

Pion absorption in light nuclei

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Cross sections have been measured for proton emission following π^+ absorption on ${}^6\text{Li}$, ${}^7\text{Li}$, ${}^9\text{Be}$, ${}^{10}\text{B}$, ${}^{11}\text{B}$, ${}^{12}\text{C}$, and ${}^{13}\text{C}$, for pion kinetic energies of 50, 100, 140, and 180 MeV using a large-solid-angle bismuth germanate detector array.

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I. INTRODUCTION

Pion absorption has been studied extensively during the past decade [1, 2]. Much of the effort has been directed toward the study of final states including two nearly coplanar outgoing protons with the goal of determining the absorption mechanism. Considerable evidence, primarily from studies of the helium isotopes [2], has been obtained that a significant fraction of the absorption is due to absorption on three nucleons. While there is general agreement that the three-body mechanism exists and becomes increasingly important as the pion energy increases, there is disagreement on the exact magnitude. Studies of ${}^3\text{He}$ and ${}^6\text{Li}$ [3] indicate that three-body absorption is about $\frac{1}{3}$ of the total absorption cross section in the $\Delta(1232)$ resonance region, but the studies of ${}^4\text{He}$ indicate a value of only about 10%. Both the ${}^4\text{He}$ and ${}^6\text{Li}$ data indicate that absorption on a pnn triplet is about twice as likely as on a ppn triplet, but the comparison of π^+ and π^- absorption on ${}^3\text{He}$ indicates they are of approximately equal importance.

Interpretation of the absorption data on heavier nuclei is complicated by initial and final state interactions (ISI/FSI). These interactions obscure the true absorption mechanism and there is no reliable method for correcting for them, or even agreement on their relative importance.

A systematic study of absorption as a function of mass could help understand these processes and be easier to interpret theoretically. However, only the helium isotopes, ${}^6,7\text{Li}$ and ${}^{12}\text{C}$ have been studied in any detail [3–12] with the exception of a measurement at 76 MeV [5]. We report here the results of a study of absorption leading to two or more energetic protons on all stable isotopes from ${}^6\text{Li}$ to ${}^{13}\text{C}$.

II. EXPERIMENT

The experiment was performed at the Clinton P. Anderson Meson Physics Facility (LAMPF) using a large solid angle detector, the BGO ball. The experimental setup is similar to that of Ref. [3], where the BGO (bismuth germanate) ball has been described in detail. There were some important changes from the previous experiment. The beam was counted by a single 5 mm square scintillator which was placed a few mm upstream of the target. The scintillator was 100 mg/cm² thick and 13% deuterium by weight. This allowed us to monitor the energy calibration of the detectors by observing the $\pi^+d \rightarrow pp$ reaction, as well as check for differences in the normalization between runs. The targets were supported on a 1 cm diameter paper tube inserted from the front of the ball. This gave more reproducible positioning of the target than the previous support. The targets were ${}^6\text{Li}$, ${}^7\text{Li}$, ${}^9\text{Be}$, ${}^{10}\text{B}$, ${}^{11}\text{B}$, ${}^{12}\text{C}$, and ${}^{13}\text{C}$, with thicknesses of 230, 340, 260, 200, 200, 277, and 206 mg/cm² thick. A 197 mg/cm² CD₂ target was used for energy calibrations. The targets were 99% isotopically pure except for the ${}^{13}\text{C}$, which was 90% pure. The target thicknesses were known to about 5% except for the boron isotopes which were known to about 15%. The overall normalization uncertainty is about 10%.

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III. RESULTS

Most of the absorption cross section leads to two or more protons in the final state. Our analysis here is based on those cross sections and the energy spectra of those events.

The observed cross sections are listed in Table I. No corrections for missing solid angle or losses due to nuclear reactions in the BGO have been made. The statistical error on the $2p$ cross section is generally 1% or less. Because the normalization uncertainty is about 10% (15% for $^{10,11}\text{B}$), the values have been rounded to two significant figures and no statistical error is given. The cross sections listed are $2p$, which means two detected protons each with an observed energy greater than 22 MeV. This cross section does not include events with a detected pion, two or more neutrals (which are mainly due to charge exchange), or events with more than two protons. $2p$ QD (quasideuteron) is the same as the $2p$ cross section with the additional requirement that the two protons have an opening angle greater than 140° . $3p$ is the same as $2p$ except that three protons each with observed energies greater than 22 MeV were detected. $2pn$ is a subset of the $2p$ cross section and requires the detection of a neutral particle in addition to the two protons. Finally,

deuteron is the cross section for events containing at least one deuteron and at least one proton.

