Quasiparticle properties of the quarks of the Nambu–Jona-Lasinio model

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In spite of the apparent limitations of the model, in recent years there have been many applications of the Nambu–Jona-Lasinio (NJL) model in the study of hadron structure and in the study of the behavior of nuclear matter at finite temperature and density. A number of researchers have studied a generalized SU(3) version of the NJL model. For example, Vogl, Lutz, Klimt, and Weise [Nucl. Phys. **A516** 469 (1990)] have performed extensive calculations that include a calculation of a scalar form factor of a constituent quark, $F_s(q^2)$, and a calculation of a quark sigma term σ_q. (In their work, the latter quantity is related to the nucleon sigma term σ_N as in a constituent quark model: $\sigma_N=3\sigma_q$.) These calculations are made in what may be termed a sigma-dominance approximation. In the work reported here, we review the important role played by the nucleon sigma term in understanding the behavior of the quark condensate in the presence of matter. We make use of the original SU(2) version of the NJL model to study how various quark properties are modified when we take into account the dressing of the constituent quarks by the pion, the Goldstone boson of the model. We calculate the quark self-energy arising from emission and absorption of a pion and also show how the calculation of the scalar form factor of the quark and σ_q are modified due to the coupling of the quark to the pion. The correction terms considered here serve to reduce the value of σ_q by a small amount relative to the value obtained in the simplest version of the sigma dominance model. For example, for a Euclidean momentum cutoff, $\Lambda=1050$ MeV, the uncorrected result is $\sigma_N=54.6$ MeV. That value is then reduced to $\sigma_N=51.5$ MeV, if the corrections due to the pion "dressing" are included. It is also found that the residue at the quasiparticle pole of the quark propagator Z is about 0.86 when the coupling to the pion field is taken into account.

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I. INTRODUCTION

In this work we calculate corrections to the properties of quarks of the Nambu–Jona-Lasinio (NJL) model that arise when the dressing of quarks by the pion is taken into account. Part of our motivation is to understand how the scalar form factor of a nucleon is to be calculated, since that form factor (at zero-momentum transfer) serves to fix the value of the nucleon sigma term, σ_N. That quantity governs the behavior of the quark condensate in nuclear matter and therefore appears as a fundamental ingredient in our understanding of field-theoretic models of nuclear structure. In a recent study of the behavior of quark condensates in matter [1] it was found useful to define a condensate order parameter $\bar{\sigma}$ such that

$$\frac{\bar{\sigma}}{f_\pi}=\left[\frac{f_\pi^*}{f_\pi}\right]^2 . \tag{1.1}$$

Here f_π^* is the value of the pion decay constant in matter. Making use of the Gell-Mann-Oakes-Renner relation in nuclear matter [2,3], we had, to first order in the baryon density,

$$m_\pi^2 f_\pi \bar{\sigma}=-m_q^0\langle NM|\bar{q}q|NM\rangle \tag{1.2}$$

$$=-m_q^0(\langle 0|\bar{q}q|0\rangle+\langle N|\bar{q}q|N\rangle\rho_B) . \tag{1.3}$$

Here, $\langle 0|\bar{q}q|0\rangle=\langle 0|\bar{u}u+\bar{d}d|0\rangle$ is the vacuum value of the condensate, m_q^0 is the average current quark mass of the up and down quarks, and ρ_B is the baryon density. Further, $\langle N|\bar{q}q|N\rangle$ is the value of the scalar form factor of the nucleon at zero-momentum transfer. With the definition $\bar{\sigma}=f_\pi+\sigma$, Eq. (1.3) yields [3–5]

$$\sigma=-(\sigma_N/m_\pi^2 f_\pi)\rho_B \tag{1.4}$$

or

$$\frac{\bar{\sigma}}{f_\pi}=\left[1-\frac{\sigma_N}{m_\pi^2 f_\pi^2}\rho_B\right] . \tag{1.5}$$

Thus, we see that the rate of change of the condensate order parameter with density is governed by the nucleon sigma term $\sigma_N=m_q^0\langle N|\bar{q}q|N\rangle$. A recent analysis gives $\sigma_N=45\pm8$ MeV [6], although somewhat larger error estimates have also been given. [We note that Eq. (1.5) is a model-independent result valid at low density [3–5]. The work of Ref. [4] supports the use of this relation at nuclear matter densities.]

One may make contact with relativistic models of nuclear matter by identifying the field σ of Eq. (1.4) with the sigma field of the relativistic models [7–9]

$$\sigma\simeq-(G_{\sigma NN}/m_\sigma^2)\rho_B . \tag{1.6}$$

[In Eq. (1.6) we have neglected the small difference between the nucleon baryon and scalar densities, ρ_B and ρ_S. Further discussion of the relation between Eqs. (1.5) and (1.6) may be found in Refs. [1] and [3].] If one accepts that identification, it is clear that knowledge of the value of σ_N is important for field-theoretic models of nuclear structure. In this work we report the results of a calculation of σ_q making use of the NJL model [10]. We then follow the work of Ref. [11] and set $\sigma_N = 3\sigma_q$.

It is of interest to note that in lattice simulations of QCD it is found that the sea and valence quarks make comparable contributions to σ_N [12]. A somewhat analogous situation emerges when one studies the"dressing" of bare quarks by mesonic excitations in quark models of nucleon structure. In such models there is a two-step process to be considered. For example, we may use the NJL model and start with current quarks that have small masses (of the order of several MeV). One then solves the gap equation to determine a constituent quark mass. This *massive* quark can now be "dressed" by coupling to low-lying excitations such as the pion (the Goldstone boson) or the sigma meson of the NJL model. Other authors [4,11] have concentrated on the quark's coupling to the sigma field when calculating the scalar form factor, for example. [See Fig. 1(a)]. The approximation used may be said to define a "sigma-dominance" model for the evaluation of σ_q. Our calculation will be seen to contain additional elements. (See Fig. 1.)

We may use the matrix element of the quark scalar density taken between quarks states to define a form factor [10],

$$\langle \mathbf{p}'s'\tau=\tfrac{1}{2}|\bar{u}(0)u(0)+\bar{d}(0)d(0)|\mathbf{p},s,\tau=\tfrac{1}{2}\rangle = \bar{u}(\mathbf{p}',s')u(\mathbf{p},s)F_s((p-p')^2) \ . \quad (1.7)$$

Here s and τ are spin and isospin indices, respectively, and the $u(\mathbf{p},s)$ are Dirac spinors, with $\bar{u}(\mathbf{p},s)u(\mathbf{p},s)=1$. The normalization is such that $F_s(0)=1$, if only the first diagram in Fig. 1(a) is considered. Diagrams of the type

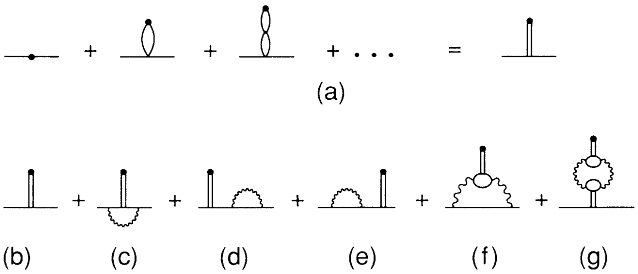

FIG. 1. Calculations of the scalar form factor of a constituent quark. (a) Here the solid dot represents the operator $\bar{u}(0)u(0)+\bar{d}(0)d(0)$. This series of terms defines a sigma-dominance approximation in which the scalar form factor is given by $F_S(0)=1+J_1=[1-G_S J_{SS}(0)]^{-1}$. (See text). Single lines denote a quark propagator. (b) The sigma dominance model for the form factor as in (a). (c) A vertex correction where the wavy line denotes a pion. This term has the value $(1+J_1)J_2$ in the notation of this work. (d) and (e) Wave-function-renormalization corrections equal to $(1+J_1)(J_3+J_4)$. (f) Correction to the scalar form factor equal to $(1+J_1)(J_5)$. The wavy line denotes a pion. (g) Correction to the scalar form factor of value $(1+J_1)(J_6)$. This appears as a self-energy correction to the sigma propagator in the sigma-dominance model.

