quark contribution. From Table III it is evident that the confining dynamical mass and related nonlocal coupling produce a 15% reduction from the purely local nonconfining (CM-LC) value. This is due to the finite range of the vector self-energy amplitude $A(p^2)$ and the smaller effective range of the confined quark states. Since g_A is usually overestimated in local chiral soliton models, the appearance of a reduction mechanism whose origin lies in the composite nature of the confining vacuum condensate $\langle \bar{q}(x)q(y)\rangle$ and associated meson modes is an interesting phenomenon. The greater reduction in g_A that is produced by introducing only the nonlocal meson coupling can be attributed solely to the increase in the wave-function renormalization constant Z. We recall that $A=2$ in both calculations that have a constant quark mass and the wave functions are essentially identical. The confining dynamical mass actually decreases Z, but the overall decrease in g_A is due to the momentum falloff of the amplitude A and the confining behavior of the wave-function components.

V. SUMMARY

We have explored a mean-field solution for the nucleon in a model where valence quarks are confined by the nonlocal structure of the quark scalar condensate generated by dynamical chiral-symmetry breaking. Associated with this mechanism is a composite, extended scalar $\bar{q}q$ field which is the chiral partner of the Goldstone pion. The employed quark-meson action has it origin in the global color-symmetry model where quarks interact via a current-current term mediated by a parameter function to represent the finite-range effective two-point gluon function. The model is defined in Euclidean metric. The meson fields that arise from bosonization are extended objects even at the tree level employed here. Only the scalar meson mode is kept in this initial numerical study.

The self-consistent mean scalar field and corresponding quark states are calculated in the presence of the nonlocal coupling dictated by the scalar term of the vacuum quark self-energy function. A combination of momentum- and position-space numerical methods is found to be convenient for this problem. The invariant four-space structure of the self-energy induces a self-consistent dependence of the quark-meson vertex upon the quark energy and three-momentum. Despite these unusual dynamical features, the obtained nucleon mass, size, and constituent quark wave functions are very acceptable for a pion-free model with one free parameter. The separate effect of the confinement mechanism and nonlocal coupling is presented. Both mechanisms increase the binding and raise the meson field kinetic energy, while the confining mechanism significantly reduces the nucleon size. The nucleon axial coupling constant g_A is calculated, and it is found that both the confining dynamical mass function and nonlocal coupling serve to reduce the value below the local, nonconfining limit. Local chiral soliton models usually overestimate g_A, and it is interesting that there is a reduction mechanism arising from the composite nature of the confining vacuum quark condensate and associated extended meson mode.

The field content of this nontopological soliton is quite primitive, but the dynamics is rather novel. It remains to be seen whether the introduction of the pion field will allow the presently successful features to survive. No new parameters are needed to include the pion in this model since there is a natural place for it due to the hidden chiral symmetry maintained in the original derivation. A crucial element in implementing a model of this type is knowledge of the vacuum quark propagator in the timelike region. We have used a simple confining form generated from an infrared momentum delta function as the effective gluon propagator in a Schwinger-Dyson equation. For a more realistic case, the analytic continuation is difficult in general and very little work in that direction is available. One exception is provided by the recent work of Burden, Roberts, and Williams [16] in which dressing of the quark-gluon vertex is included along with the delta-function gluon propagator.

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