The most striking feature of these cross sections is the nearly constant, within the normalization uncertainty, of the cross sections for $2p$ for ^6Li to ^9Be and for ^{11}B to ^{13}C , with ^{10}B between the two sets. If the only interaction were simple quasideuteron absorption (QDA) one would expect an increase with mass. The fact that ^9Be is within 10% of ^7Li is particularly hard to understand in terms of a simple QDA model. The other cross sections show a similar behavior. The fraction of the cross section which displays QD kinematics decreases smoothly with increasing mass and with increasing energy for each mass.

Next we look at the missing energy spectra. The missing energy is defined as

$$E_{\text{miss}} = T_\pi + m_\pi - E_b - \sum T_{\text{prot}}$$

where $\sum T_{\text{prot}}$ is the sum of the proton energies and E_b is the binding energy of the least bound np pair. E_{miss} is shown for the $2p$ final state in Fig. 1 and for the $3p$ final state in Fig. 2. The points are for ^6Li , the solid line is ^9Be , and the dashed line is ^{12}C . The lines are not fits to the data, but simply connect the data points.

TABLE I. Observed cross sections in mb with an uncertainty of 10% except for $^{10,11}\text{B}$ which have an uncertainty of 15%. See text for description of cross sections.

Energy	Nucleus	$2p$	$2p$ QD	$3p$	$2pn$	Deuteron
50	^6Li	15	11	0.23	0.11	2.4
	^7Li	15	10.0	0.13	0.11	3.1
	^9Be	12	7.8	0.19	0.06	2.6
	^{10}B	15	8.0	0.29	0.08	3.0
	^{11}B	19	10	0.27	0.18	4.1
	^{12}C	19	10	0.27	0.11	3.1
	^{13}C	18	9.0	0.33	0.13	3.7
100	^6Li	29	20	1.0	0.58	4.7
	^7Li	27	17	0.69	0.57	5.2
	^9Be	26	15	0.83	0.52	5.2
	^{10}B	33	18	1.5	0.68	7.6
	^{11}B	45	23	1.3	0.80	9.9
	^{12}C	43	22	1.8	0.62	8.4
	^{13}C	44	22	1.8	0.92	9.8
140	^6Li	31	20	2.1	1.4	5.6
	^7Li	32	19	1.8	1.4	6.5
	^9Be	32	18	1.8	1.3	6.2
	^{10}B	35	17	2.8	1.6	10
	^{11}B	49	24	3.3	2.0	11
	^{12}C	47	22	3.8	1.9	9.6
	^{13}C	47	21	3.5	2.0	12
180	^6Li	31	18	3.4	2.4	5.7
	^7Li	30	16	2.5	2.1	5.7
	^9Be	29	15	2.7	1.9	6.5
	^{10}B	38	17	4.9	2.7	9.5
	^{11}B	50	21	5.0	3.4	12
	^{12}C	49	22	5.7	2.8	10

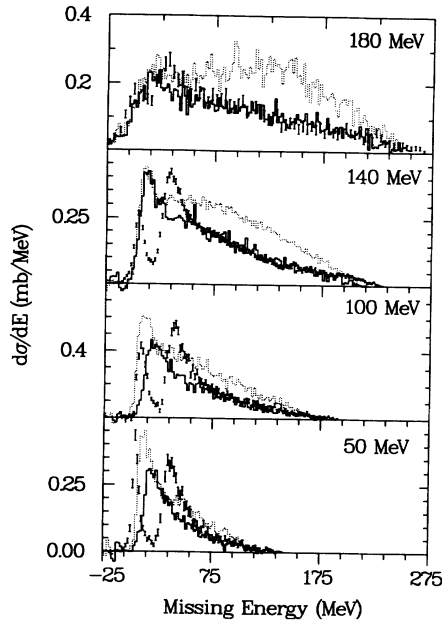


FIG. 1. $2p$ missing energy spectrum for ${}^6\text{Li}$ (points), ${}^9\text{Be}$ (solid line), ${}^{12}\text{C}$ (dashed line) at 50, 100, 140, and 180 MeV.