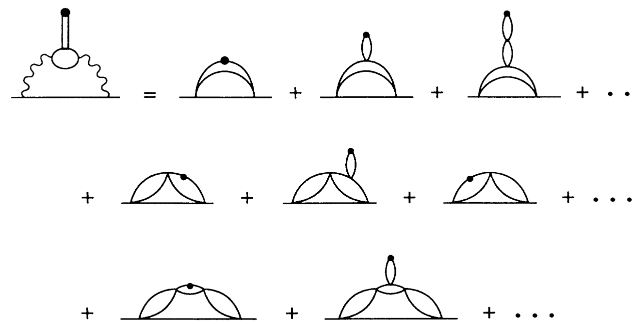

FIG. 2. The evaluation of the process shown in Fig. 1(f) is represented in term of quark propagators. The representation in terms of a pion propagator is an approximation to the sum of diagrams on the right-hand side.

shown in Fig. 1(a) were considered in Ref. [11] and may be said to stress the sigma-dominance aspect of the calculation of the quark scalar form factor, $F_s(q^2)$. Our main concern here is to go beyond the sigma-dominance calculation and to consider the role of the diagrams Figs. 1(c)–1(g). Here, Fig. 1(b) represents the diagrams summed in Fig. 1(a). Figure 1(c) represents a vertex correction which may be seen to be largely canceled by the wave-function renormalization diagrams, Figs. 1(d) and 1(e). Figures 1(f) and 1(g) are further corrections to the sigma-dominance result due to the presence of the pion "cloud." Note that Fig. 1(g) appears as a self-energy correction to the sigma propagator in the sigma-dominance model. The approximation used in Figs. 1(f), for example, can be somewhat better understood by reference to Fig. 2, where we show the process of Fig. 1(f) written out in terms of quark propagators. The wavy line (a pion propagator) denotes an approximation to the chain of quark-antiquark bubbles in the channel with the quantum numbers of the pion, etc. We remark at this point that the diagrams that we could draw in which the pion propagator is replaced by a sigma propagator are significantly smaller than the corresponding diagrams that contain the pion propagator (because of the small mass of the pion) and we drop such diagrams from further consideration.

The organization of our work is as follows. In Sec. II, we review the generation of mass via the gap equation of the NJL model. We go on to calculate corrections to the quark self-energy due to emission and absorption of pions. We also review the calculation of f_π in the NJL model and discuss the quark-quark interaction in terms of meson exchange. In Sec. III, we describe the calculation of the sigma term σ_q of a constituent quark. Finally, Sec. IV contains some further discussion and conclusions.

II. PROPERTIES OF "DRESSED" QUARKS OF THE NAMBU-JONA-LASINIO MODEL

A. The NJL MODEL

For completeness, we review some well-known features of the Nambu–Jona-Lasinio model. The Lagrangian is

TABLE I. Results of calculations for the Nambu–Jona-Lasinio model. Here Λ, m_q^0, and G_S are parameters of the model adjusted so that $f_\pi = 93$ MeV and $m_\pi = 138$ MeV. Here $F_S(0) = [1 - G_S J_{SS}(0)]^{-1}$.

Λ (MeV)	800	850	900	1000	1050
m_q^0 (MeV)	8.0	7.3	6.6	5.5	5.1
$m_q = m_q^{\rm con} + m_q^0$	390	325	295	260	245
G_S (GeV^{-2})	16.5	12.7	10.6	7.91	6.97
m_σ (MeV)	787	632	592	520	490
$1 - G_S J_{SS}(0)$	0.67	0.51	0.42	0.32	0.28
$1 - G_S J_{PP}(0)$	0.021	0.021	0.021	0.021	0.021
$g_{\sigma qq}$	3.91	3.17	2.97	2.58	2.44
$g_{\pi qq}$	3.79	3.39	3.10	2.68	2.51
f_π (MeV)	93.0	93.0	93.0	93.0	93.0
$-2m_q^0\langle 0\|\bar{u}u\|0\rangle$ (MeV^4)	1.84×10^8	1.82×10^8	1.79×10^8	1.76×10^8	1.76×10^8
$F_S(0)$	1.49	1.96	2.38	3.12	3.57
σ_q (MeV)	11.9	14.3	15.7	17.2	18.2
σ_N (MeV)	35.8	42.9	47.1	51.5	54.6

$$\mathcal{L}(x) = \bar{q}(i\not{\partial} - m_q^0)q + (G_S/2)[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] . \quad (2.1)$$

In the limit of zero current quark mass ($m_q^0 = 0$), the constituent mass $m_q^{\rm con}$ is given by

$$m_q^{\rm con} = -G_S\langle 0|\bar{q}q|0\rangle , \quad (2.2)$$

while the vacuum condensate is

$$\langle 0|\bar{q}q|0\rangle = \langle 0|\bar{u}u + \bar{d}d|0\rangle \quad (2.3)$$

$$= (-1)n_c n_f \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\frac{i}{\not{k} - m_q^{\rm con} + i\epsilon}\right] . \quad (2.4)$$

Here, the number of colors is $n_c = 3$ and the number of flavors is $n_f = 2$. Integrals such as that in Eq. (2.4) are regulated by passing to a Euclidean momentum space where the maximum value of the momentum is Λ. We can solve for G_S in terms of Λ and $m_q^{\rm con}$,

$$G_S = \frac{4\pi^2}{n_c n_f}\left[\Lambda^2 - (m_q^{\rm con})^2 \ln\left[\frac{\Lambda^2 + (m_q^{\rm con})^2}{(m_q^{\rm con})^2}\right]\right]^{-1} . \quad (2.5a)$$

In the case that $m_q^0 \neq 0$, we have

$$G_S = \frac{4\pi^2}{n_c n_f}\left[\frac{m_q^{\rm con}}{m_q}\right]\left[\Lambda^2 - m_q^2 \ln\left[\frac{\Lambda^2 + m_q^2}{m_q^2}\right]\right]^{-1} , \quad (2.5b)$$

where $m_q \equiv m_q^{\rm con} + m_q^0$. (See Table I.) In Figs. 3 and 4 we show G_S and $\langle 0|\bar{u}u|0\rangle$ as functions of $m_q^{\rm con}$ for $m_q^0 = 0$. Note that a typical value of the condensate obtained from QCD sum-rule studies is

$$\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = (-240 \pm 25 \text{ MeV})^3 .$$

Now we follow the authors of Refs. [4] and [11] and define a *quark* sigma term for an up quark, for example,

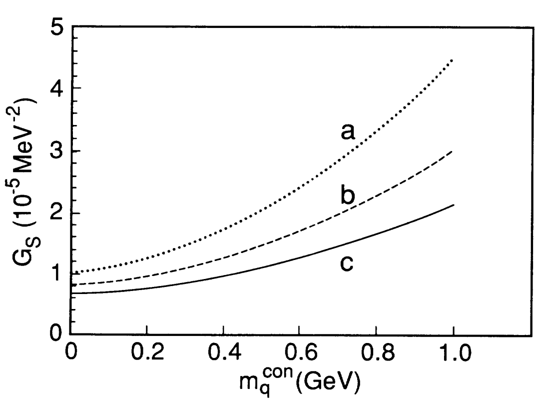

FIG. 3. The value of the coupling constant G_S is shown as a function of the constituent quark mass for the case $m_q^0 = 0$. (a) $\Lambda = 0.8$ GeV, (b) $\Lambda = 0.9$ GeV, (c) $\Lambda = 1.0$ GeV.

