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Realistic estimate of incomplete fusion excitation function in nucleus-nucleus collisions

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Incomplete-fusion excitation functions are calculated for the systems $^{40}Ca+^{40}Ca$ and $^{48}Ca+^{200}Hg$ taking into account the temperature dependence of the free interaction energy and a realistic dynamical treatment of preequilibrium emission of nucleons before the fusion process. The inhibition to fusion of the hot rotating composite due to opening of fission channels is also investigated.

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It is well known that with increasing bombarding energy above the Coulomb barrier, complete fusion gradually gives way to incomplete fusion [1]. At energies ~40 MeV nucleon, the cross section for incompletely fused events is also reported to disappear [2] in recent nuclear collision experiments. This disappearance finds explanation in the nature of the equation of state at subnuclear density and in the choice of an in-medium nucleon-nucleon cross section in dynamical Boltzmann-Uehling-Uhlenback- (BUU-) type calculations [3]. It has also been explained in thermodynamic models by invoking liquid-gas phase instability at these energies [4].

A simple explanation for the disappearance of the fusion cross section has recently been offered [5] in the dynamical one-dimensional potential pocket model using a temperature-dependent interaction potential between reacting nuclei. During close collisions, a large amount of energy is transferred from the relative motion into the intrinsic excitations of the nuclei, a significant fraction of which may remain in the thermal mode, thus raising the temperature of the nuclei. The temperature generally raises the fusion barrier, and at a certain temperature denoted the "critical fusion temperature" (~5–7 MeV, depending on the mass and charge of the system), the barrier disappears completely [6]. Fusion cannot occur on such a potential profile. Thus a quantitative estimate of the energy at which fusion disappears calls for a subtle understanding of the excitation energy the interacting systems receive on their path to fusion. In Ref. [5] it has been shown that when all the energy lost from relative motion is assumed to be locked in the intrinsic excitations of the nuclei, fusion disappears at ~25 MeV/nucleon in contrast to ~40 MeV/nucleon as found experimentally. It is, however, well known that at energies in the vicinity of Fermi energy, a considerable amount of energy is carried away by preequilibrium particles. To have a qualitative feeling of the effect of this missing energy on the fusion process, it is assumed in Ref. [5] that at each stage of the fusion trajectory a nominal 20% of the total excitation energy is carried away by preequilibrium particles keeping the mass and charge of the reacting fragments unaltered. The critical fusion energy is then increased from ~25 to ~45 MeV/nucleon. A reliable estimate of the critical energy for the disappearance of fusion thus calls for a realistic treatment of preequilibrium emission along the fusion path on the temperature-dependent potential profile. In this Brief Report, we deal with these aspects of the fusion dynamics quantitatively in the model of promptly emitted particles (PEP's) [7].

In the PEP model, exchanged nucleons between two closely interacting nuclei get a kinematic boost in the recipient nucleus because of coupling of the nuclear relative velocity with the intrinsic nucleonic velocity. Depending on in-medium two-body collisions and energy restrictions [7], some of the exchanged and struck nucleons may be emitted in the continuum as preequilibrium particles which remove mass, energy, and angular momentum from the interacting nuclear system. Exchanged nucleons that are absorbed increase the intrinsic excitation energy and collective angular momentum of the nuclei. Assuming instantaneous thermalization of the excitation energy, the thermal energy at any time t is given by

$$E_T^*(t)=E^*(t)-E_{\rm rot}(t)\ , \qquad (1)$$

where $E_{\rm rot}$ refers to the collective rotational energy and E^* is the total intrinsic excitation. The temperature T of the system is then given by $E_T^*=aT^2$, where the level density parameter a is taken as $a=A/8$, A being the instantaneous mass number of the system. For asymmetric systems the division of excitation energy between the reacting partners may not be according to mass [8], but for simplicity, we have assumed the nuclei to have the same temperature. Instantaneous thermalization is a questionable assumption, but to handle the changing momentum distributions of the constituent nucleons of the reacting nuclei due to flow of mass and energy from exchange, we assume the momentum distributions to be diffuse Fermi distributions at each instant; i.e., effectively, they are simulated by a dynamical temperature.