Other nuclei have been omitted for the sake of clarity. For the two lowest energies the large missing energy parts of the spectra are fairly similar. For 140 MeV pions the ${}^{12}\text{C}$ has a distinct change in shape compared to the light nuclei, with a much larger fraction of the cross section at higher missing energy. This trend continues at 180 MeV, although the light nuclei also show an increase in the fraction of high missing energy. ${}^7\text{Li}$ is similar to ${}^9\text{Be}$. ${}^{11}\text{B}$ and ${}^{13}\text{C}$ are similar to ${}^{12}\text{C}$, while ${}^{10}\text{B}$ is in between the two sets. The three proton missing energy does not

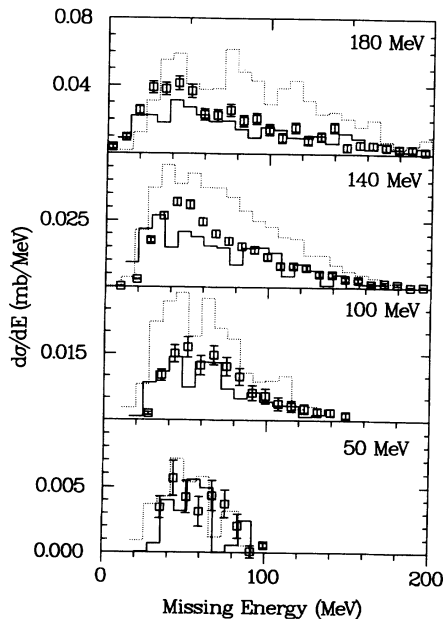


FIG. 2. $3p$ missing energy spectrum for ${}^6\text{Li}$ (points), ${}^9\text{Be}$ (solid line), ${}^{12}\text{C}$ (dashed line) at 50, 100, 140, and 180 MeV.

show a similar trend. The statistics are not as good, but the shape of the spectrum does not appear to change much with increasing mass.

The previous study of ${}^6\text{Li}$ [3] showed a large cross section for the $2pn$ final state at higher energies. That trend is seen here as well. With 180 MeV pions the cross section for $2pn$ is nearly as large as that for $3p$, although the neutron detection efficiency is small (25% maximum). Since the solid angle coverage is the same, this indicates that the $2pn$ cross section is substantially larger than the $3p$ cross section. This appears to be consistent with the large fraction of the $2p$ missing energy spectrum at high energy. There is the possibility that these may be partially due to single charge exchange in which one photon is not detected and an ISI or FSI gives two protons in the final state. Due to kinematical considerations this is only a possible contamination for the pion energies of 140 and 180 MeV. Because of the large solid angle coverage and high detection efficiency for photons, about 80% of the π^0 decays will result in both photons being detected. This indicates that $2pn$ cross section which is due to $2p2\gamma$ should be at most about $\frac{1}{4}$ of the $2p2n$ cross section. We find that even with 180 MeV pions the $2p2n$ cross section is only about 25% of the $2pn$ cross section, which would indicate that the charge exchange contamination of the $2pn$ cross section is at the 5% level, and is negligible for lower energies. Other contamination, such as accidentals, or misidentified protons, are also estimated to be 10% or less of the observed $2pn$ cross section, as discussed in Ref. [3].

We also note that there is substantial cross section for deuteron emission. The angular distributions of the pd final state are similar to that of the pp leading us to believe that it is largely due to pp final states in with a pickup of a neutron by one of the final state protons. However, we have no good model of this process at this time to estimate the expected cross section.

We can estimate the total cross section for absorption by comparing the observed deuterium cross section with the accepted value [14], and by modeling the interaction to estimate the fraction. The comparison with deuterium can be done by measuring the cross section in the peak and by subtracting the carbon cross section from the CD_2 . In the first case protons reacting in the BGO will be lost. In the second case they will be lost only if the energy loss is so great that the proton falls outside the cuts on the dE vs E curve. The correction factors for the two cases are 2.0/1.8, 2.3/1.8, 3.4/2.0, and 2.9/1.6.

We have estimated the correction factors using a Monte Carlo phase space method using FOWL [13] described in an earlier paper [3], although using a somewhat simpler analysis. Rather than making a detailed model of many final states we have compared with phase space of three final states: $2pn$, $3pn$, and $2p2n$. The correction factors for two proton inclusive cross section are given in Table II. The values are in reasonable agreement with the deuterium correction factors. We should note that although phase space is more isotropic in the laboratory and has lower proton energies than deuterium, both of which should give a smaller correction factor, there are

TABLE II. Correction factors for proton inclusive cross section based on phase space of various final states.