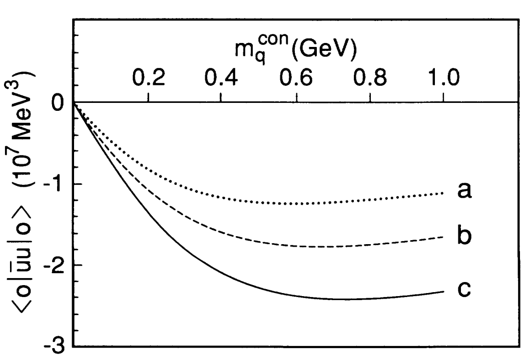

FIG. 4. The value of the quark condensate $\langle 0|\bar{u}(0)u(0)|0\rangle$ is given as a function of the constituent quark mass for the case $m_q^0 = 0$. (a) $\Lambda = 0.8$ GeV, (b) $\Lambda = 0.9$ GeV, (c) $\Lambda = 1.0$ GeV.

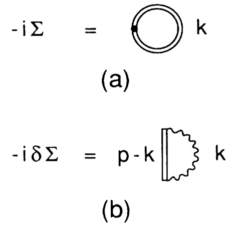

FIG. 5. (a) Nonlinear equation for the quark self-energy. The double line denotes the propagator $iS(k)=i(\not k-\Sigma+i\epsilon)^{-1}$. Here $m_q^0=0$. Expressions for finite values of m_q^0 are given in the text. (b) Self-energy correction for the quark propagator calculated with the value of $m_q^{\rm con}$ determined in (a). Here the wavy line denotes a pion. [See Eqs. (2.21) and (2.22).]

$$\sigma_q=m_q^0\langle \mathbf{p},s,\tfrac{1}{2}|\bar{q}q|\mathbf{p},s,\tfrac{1}{2}\rangle \tag{2.6}$$

$$=2m_q^0\langle \mathbf{p},s,\tfrac{1}{2}|\frac{\bar{u}u+\bar{d}d}{2}|\mathbf{p},s,\tfrac{1}{2}\rangle \ . \tag{2.7}$$

The authors of Refs. [4] and [11] set $\sigma_N=3\sigma_q$, as is appropriate in a constituent quark model of nucleon structure. We will use the same definition here, although corrections to that relation may be considered.

B. The quark self-energy

In this subsection we discuss corrections to the quark propagator due to the emission and absorption of a pion. We will calculate these corrections using perturbation theory, although much more complex calculations may be envisioned.

In the NJL model, the quark propagator is

$$iS_0(p)=\frac{i}{\not p-(m_q^{\rm con}+m_q^0)+i\epsilon} \ . \tag{2.8}$$

We now consider a more general form of the self-energy so that

$$iS(p)=\frac{i}{\not p-[\tilde{\Sigma}(p)+m_q^0]+i\epsilon} \ . \tag{2.9}$$

Here $\tilde{\Sigma}(p)$ is to be obtained by considering both the mean-field vacuum contribution and the emission and absorption of a pion. (See Fig. 5.) One might attempt to solve an equation of the form

$$-i\tilde{\Sigma}(p)=(-1)iG_S n_f n_c\int\frac{d^4k}{(2\pi)^4}{\rm Tr}\left[\frac{i}{\not k-\tilde{m}_q+i\epsilon}\right] -3g_{\pi qq}^2\int\frac{d^4k}{(2\pi)^4}\gamma_5\frac{1}{\not p-\not k-\tilde{m}_q+i\epsilon} \times\gamma_5\frac{1}{k^2-m_\pi^2+i\epsilon} \ . \tag{2.10}$$

(Here the quantities with a tilde refer to the NJL model extended to include the effects of the pion dressing of the quark propagator.) We have not used Eq. (2.10), since there are a number of problems associated with such an equation. For example, in the absence of the second term on the right-hand side in Eq. (2.10) (that is, for the standard gap equation) there is a close relation between the gap equation and the Bethe-Salpeter equation that yields the pion mass. That relation is lost, if one tries to solve

TABLE II. Results obtained for the NJL model including effects of pion dressing of the quark. Here quantities without a tilde are unchanged from Table I. In this calculation m_π is fixed to be 138 MeV.

Λ (MeV)	800	850	900	1000	1050
m_q^0 (MeV)	8.0	7.3	6.6	5.5	5.1
$\tilde{m}_q$ (MeV)	470	388	349	302	281
G_S (GeV^{-2})	16.5	12.7	10.6	7.91	6.97
$\tilde{m}_\sigma$ (MeV)	969	793	714	616	547
$1-G_S\tilde{J}_{SS}(0)$	0.825	0.64	0.54	0.40	0.33
$1-G_S\tilde{J}_{PP}(0)$	0.021	0.021	0.021	0.021	0.021
$\tilde{g}_{\sigma qq}$	4.33	3.53	3.16	2.74	2.51
$g_{\pi qq}$	3.79	3.39	3.10	2.68	2.51
$\tilde{f}_\pi$ (MeV)	95.0	97.4	98.8	99.7	99.2
$-2m_q^0\langle 0\|\bar{u}u\|0\rangle$ (MeV^4)	1.94×10^8	1.97×10^8	1.97×10^8	1.95×10^8	1.94×10^8
$m_\pi^2\tilde{f}_\pi^2$ (MeV^4)	1.72×10^8	1.81×10^8	1.86×10^8	1.90×10^8	1.88×10^8
$1+J_1$	1.21	1.56	1.85	2.50	3.03
J_2			0.19	0.17	0.16
J_3+J_4			−0.18	−0.16	−0.15
J_5			−0.126	−0.076	−0.058
J_6			0.32	0.20	0.16
$F_S(0)$			2.23	2.84	3.37
σ_q (MeV)			14.7	15.6	17.2
σ_N (MeV)			44.1	46.8	51.5

Eq. (2.10). Somehow Eq. (2.10) would have to be related to an equation that determines the pion mass, such that we would have $m_\pi=0$, if $m_q^0=0$. Since we have not be able to reformulate the theory such that Eq. (2.10) could be used, we chose a perturbative approach. In the perturbative approach, we first solve the standard gap equation, as well as the Bethe-Salpeter equation, with $m_q^0\neq 0$. In that way, we obtain a finite value for the pion mass. Once the pion is made massive through explicit symmetry braking ($m_q^0\neq 0$), we calculate various corrections containing pion loops. This procedure has some justification, *a posteriori,* in that the effects we calculate using perturbation theory are small. (See Tables I and II.)