Once the common temperature of the nuclei is known, their temperature-dependent interaction potential is evaluated in a sudden approximation employing energy-density formalism. It is given as

$$F_I(R,T)=\int d^3r\,[F(\rho_1+\rho_2,T)-F(\rho_1,T)-F(\rho_2,T)]\ , \quad (2)$$

where ρ_1 and ρ_2 are the temperature-dependent densities of the two nuclei taken as Woods-Saxon distributions and R is the distance between their centers. The kinetic energy and entropy terms in the free energy F are evaluated in the extended Thomas-Fermi (ETF) approximation taken up to second order and the nuclear part of the potential-energy density is calculated employing the Skyrme interaction SkM*. For the Coulomb and nuclear interactions, the details are given in Ref. [5]. The force between the nuclei is then calculated as

$$K=-\left[\partial F_I/\partial R\right]_T\ . \quad (3)$$

At finite relative angular momentum, the centrifugal contribution is added. The cross section for the hot composite formation is then determined in the sharp cutoff approximation as

$$\sigma_h=\pi\lambda\!\!\!^{-2} l_{\max}(l_{\max}+1)\ . \quad (4)$$

Here $l_{\max}$ refers to the maximum orbital angular momentum of the classical trajectories that are trapped in the effective temperature-dependent interaction potential and $\lambda\!\!\!^{-}$ is the asymptotic de Broglie wavelength of the system. These trajectories are obtained by solving the Euler-Lagrange equations of motion [9]. The nonconservative force is taken care of through the particle-exchange-induced energy and angular momentum loss from the relative motion. This dissipative force agrees fairly well with the proximity friction [10].

The hot composite, so formed, at relatively high bombarding energies contains large angular momentum in general and may thus become unstable against fission. If so, such an event cannot be regarded as a fusion event; so fusion dynamics in the context of the pocket model cannot alone determine the fusion excitation function. It is well known that the fission barrier decreases steadily with both temperature [11] and angular momentum [12]. To determine the stability of the hot rotating composite against fission, we have therefore made a simultaneous study of the dependence of the fission barrier on temperature and angular momentum and have then calculated the incomplete-fusion excitation function.

To calculate the fission barrier, we describe the deformation coordinate by the y parameter of the Nix family [13] that runs through saddle-point shapes of nuclei with fissilities $x=1-y$. Here $y=0$ refers to zero deformation and $y=1$ corresponds to two tangent spheres. We consider only symmetric fission. The free energy is calculated with the temperature-dependent droplet-model parameters given by Guet, Strumberger, and Brack [14]. The moment of inertia is given by the rigid-body value. The change in free energy due to deformation is given by

$$\delta F_L=F_s(B_s-1)+C_1Z^2A^{-1/3}(B_C-1)+a_C A^{1/3}(B_k-1)+L^2\hbar^2[B_{\rm rot}^{-1}-1]/[2\mathcal{J}(0)]\ , \quad (5)$$

where the first, second, third, and fourth terms on the right-hand side correspond to surface, Coulomb, curvature, and rotational contributions, respectively. The shape functions B_s, B_C, B_k, and $B_{\rm rot}$ are temperature independent and are easily calculated [13]. The temperature dependence of the other parameters in Eq. (5) are known [14]. The deformation energy and hence the fission barrier as a function of both temperature and angular momentum can then be evaluated. If $l_{\max}$, the maximum angular momentum corresponding to hot-composite formation, is greater than the limiting angular momentum $l_{\rm lim}$ at which the fission barrier of the said composite vanishes, then the events corresponding to the trajectories in the angular momentum range $l_{\rm lim}$ to $l_{\max}$ cannot be considered as fusion events since the composite would undergo fission almost instantaneously. The fusion cross section is then given by

$$\sigma_f=\pi\lambda\!\!\!^{-2} l_{\rm cr}(l_{\rm cr}+1)\ , \quad (6)$$

where $l_{\rm cr}$ is the minimum of $l_{\max}$ and $l_{\rm lim}$.