Energy	$2pn$	$3pn$	$2p2n$
50	2.0	2.2	3.2
100	2.1	2.3	2.6
140	2.0	2.3	2.4
180	2.1	2.4	2.3

also more protons lost due to the energy cutoff and higher energy protons which do react have a greater chance of remaining in the proton cut than lower energy protons due to the slowly changing dE at higher energies, both of which lead to a larger correction factor. Except for $2p2n$ at 50 MeV, the resulting correction factors are within 20% of 2.2 independent of the final state. The previous more detailed study of ${}^6\text{Li}$ also gave an estimated correction factor of about two at all energies. The pd data appear to originate from initial pp states, so that should be included in the $2p$ cross section before corrections are made. Our estimate of the total cross sections leading to two or more protons in the final state is then 2.2 times the $2p + pd$ cross section, with an uncertainty of about 20%.

We can also estimate the expected $2pn$ cross sections. We have assumed a neutron detection efficiency of zero for neutrons below 30 MeV, 25% for neutrons above 100 MeV, and linear increase from zero to 25% between 30 and 100 MeV. This is probably a slight overestimate of the efficiency, but we wish to use it only as a guide of expected cross sections. Using a correction factor of two to the $2p$ cross sections and assuming the total cross section is $2pn$ would predict observed $2pn$ cross sections of about 0.9 mb at 50 MeV to 6 mb at 180 MeV for ${}^{12}\text{C}$, compared to actual values of 0.1 and 2.8 mb, which indicates that our observed $2pn$ cross sections are not unreasonably large, although there are of course considerable uncertainties in this estimate.

IV. COMPARISON WITH PREVIOUS DATA

Measurements of ${}^6\text{Li}$ [3] and ${}^{12}\text{C}$ [4] were previously made with the BGO ball. The ${}^6\text{Li}$ results are in good agreement, with the most significant discrepancy between the current 140 MeV data and the previous 150 MeV data, in which the previous measurement was about 20% larger. The values at other energies agree within 10%. The current ${}^{12}\text{C}$ measurement is systematically about 15% smaller than the previous measurement. The target used in this measurement was 277 mg/cm² compared with 100 mg/cm² previously. Part of the discrepancy can be explained by the larger energy loss in the target, giving an effectively higher cutoff energy, which we estimate would give about a 5% smaller cross section. Except for the uncertainty in the target thickness we have no other explanation for the discrepancy.

Favier *et al.* [5] measured absorption on ${}^6,{}^7\text{Li}$, ${}^9\text{Be}$, natural boron, and natural carbon at 76 MeV with two

TABLE III. Comparison with results of Favier [5], in microbarns, with the current results at 100 MeV normalized to the ${}^6\text{Li}$ value of Favier.

Nucleus	Favier	Normalized
${}^6\text{Li}$	170±12	170
${}^7\text{Li}$	150±15	144
${}^9\text{Li}$	140±30	128
B	110±25	187
${}^{12}\text{C}$	160±35	187

detectors at $\pm 90^\circ$ to the beam. They integrated over the solid angle of their detectors to get the values given in Table III. We have normalized our $2p$ QD data of 100 MeV to the ${}^6\text{Li}$ value of Favier, also shown in Table III. The general features are well reproduced with the exception of boron where their value is about 40% lower than our value. The Favier boron value also seems unusually low compared to their other data.

Yokota *et al.* [12] measured absorption on ${}^6,{}^7\text{Li}$ and ${}^{12}\text{C}$ for pion energies of 70, 130, and 165 MeV. They measured coplanar protons at several angle combinations and integrated results. Their results are given in Table IV. They found a decreasing cross section with increasing mass with values which are clearly significantly smaller than even our measured values and are about $\frac{1}{2}$ to $\frac{1}{5}$ of our estimated total. The Yokota detectors had a vertical angular extent of less than $\pm 9^\circ$. As we have seen from the great reduction in cross section from $2p$ to $2p$ QD, this strict coplanarity requirement will give a much reduced cross section and affects carbon more than lithium. Our previous study of ${}^6\text{Li}$ found that for 150 MeV pions the Yokota measurement would give only $\frac{1}{4}$ the actual cross section. We do not have sufficient angular resolution to make a more detailed comparison.

Total cross sections have been measured by Ashery *et al.* [15] at 85, 125, 165, and 205 MeV for natural lithium

TABLE IV. Comparison with results of Yokota [12] (Y), Ashery [15] (A), and Navon [16] (N). Ashery and Navon used natural lithium. Our estimated totals are also given (BGO) with uncertainties of 20%, and we linearly interpolated our cross sections to the energies of Ashery. All values in millibarn.