We now consider a perturbative correction to the quark mass and write

$$-i\tilde{\Sigma}(p)\simeq(-1)iG_S n_f n_c i\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\frac{i}{\not k=(m_q^{\rm con}+m_q^0)+i\epsilon}\right]$$

$$-3g_{\pi qq}^2\int\frac{d^4k}{(2\pi)^4}\gamma_5\frac{1}{\not p-\not k-(m_q^{\rm con}+m_q^0)+i\epsilon}\gamma_5\frac{1}{k^2-m_\pi^2+i\epsilon} \tag{2.11}$$

$$\simeq -i[m_q^{\rm con}+\delta\Sigma(p)]\ , \tag{2.12}$$

where $m_q^{\rm con}$ is the constituent mass obtained in the mean-field approximation. Here, $-i\delta\Sigma(p)$ is given by the second term in Eq. (2.11). It is also useful to define the functions $A(p^2)$ and $B(p^2)$ such that

$$\delta\Sigma(p)=A(p^2)+B(p^2)\not p \tag{2.13}$$

$$=(\not p-\tilde{m}_q)B(p^2)+\tilde{A}(p^2)\ . \tag{2.14}$$

Then, with $m_q=m_q^{\rm con}+m_q^0$,

$$iS(p)=\frac{i}{[\not p-m_q-\tilde{A}(p^2)]-(\not p-\tilde{m}_q)B(p^2)} \tag{2.15}$$

$$=\frac{i}{(\not p-\tilde{m}_q)[1-B(p^2)]-[\tilde{A}(p^2)-\tilde{A}(\tilde{m}_q^2)]} \tag{2.16}$$

$$=\frac{i[1-B(p^2)]^{-1}}{\not p-\tilde{m}_q-\Sigma_f(p^2)}\ . \tag{2.17}$$

Here, $\tilde{m}_q=m_q+\tilde{A}(\tilde{m}_q^2)$ and

$$\Sigma_f(p^2)=-\frac{\tilde{A}(\tilde{m}_q^2)-\tilde{A}(p^2)}{1-B(p^2)}\ . \tag{2.18}$$

(See Fig. 6). We then have, with $Z(p^2)\equiv[1-B(p^2)]^{-1}$,

$$S(p)=\frac{Z(p^2)}{\not p-m_q-\Sigma_f(p^2)+i\epsilon}\ . \tag{2.19}$$

Now, near the pole ($\not p=\tilde{m}_q$), we have

$$S(p)|_{\not p\to\tilde{m}_q}=\frac{Z(\tilde{m}_q^2)}{\not p-\tilde{m}_q+i\epsilon}\ , \tag{2.20}$$

since $\Sigma_f(\tilde{m}_q^2)=0$. (See Fig. 6.)

We now turn to the evaluation of $\delta\Sigma(p)$. We have

$$i\delta\Sigma(p)=3g_{\pi qq}^2 i\int\frac{d^4k}{(2\pi)^4}\gamma_5\frac{\not p-\not k-m_q}{(p-k)^2-m_q^2+i\epsilon}\gamma_5\frac{1}{k^2-m_\pi^2+i\epsilon} \tag{2.21}$$

$$=-3g_{\pi qq}^2\int_0^1 dx\int\frac{d^4k}{(2\pi)^4}\frac{(\not k-\not p+m_q)}{\{[(p-k)^2-m_q^2]x+(1-x)(k^2-m_\pi^2)\}^2}\ , \tag{2.22}$$

where we have used a Feynman parametrization of the integral. We find

$$A(p^2)=\frac{3g_{\pi qq}^2}{16\pi^2}m_q\int_0^1 dx\, x\left[\ln\left[\frac{\Lambda^2+E}{E}\right]-\left[\frac{\Lambda^2}{\Lambda^2+E}\right]\right] \tag{2.23}$$

and

$$B(p^2)=\frac{3g_{\pi qq}^2}{16\pi^2}\int_0^1 dx(x-1)\left[\ln\left[\frac{\Lambda^2+E}{E}\right]-\left[\frac{\Lambda^2}{\Lambda^2+E}\right]\right], \tag{2.24}$$

with

$$E=p^2x^2-(p^2-m_q^2)+m_\pi^2(1-x)\ . \tag{2.25}$$

In Figs. 7 and 8 we show $A(p^2)$ and $B(p^2)$ for various values of Λ. Figure 9 exhibits $Z(p^2)=[1-B(p^2)]^{-1}$ as a function of p^2 for $\Lambda=1$ GeV. For the results shown in Figs. 7–9 we have put $m_q=312$ MeV, $m_\pi=140$ MeV, and $g_{\pi qq}=3.0$. The value of $B(p^2)$ at the pole is $B(\tilde{m}_q^2)\simeq-0.16$, which yields $Z(\tilde{m}_q^2)\approx 0.86$. (The value of B at the pole will be needed in Sec. III.) Values of $\tilde{m}_q$ are given in Table II, and by comparing Tables I and II, we see that $\tilde{m}_q$ is shifted upward from $m_q=m_q^{\rm con}+m_q^0$ by about 40 MeV for $\Lambda\sim 1$ GeV. We see that the inclusion of pion "dressing" leads to values of $\tilde{m}_q$ that are similar to those used in the constituent quark model, if 900 MeV $<\Lambda<$ 1000 MeV. (See Table II.)

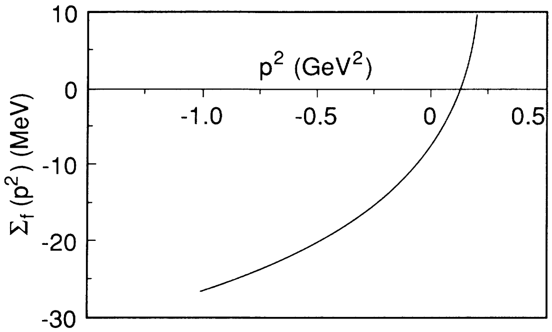

FIG. 6. The quantity $\Sigma_f(p^2)$ is shown as a function of p^2. Note that $\Sigma_f(\tilde{m}_q^2)=0$. Here $\Lambda=1$ GeV, $m_q^0=0$, $m_q^{\rm con}=312$ MeV, and $g_{\pi qq}=3$.

C. The quark-quark interaction

We note that the quarks of the NJL model may interact by exchanging a number of $\bar{q}q$ pairs, as shown in Fig. 10. In the scalar-isoscalar channel we see that the quark-antiquark scattering amplitude M is proportional to $G_S/[1-G_SJ_{SS}(q^2)]$, while in the case $\bar{q}q$ pairs with pion quantum numbers are exchanged, the amplitude is proportional to $G_S/[1-G_SJ_{PP}(q^2)]$. [We have used the notation of Ref. [4] for the quantities $J_{SS}(q^2)$ and $J_{PP}(q^2)$.] Inspection of Fig. 10 leads to the expression

$$J_{SS}(q^2)=(-1)i^3n_cn_f\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[S\left[k+\frac{q}{2}\right]S\left[k-\frac{q}{2}\right]\right] \tag{2.26}$$

$$=4in_cn_f\int_0^1dx[K_1(A)+2AK_2(A)]\,, \tag{2.27}$$

where

$$K_1(A)=-\frac{i}{16\pi^2}\left[\Lambda^2-A\ln\left[\frac{\Lambda^2+A}{A}\right]\right], \tag{2.28}$$

$$K_2(A)=\frac{i}{16\pi^2}\left[\ln\left[\frac{\Lambda^2+A}{A}\right]-\left[\frac{\Lambda^2}{\Lambda^2+A}\right]\right], \tag{2.29}$$

and $A=m_q^2-x(1-x)q^2$. In the case of the pion, we have

$$J_{PP}(q^2)=(-1)i^3n_cn_f\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma_5S\left[k+\frac{q}{2}\right]\gamma_5S\left[k-\frac{q}{2}\right]\right] \tag{2.30}$$

$$=4in_cn_f\int_0^1dx[K_1(A)+(2A-2m_q^2)K_2(A)]\,, \tag{2.31}$$

with K_1, K_2, and A as defined as above. The quantities $[1-G_SJ_{SS}(q^2)]$ and $[1-G_SJ_{PP}(q^2)]$ are shown in Figs. 11 and 12. Note that $[1-G_SJ_{SS}(q^2)]$ has a zero at $q^2=m_\sigma^2$ and the zero of $[1-G_SJ_{PP}(q^2)]$ is at $q^2=m_\pi^2$.