The calculated fusion excitation functions for the systems ^{40}Ca+^{40}Ca and ^{48}Ca+^{200}Hg are displayed in Figs. 1 and 2. These systems are chosen as representative cases for the formation of medium-heavy and heavy hot com-

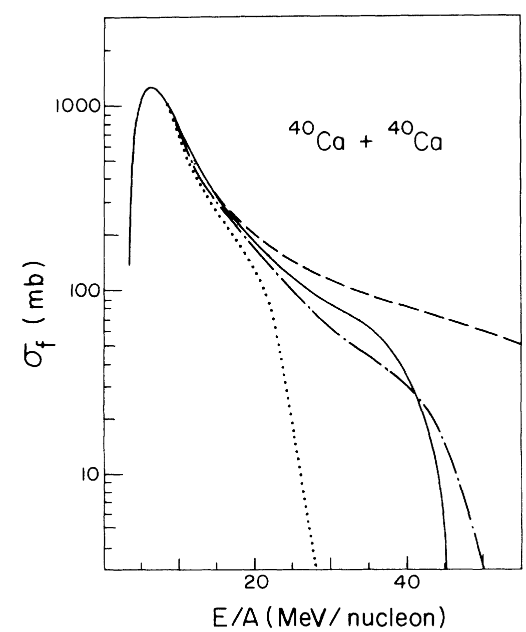

FIG. 1. Fusion excitation function for the system ^{40}Ca+^{40}Ca. The dashed line represents calculations with a temperature-independent ($T=0$) potential, the dotted line corresponds to calculations with a temperature-dependent potential ignoring preequilibrium emission, the dot-dashed line refers to those with the inclusion of an effective reduction of temperature in a schematic treatment (see text), and the solid line refers to calculations ($T\neq 0$) with a realistic treatment of prompt particle emission.

posites. The dashed lines correspond to results obtained in the conventional one-dimensional pocket model with temperature-independent SkM* interaction. Fusion is then found to persist even up to ~100 MeV/nucleon. The dotted lines correspond to calculations with suppression of prompt emission employing a temperature-dependent interaction potential as given by Eq. (2). As the interacting ions come close together, both nuclei become excited because of particle exchange and the system then moves on a dynamically changing temperature-dependent potential profile. The critical fusion temperature T_{CF} for ^{40}Ca+^{40}Ca is found to be ~7 MeV, and for ^{48}Ca+^{200}Hg, it is ~5 MeV. On the fusion path, these temperatures are reached for the first system at ~25 MeV/nucleon and for the second system at ~20 MeV/nucleon. This is reflected in the disappearance of the fusion cross section at these bombarding energies as shown by the dotted lines. Emission of prompt particles denudes the system of excitation energy and thus lowers the temperature of the reacting nuclei. In an *ad hoc* fashion, when it is assumed that because of PEP emission the excitation energy of the system is reduced by 20% at each stage of the collision, the critical fusion energy (energy above which fusion disappears) is shifted to ~45 MeV/nucleon for ^{40}Ca+^{40}Ca and ~30 MeV/nucleon for ^{48}Ca+^{200}Hg. These are shown as dot-dashed-dotted lines in Figs. 1 and 2. A realistic evaluation of excitation energy and thus temperature with dynamically changing mass and charge variables from prompt particle emission leaves the excitation function for ^{40}Ca+^{40}Ca nearly unchanged, but changes it for ^{48}Ca+^{200}Hg considerably, as shown by the solid lines in Figs. 1 and 2.