Energy/Ref.	${}^6\text{Li}$	${}^7\text{Li}$	${}^{12}\text{C}$
50/N		28±21	
50/BGO		40	
85/A		44±20	109±20
85/BGO		60	90
125/A		114±26	166±26
125/BGO		80	120
165/A		124±30	194±36
165/BGO		80	130
205/A		59±33	157±30
70/Y	20.2	17.5	15.1
130/Y	24.9	21.6	15.7
165/Y	25.2	22.4	15.9

and carbon, and Navon *et al.* [16] measured 50 MeV. Both results are shown in Table IV with comparisons to our data. As explained in Sec. III, we estimated total cross section by multiplying the observed $2p + pd$ cross sections by 2.2 with an uncertainty of about 20%. To make comparisons at the same energies as the data of Ashery *et al.* we have linearly interpolated our data to the same energies. We are consistent with the data of Navon *et al.* within the very large error. Our values are generally systematically smaller than those of Ashery which may indicate that pn or nn final states are significant, since we are insensitive to that part of the cross section.

V. DISCUSSION

There are two major features seen from these data. First, there is a relatively small difference in cross section of the lithium isotopes and beryllium and between the carbon isotopes and ^{11}B . Second, the shape of the missing energy spectrum is very similar for the lithium isotopes and ^9Be in the large missing energy region and these are distinctly different from the shape of the higher mass nuclei.

The first question is what ratios of cross sections would be expected simply from QDA on relative $l = 0$, $S = 1$, $T = 0$ pairs. Zheng and Zamick [17] have made a shell model calculation of the number of such pairs in the p shell. Assuming three pairs in the s shell and no cross shell absorption the number of pairs is 3.95, 4.5, 6.5, 6.3, 8.7, 8.8, and 11.6 for ^6Li to ^{13}C in order of mass. This predicts that ^9Be should have a 65% larger cross section than ^6Li and ^{12}C should be 2.2 times as large. At 50 MeV this is clearly not the case, although by 180 MeV the ^{12}C to ^6Li ratio is 1.6. The relatively small ^9Be and ^{13}C cross sections are difficult to understand in this context.

The missing energy spectra show very similar spectra for ^6Li and ^9Be , except in the low missing energy region, as expected from the very weakly bound p -shell pair in ^6Li . If the large missing energy region were due primarily to ISI/FSI it might be expected that ^9Be would show a distinct difference in shape since it has an additional proton and two additional neutrons. Yet this does not seem to have a significant effect on the spectrum. In contrast, ^{12}C does have a distinct difference in shape which becomes more pronounced at higher energy. In addition, boron and carbon should have a larger fraction of the absorption in the p shell compared to lithium which would be expected to give relatively low missing energies. For

the energies of the nucleons produced from absorption the nucleon-nucleon cross sections are decreasing with increasing pion energy. This, along with the similarity of the lithium and beryllium spectra, would argue against FSI as the main cause of the large missing energy region. It is unclear whether ISI or FSI would significantly change the cross section ratios, given our large acceptance and relatively low energy threshold. Because of the dominance of the Δ in this region, giving a nine times greater πp than πn elastic cross section, the apparently large ppn final state is also difficult to understand in terms of a simple ISI followed by absorption.

We have made some investigation of the absorption in the $A(\pi, pp)A - 2$ reaction, with the final nucleus in the ground state, using the distorted wave code THREEDEE [18]. For 100 MeV pions, the estimate of the cross section for the plane wave and distorted wave calculations showed only a small difference for ^6Li , but for ^9Be and ^{12}C the distorted wave calculation was 41% and 27%, respectively, of the plane wave calculation. This indicates distortions may be quite important and perhaps can explain some of the apparent suppression of the higher mass cross sections.

VI. CONCLUSIONS

The first systematic study of pion absorption on light nuclei has been presented. The results indicate that there is an important part of the absorption cross section in which the energy is shared among more than two nucleons. The systematics of the missing energy spectra and two proton emission cross sections are not consistent with the three-body part of the cross section being due primarily to ISI or FSI. The ratios of the cross sections are not consistent with the expectations of QDA in a shell model calculation, but this does not include distortions, which may be significant. There appears to be substantial cross section for the ppn final state which would indicate that the absorption involving pnn triplets is significantly greater than on ppn triplets. A complete understanding of these data will require better theoretical calculations of the absorption process than currently exist.

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