It is of interest to describe the interaction of a $\bar{q}q$ or a qq pair as proceeding through the interchange of pion or sigma fields. Therefore, we introduce momentum-dependent coupling parameters, $g_{\sigma qq}(q^2)$ and $g_{\pi qq}(q^2)$:

$$\frac{g_{\sigma qq}^2(q^2)}{q^2-m_\sigma^2}=-\frac{G_S}{1-G_SJ_{SS}(q^2)} \tag{2.32}$$

and

$$\frac{g_{\pi qq}^2(q^2)}{q^2-m_\pi^2}=-\frac{G_S}{1-G_SJ_{PP}(q^2)}\,. \tag{2.33}$$

We further define the coupling constants at $q^2=0$:

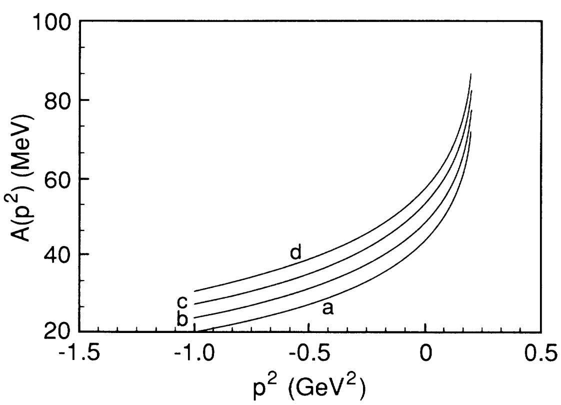

FIG. 7. The figure represents $A(p^2)$ for various values of Λ. Here $m_q^0=0$, $m_q=312$ MeV, and $g_{\pi qq}=3$. (a) $\Lambda=0.9$ GeV, (b) $\Lambda=1.0$ GeV, (c) $\Lambda=1.1$ GeV, (d) $\Lambda=1.2$ GeV.

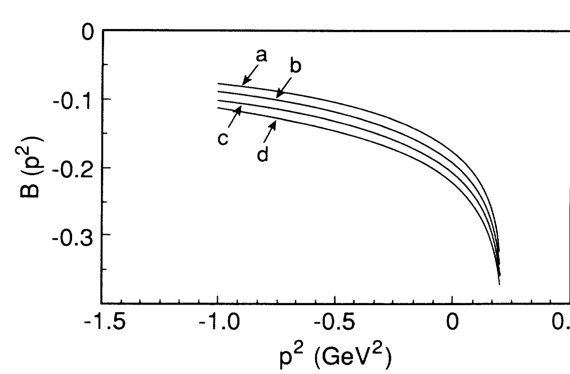

FIG. 8. The figure represents $B(p^2)$ for various values of Λ. Here $m_q^0=0$, $m_q=312$ MeV, and $g_{\pi qq}=3$. (a) $\Lambda=0.9$ GeV, (b) $\Lambda=1.0$ GeV, (c) $\Lambda=1.1$ GeV, (d) $\Lambda=1.2$ GeV.

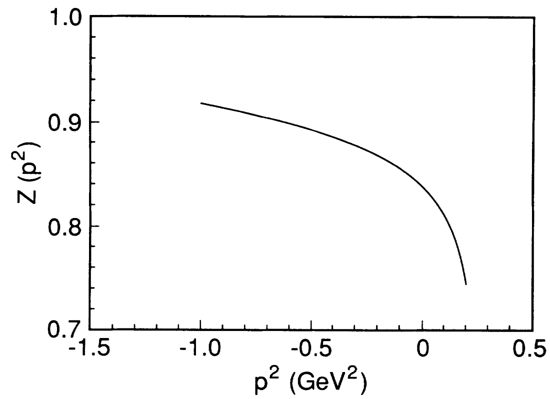

FIG. 9. The quantity $Z(p^2)=[1-B(p^2)]^{-1}$ is shown as a function of p^2. Here $\Lambda=1$ GeV, $m_q^0=0$, $m_q^{\rm con}=312$ MeV, and $g_{\pi qq}=3$.

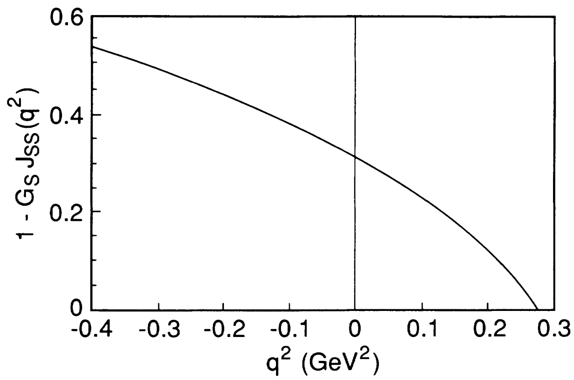

FIG. 11. The quantity $1-G_S J_{SS}(q^2)$ is shown as a function of q^2. Here $\Lambda=1$ GeV, $m_q=m_q^{\rm con}+m_q^0=258$ MeV, $m_\sigma=525$ MeV, and $m_q^0=5.5$ MeV. Note that $[1-G_{SS}J_{SS}(m_\sigma^2)]=0$.

$$g_{\sigma qq}=m_\sigma\left[\frac{G_S}{1-G_S J_{SS}(0)}\right]^{1/2} \tag{2.34}$$

and

$$g_{\pi qq}=m_\pi\left[\frac{G_S}{1-G_S J_{PP}(0)}\right]^{1/2} . \tag{2.35}$$

Values of the quantities defined in Eqs. (2.34) and (2.35) are to be found in Table I. Inspection of Figs. 11 and 12 show that to a good approximation $g_{\pi qq}(q^2)$ may be replaced by $g_{\pi qq}=g_{\pi qq}(0)$. The situation is somewhat different in the case of the scalar channel where $g_{\sigma qq}(q^2)$ has some dependence on q^2. [It is worth noting that, if we consider the T matrix for scalar exchange in the vicinity of $q^2=0$, the corresponding force would be of somewhat shorter range than that obtained from $(m_\sigma)^{-1}$.]

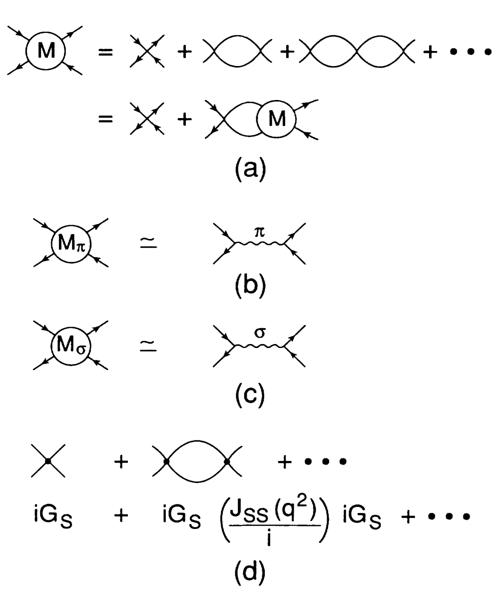

FIG. 10. (a) The equation determining the quark-antiquark scattering matrix in the NJL model is shown. (b) Quark-antiquark scattering amplitude approximated by an s-channel pole (pion). (c) Quark-antiquark scattering amplitude approximated by a sigma s-channel pole. (d) Consideration of the first two terms of the series in (a) allow us to identify the quantity $J_{SS}(q^2)$. (We use the notation of Refs. [4] and [10].)