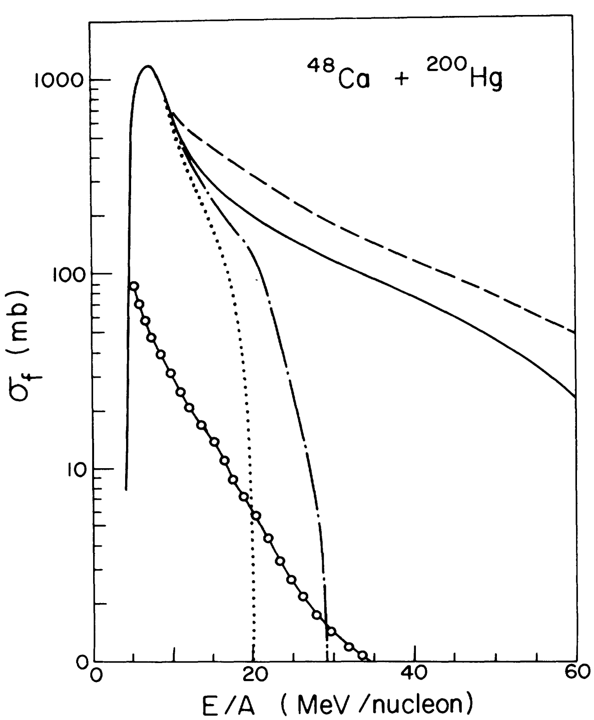

FIG. 2. Same as in Fig. 1 for the system ^{48}Ca+^{200}Hg. Cross sections represented by the line with open circles refer to the maximum angular momentum sustained by the hot residues formed after loss of mass and energy due to preequilibrium emission.

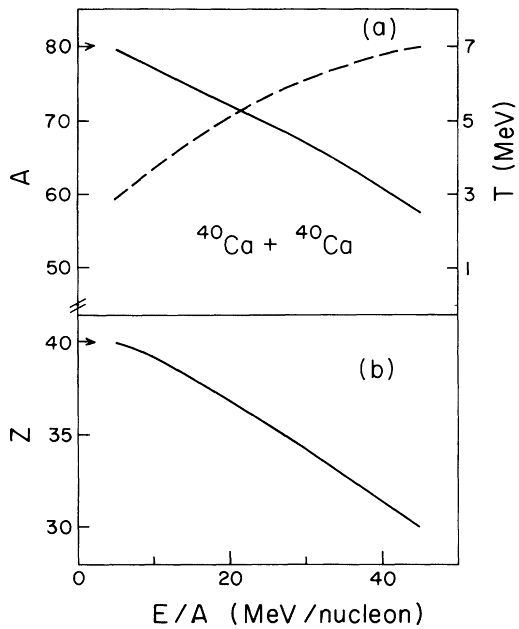

FIG. 3. Mass and charge of the hot residues formed as a function of bombarding energy for the system ^{48}Ca+^{200}Hg. The arrows on the ordinate refer to the initial mass and charge. The dashed line in (a) displays the temperature of the hot residue. Mass, charge, and temperature correspond to l_{max} as defined in the text.

For the dynamical trajectory calculations with one- and two-body PEP's leading to fusion, we follow the prescription given in Ref. [7]. In Fig. 3 we display as a function of incident energy the final mass and charge of the fused composite for the reaction ^{48}Ca+^{200}Hg corresponding to the trajectory with angular momentum l_{max}. With increasing energy, since the system loses a large number of preequilibrium nucleons, the temperature of the system is controlled by a sensitive interplay of the mass and energy carried away by them. The dashed line in Fig. 3(a) represents the temperature T of the fused residue and A and Z refer to its mass and charge. For the system ^{40}Ca+^{40}Ca, the behavior of T, A, and Z is nearly the same as for the heavier system. We note from Figs. 1 and 2 that for the system ^{40}Ca+^{40}Ca, the realistic excitation function does not differ substantially from the dot-dashed line representing schematic treatment of prompt emission, whereas for the system ^{48}Ca+^{200}Hg, they differ appreciably. The Coulomb interaction plays a very crucial role for such heavy systems. Loss of protons through particle emission dilutes the Coulomb potential, and this enhances the critical fusion energy. Moreover, since this is a highly asymmetric system, the per particle excitation energy and hence the temperature of the dinuclear complex is comparatively lower, which may also have an effect for this observation.