Once we have calculated $g_{\sigma qq}$, we can ask how one is calculate the sigma field in nuclear matter. In Sec. I we have presented two ways such a calculation may be performed that are represented by Eqs. (1.4) and (1.6). For example, with $\sigma_N=50$ MeV, Eq. (1.4) yields $\sigma=-37$ MeV, while with $G_{\sigma NN}=9.45$ and $m_\sigma=550$ MeV, Eq. (1.6) yields $\sigma=-41$ MeV. (The values used here for $G_{\sigma NN}$ and m_σ are typical of values determined in applications of the one-boson-exchange model of nuclear forces.)

We now consider a third calculation that makes use of $g_{\sigma qq}$. In describing how $g_{\sigma qq}$ is to be used, it is important to avoid double counting. By study of a diagrammatic representation of the processes contributing to sigma exchange, one sees that the simplest calculation would have three valence quarks in the nucleon as the source of the sigma field. Thus, we would have

$$\sigma=-(g_{\sigma qq}/m_\sigma^2)\langle N|\bar{q}q|N\rangle_{\rm val}\rho_B \tag{2.36}$$

with $\langle N|\bar{q}q|N\rangle_{\rm val}\simeq 3$. Then using $g_{\sigma qq}=2.75$, which is

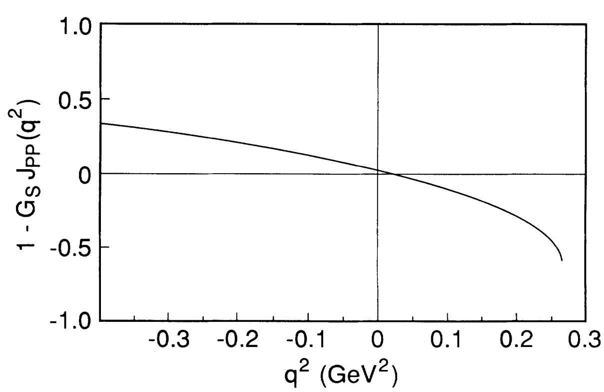

FIG. 12. The quantity $[1-G_S J_{PP}(q^2)]$ is shown as a function of q^2. Here $\Lambda=1$ GeV, $m_q=m_q^{\rm con}+m_q^0=258$ MeV, and $m_\pi=138$ MeV. Note that $[1-G_S J_{PP}(m_\pi^2)]=0$.

the value for $\Lambda=1$ GeV given in Table II, we find $\sigma=-36$ MeV. That is in general agreement with the values of -37 and -47 MeV quoted above. (The use of $\langle N|\bar{q}q|N\rangle_{\rm val}=3$ is in correspondence with those applications made using the constituent quark model, where one sets $G_{\sigma NN}=3g_{\sigma qq}$). Note that the values of σ_N calculated for $900<\Lambda<1050$ MeV (see Table I) are in general accord with the parametrization of the sigma field in Dirac phenomenology [9], relativistic Brueckner Hartree-Fock [7] and the one-boson-exchange model of nuclear forces.

D. Calculation of f_π

We calculate f_π using the relation

$$f_\pi p^\mu=-iN\sqrt{n_c/2}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma^5\gamma^\mu S\left[k+\frac{p}{2}\right]\gamma^5 S\left[k-\frac{p}{2}\right]\right], \tag{2.37}$$

where p^μ is the momentum of the pion, N is a normalization constant, and $n_c=3$. (See Fig. 13.) This expression may be evaluated in the point rest frame, where $p^\mu=(m_\pi,0,0,0)$, to obtain

$$f_\pi=-4iNm_q\sqrt{n_c/2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{[(k+p/2)^2-m_q^2][(k-p/2)^2-m_q^2]} \tag{2.38}$$

$$=N\frac{m_q}{4\pi^2}\sqrt{n_c/2}\int_0^1 dx\left[\ln\left[\frac{\Lambda^2+A}{A}\right]-\left[\frac{\Lambda^2}{\Lambda^2+A}\right]\right] \tag{2.39}$$

with

$$A(p^2)=m_q^2-x(1-x)p^2\ , \tag{2.40a}$$

$$A(m_\pi^2)=m_q^2-x(1-x)m_\pi^2\ . \tag{2.40b}$$

The normalization parameter N is obtained by evaluating a form factor (see Fig. 13). After evaluating the isospin factors, we have

$$\mathcal{F}(q^2)(p'+p)^\mu=|N|^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[S(k-p/2)\gamma_5 S(k+p/2)\gamma^\mu S(k+q+p/2)\gamma^5]\ . \tag{2.41}$$

Setting $\mathcal{F}(0)=1$, we obtain

$$\frac{1}{N^2}=i\int\frac{d^4k}{(2\pi)^4}\frac{1}{D}\left[\left[3m_q^2-3k^2+\frac{p^2}{4}-p\cdot k\right]+\frac{2p\cdot k}{p^2}\left[m_q^2-k^2+\frac{3}{4}p^2+p\cdot k\right]\right], \tag{2.42}$$

where, with $l=k+p/2$, $D=(l^2-m_q^2)^2[(p-l)^2-m_q^2]$. Finally,

$$\frac{1}{N^2}=\frac{1}{8\pi^2}\int_0^1 dx\,x\left\{\frac{\Lambda^4}{A(\Lambda^2+A)^2}\left[m_q^2(2-x)+p^2x(1-x)^2-\left[2-\frac{3x}{2}\right]\right]+\left[\ln\left[\frac{\Lambda^2+A}{A}\right]-\left[\frac{\Lambda^2}{\Lambda^2}+A\right]\right](4-3x)\right\}, \tag{2.43}$$

where $A(m_\pi^2)$ is given in Eq. (2.40b).

III. QUARK SCALAR FORM FACTOR AT ZERO-MOMENTUM TRANSFER

In this section we describe the calculation of the scalar form factor of a constituent quark at zero-momentum transfer. We denote that quantity as $F_S(0)$ and write

$$F_S(0)=(1+J_1)(1+J_2+J_3+J_4+J_5+J_6)\ . \tag{3.1}$$

Here, $(1+J_1)=[1-G_SJ_{SS}(0)]^{-1}$. The other terms in Eq. (3.1) will be defined below.

A. A sigma-dominance model

The quantity $(1+J_1)$ represents the contribution to $F_S(0)$ of the sum of the diagrams shown in Fig. 1(a). We may define a sigma-dominance model by relating $[1-G_SJ_{SS}(q^2)]$ to the propagator for a sigma meson. Thus we have, using Eq. (2.32),

$$\frac{1}{1-G_SJ_{SS}(q^2)}=-\frac{g_{\sigma qq}(q^2)}{G_S}\left[\frac{i}{q^2-m_\sigma^2+i\epsilon}\right]\times[-ig_{\sigma qq}(q^2)]\ . \tag{3.2}$$

from which we can infer how to obtain matrix elements of the quark scalar density from Feynman diagrams containing a sigma propagator. [See Fig. 1(a), for example.] Equation (3.2) is consistent with the observation that in a bosonization of the NJL model we would introduce the field

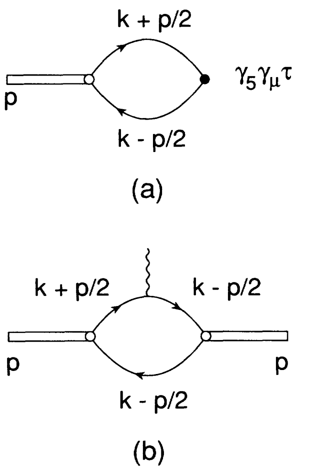

FIG. 13. (a) Calculation of the pion decay constant. Here the double line denotes an on-mass-shell pion. (b) Calculation of a pion form factor that serves to determine the normalization constant in the pion vertex function for $\pi \to q+\bar{q}$.

$$\bar{\sigma} = -\frac{G_S}{g_{\sigma qq}}(\bar{u}u+\bar{d}d) \ . \tag{3.3}$$

In vacuum, Eq. (3.3) would read

$$g_{\sigma qq} f_\pi = -G_S \langle 0|\bar{u}u+\bar{d}d|0\rangle \ . \tag{3.4}$$

The right-hand side of Eq. (3.4) was used to define the constituent mass $m_q^{\rm con}$. With that in mind, we see that we obtain the Goldberger-Treiman relation $m_q^{\rm con} = g_{\sigma qq} f_\pi$ from Eq. (3.4). Recalling that $\bar{\sigma} = f_\pi + \sigma$, we obtain from Eq. (3.3) the mean-field relation in nuclear matter

$$\sigma = -(G_S/g_{\sigma qq})\langle N|\bar{u}u+\bar{d}d|N\rangle \rho_B \ . \tag{3.5}$$

If we recall Eq. (1.4), we see that we also have the relation

$$\frac{G_S}{m_q^0 g_{\sigma qq}} = \frac{1}{m_\pi^2 f_\pi} \tag{3.6}$$

among the parameters of the model if the Gell-Mann-Oakes-Renner relation is valid.

We now consider the calculation of $(1+J_1)$. As noted above

$$1+J_1 = \frac{1}{1-G_S J_{SS}(0)} \ , \tag{3.7}$$

where we have (see Fig. 10)

$$G_S J_{SS}(0) = I_1 n_c n_f (-1) i^2 (iG_S) \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[S(k)S(k)] \ . \tag{3.8}$$

Here $I_1 = 1$ is the isospin factor.

We find

$$G_S J_{SS}(0) = 4 n_c n_f G_S i \int_0^1 dx [K_1(\tilde{m}_q^2) + 2\tilde{A} K_2(\tilde{m}_q^2)] \ , \tag{3.9}$$

with

$$K_1(A) = -\frac{i}{16\pi^2}\left[\Lambda^2 - \tilde{A}\ln\left[\frac{\Lambda^2+\tilde{A}}{\tilde{A}}\right]\right] \ , \tag{3.10}$$

$$K_2(\tilde{A}) = \frac{i}{16\pi^2}\left[\ln\left[\frac{\Lambda^2+\tilde{A}}{\tilde{A}}\right] - \left[\frac{\Lambda^2}{\Lambda^2+\tilde{A}}\right]\right] \ , \tag{3.11}$$

and $\tilde{A} = \tilde{m}_q^2 - x(1-x)q^2$.

If we neglect pion dressing, the results of the application of the sigma-dominance model are to be found in Table I, with $\tilde{m}_q$ to be replaced by m_q in Eqs. (3.8)–(3.11). If we include pion dressing $m_q \to \tilde{m}_q$. Values for $\tilde{m}_q$ and $(1+J_1)$ may be found in Table III. [The values of $(1+J_1)$ of Table II may be compared with the values given for $F_S(0)$ in Table I.]

B. Pion dressing of a constituent quark

We now go beyond sigma-dominance model and consider the diagrams in Figs. 1(c)–1(f). The contribution of Fig. 1(c) is $(1+J_1)J_2$, while that of Figs. 1(d) and 1(e) are $(1+J_1)J_3$ and $(1+J_1)J_4$. Note that these three contributions to $F_S(0)$ largely cancel. (See Table II.) The contribution of Fig. 1(f) is equal to $(1+J_1)J_5$ and that of Fig. 1(g) is $(1+J_1)J_6$. We will consider the calculation of these various contributions to $F_S(0)$ in order.

1. Calculation of J_2

We here are concerned with the evaluation of the diagram of Fig. 1(c). We have

$$J_2 = -iI_2 g_{\pi qq}^2 \int d^4k\, \bar{u}(\mathbf{p},s) \times[\gamma_5 D(k)\gamma_5 S(p+k)S(p+k)] \times u(\mathbf{p},s) \ , \tag{3.12}$$

where $D(k) = (k^2 - m_\pi^2 + i\epsilon)^{-1}$ and I_2 is an isospin factor,

$$I_2 = \mathrm{Tr}\left[\tau_a\tau_a\left[\frac{1+\tau_3}{2}\right]\right] \tag{3.13a}$$

$$= 3 \ . \tag{3.13b}$$

We have

$$J_2 = I_2 g_{\pi qq}^2 (-i) 2\int_0^1 dx\, x\{[L+(2-x)^2\tilde{m}_q^2]K_3(L) + K_2(L)\} \ , \tag{3.14}$$

with

$$L = \tilde{m}_q^2 x^2 + (1-x)m_\pi^2 \tag{3.15}$$

and

$$K_3(L) = -\frac{i}{32\pi^2}\left[\frac{1}{L} - \frac{2\Lambda^2}{(\Lambda^2+L)^2} - \frac{L}{(\Lambda^2+L)^2}\right] \ . \tag{3.16}$$

Values of $g_{\pi qq}$, $\tilde{m}_q$, and J_2 are to be found in Table II.

2. Calculation of J_3+J_4

Here we are concerned with the evaluation of Figs. 1(d) and 1(e). These diagrams are usually called wave-function-renormalization diagrams and their value may be obtained from the knowledge of the quark self-energy. We recall that we had written $\delta\Sigma(p)=B(p^2)\not p+A(p^2)$. Then both J_3 and J_4 are equal to $B(\tilde m_q^2)/2$. Values obtained for J_3+J_4 may be found in Table II. [Note that at $\Lambda=1$ GeV, we have $Z=(1-B)^{-1}=0.86$ at the pole.] As noted earlier, $J_2+J_3+J_4\simeq 0$, as might be expected, if we recall the calculation of vertex corrections and wave-function-renormalization corrections in QED.

3. Calculation of J_5

Here we consider the calculation of the diagram of Fig. 1(f). As a first step we can evaluate the fermion loop, assuming the pion has momentum k. We define

$$H(k^2)=-2i^3g_{\pi qq}^2\int\frac{d^4l}{(2\pi)^4}\mathrm{Tr}[\gamma_5S(l)S(l)\gamma_5S(l-k)]\,, \tag{3.17}$$

where 2 is a symmetry factor. Then

$$J_5=I_5(-i)g_{\pi qq}^2\bar u(\mathbf{p},s)\times\int\frac{d^4k}{(2\pi)^4}[\gamma_5D(k)H(k^2)D(k)S(p+k)\gamma_5]u(\mathbf{p},s) \tag{3.18}$$

where I_5 is the isospin factor,

$$I_5=\mathrm{Tr}(\tau_a\tau_b)\mathrm{Tr}\left[\tau_b\tau_a\left[\frac{1+\tau_3}{2}\right]\right] \tag{3.19a}$$

$$=6\,. \tag{3.19b}$$

Now

$$H(k^2)=2g_{\pi qq}^2 i\int\frac{d^4l}{(2\pi)^4}\frac{\mathrm{Tr}[(\tilde m_q-\not l)(\tilde m_q-\not l)(\tilde m_q+\not l-\not k)]}{(l^2-\tilde m_q^2+i\epsilon)^2[(l-k)^2-\tilde m_q^2+i\epsilon]} \tag{3.20}$$

$$=16g_{\pi qq}^2 i\int_0^1 dx\, x\{[\tilde m_q^3+\tilde m_q(1-x^2)k^2]K_3(\tilde A)-\tilde m_q[K_2(\tilde A)+\tilde A K_3(\tilde A)]\}\,, \tag{3.21}$$

where $K_2(\tilde A)$ and $K_3(\tilde A)$ were defined previously. Note that $\tilde A=\tilde m_q^2-x(1-x)k^2$.