For the reaction ^{48}Ca+^{200}Hg, the heavy residues are formed with high temperature and generally high angular momentum and as such may become unstable against spontaneous symmetric fission. The limiting angular

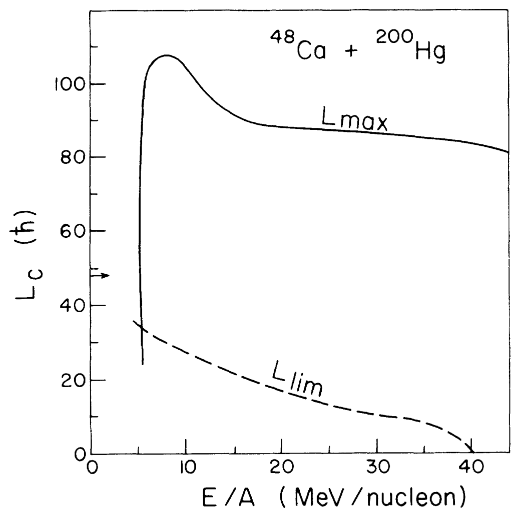

FIG. 4. Maximum angular momentum of the hot composite formed in collisions of $^{48}Ca+^{200}Hg$ at different bombarding energies. The quantity l_m refers to the maximum angular momentum corresponding to capture in the potential pocket, and $l_{\rm lim}$ refers to the limiting angular momentum corresponding to onset of spontaneous symmetric fission for the composites formed.

momentum $l_{\rm lim}$, beyond which the hot composite fissions spontaneously, is displayed in Fig. 4 as a function of incident energy by the dashed line. The temperature of the hot composite is obtained from the fusion trajectory corresponding to $l_{\rm lim}$ at the relevant bombarding energy. The arrow on the ordinate corresponds to the limiting angular momentum for $^{248}Fm(^{48}Ca+^{200}Hg)$ at zero temperature. In this figure the angular momentum $l_{\rm max}$ corresponds to the fusion excitation function given by the solid line in Fig. 2. The excitation function corresponding to $l_{\rm lim}$ is shown by the line with open circles in Fig. 2. Fusion then vanishes practically beyond 35 MeV/nucleon. Comparing the solid line with the line with open circles, it then becomes evident that fusion dynamics alone governs the fusion cross section for this heavy system only up to ~6 MeV/nucleon, beyond which fission takes over. For light systems ($A \lesssim 100$), asymmetric fission is more dominant [15] than symmetric fission. The saddle shapes for such systems are also generally triaxial [12]. These render the calculation of the limiting angular momentum ($l_{\rm lim}$) for the light system $^{40}Ca+^{40}Ca$ somewhat involved, and we have not attempted its evaluation.

To summarize, we have performed calculations of fusion excitation functions for two representative systems, namely, $^{40}Ca+^{40}Ca$ and $^{48}Ca+^{200}Hg$ with a temperature-dependent interaction potential. Emission of preequilibrium particles along the fusion path is found to have a strong influence on the fusion excitation function. For the lighter system, fusion then disappears at around 45 MeV/nucleon, close to the experimentally reported values. At such energies there are strong excitations of collective modes such as compression, but they may not be strong enough to disrupt [16] the system into several fragments at the excitation energies per particle considered here. For the heavy system, hot-composite formation persists even beyond 60 MeV/nucleon. However, such a heavy hot composite becomes unstable against spontaneous fission, and then we find that for $^{48}Ca+^{200}Hg$ fusion vanishes at ~35 MeV/nucleon. The observed disappearance of fusion near the Fermi-energy domain for both light and heavy systems may thus have a simple dynamical origin and may not necessarily be related to the bulk instability of hot nuclear matter.

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