We find that we can approximate $H(k^2)$ by

$$H(k^2)=8g_{\pi qq}^2\sum_{i=1}^{n}\frac{a_i}{1-k^2/\Lambda_i^2}\,. \tag{3.22}$$

For example, with $n=2$, we find $a_1=3.34$, $a_2=-0.34$, $\Lambda_1=707$ MeV and $\Lambda_2=3162$ MeV for the case $\Lambda=1$ GeV, $m_q=260$ MeV. Then

$$J_5=iI_5g_{\pi qq}^2\bar u(\mathbf{p},s)\int\frac{d^4k}{(2\pi)^4}\frac{\not k H(k^2)}{(k^2-m_\pi^2-i\epsilon)^2[(p+k)^2-\tilde m_q^2+i\epsilon]}u(\mathbf{p},s) \tag{3.23}$$

$$=8iI_5g_{\pi qq}^4\sum_{i=1}^{n}\tilde m_q a_i\Lambda_i^2 6\int_0^1 x\,dx\int_0^{1-x}dy\,K_4(B)(1-y)\,, \tag{3.24}$$

with

$$B=y^2\tilde m_q^2+xm_\pi^2+(1-x-y)\Lambda_i^2 \tag{3.25}$$

and

$$K_4(B)=\frac{i}{96\pi^2}\left[\frac{1}{B^2}-\frac{3\Lambda^2+B}{(\Lambda^2+B)^3}\right]\,. \tag{3.26}$$

Values obtained for J_5 are to be found in Table II.

4. Calculation of $(1+J_1)J_6$

The calculation of the diagram of Fig. 1(g) requires the evaluation of a three-loop integral. Rather than proceeding in that manner, we have calculated the discontinuity across the two-pion cut and then have calculated the real part of the diagram by means of a dispersion relation. We neglect the quark-antiquark cut that starts at $q^2=(2m_q)^2$. The two-pion cut for $q^2>(2m_q)^2$ is treated in an approximate manner. (Since we require the real part of the diagram at $q^2=0$, the result is not particularly sensitive to the treatment at large q^2.) The details of this calculation will be reported in another publication. The results obtained here appear in Table II.

We remark that there is some cancellation seen between the values of J_5 and J_6 given in Table II. Note that, if the pion momenta in diagrams of Figs. 1(f) and 1(g) took on only small values, there would be almost a complete cancellation between the values of J_5 and J_6. That observation may be understood by dividing the diagrams of Figs. 1(f) and 1(g) in two parts by drawing a horizontal line, cutting the two pion lines. The upper parts of the two diagrams are then identical. The lower parts of the diagrams are related to the calculation of off-mass-shell pion-quark scattering in a linear sigma model. Explicit evaluation of the lower parts of the two diagrams does show an exact cancellation if the pion momentum $k^\mu=0$. Corrections of order k^2/m_q^2 appear, however. In our analysis this cancellation is only partial, since the pion momenta are not particularly small. These pion momenta are limited by the single-quark loops in Figs. 1(f) and 1(g), which serve as form factors for the amplitude $\sigma \rightarrow \pi + \pi$. However, the resulting pion momenta are not small enough so as to lead to a complete cancellation of J_5 and J_6.

5. *Calculation of σ_q and σ_N*

We recall the definition $\sigma_q = m_q^0 F_S(0)$ and that we had set $\sigma_N = 3\sigma_q$. Values for σ_q and σ_N are given in Table I. In that case,

$$F_S(0) = 1 + J_1 = [1 - G_S J_{SS}(0)]^{-1} .$$

In Table II we present the values of $F_S(0)$, σ_q and σ_N for the case that pion dressing of the quark is taken into account.

IV. DISCUSSION AND CONCLUSIONS

As stressed in Sec. I, the value of σ_N is important when one studies the behavior of quark condensates in matter. Here we have investigated corrections to the model of Ref. [4] arising from pion dressing of the quark. In calculations making use of the NJL model, a value of $\Lambda \simeq 1$ GeV is usually used. For definiteness, let us consider the results for $\Lambda = 1050$ MeV given in Tables I and II. We see that $m_q = m_q^{\text{con}} + m_q^0$ increases by 36 MeV when pion dressing is considered. We also note that the Gell-Mann-Oakes-Renner relation is satisfied to about 6% (Table I) or about 3% (Table II). The condensate value, $\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle$, for $\Lambda = 1050$ MeV is $(-257\ \text{MeV})^3$ (Table I) or $(-267\ \text{MeV})^3$ (Table II). It is worth noting that, in a recent work [13], a value for the condensate of $(-260 \pm 1\ \text{MeV})^3$ was obtained by studying the mass difference between the charged and neutral pions in the NJL model. That value is in reasonable accord with the values calculated in our work.

For the case $\Lambda = 1050$ MeV, the value of σ_N is either 54.6 MeV (Table I) or $\sigma_N = 51.5$ MeV (Table II). As discussed in Sec. I, a value of σ_N of about 50 MeV is in general accord with the magnitude of the scalar fields of the relativistic Brueckner-Hartree-Fock theory or Dirac phenomenology. If the value of σ_N were about 38 Mev, only about 75% of the scalar field of the models of relativistic nuclear physics would be directly related to the scalar order parameter that describes the behavior of the quark condensate at finite baryon density. That last result would not be particularly surprising, since it is believed that virtual excitation of the delta resonance can account for about 30% of the nucleon-nucleon attraction in the scalar-isoscalar channel. (If it turns out that $\sigma_N \simeq 35$ MeV, we might question the approximation $\sigma_N = 3\sigma_q$ used in Refs. [4,11] and in this work.) Because of the large uncertainty in the value of σ_N, we suggest that further study is required to obtain a more definitive interpretation of the nature of the scalar fields of relativistic nuclear physics than that given in [1] and reviewed here.

We may remark that the approximation $\sigma_N = 3\sigma_q$ used in Refs. [4,11] and in this work is suited to a nonrelativistic constituent quark model. If the quarks were described by Dirac wave functions, with upper and lower components, we would have $\sigma_N = 3(1-2a)\sigma_q$, where a is the fraction of the wave-function-normalization integral due to the presence of the lower component in the wave function. Even small values of a, of about 0.1, serve to reduce σ_N significantly. Because of this uncertainty, we suggest that our calculations tend to give a (theoretical) upper bound for σ_N, while providing estimates of the relative importance of the effects due to pion dressing.

We remark that a model for pion dressing of the nucleon is also needed when discussing the flavor asymmetry in the light-quark sea of the nucleon [14] with the aim of explaining the Gottfried sum rule deficit reported by the New Muon Collaboration [15]. For example, the authors of Ref. [14] calculate that the probability of a quark to emit a pion is $3a/2$, with $a = 0.083$. The value of 0.12 for that probability is reasonable accord with our value for $B = -0.15$, obtained for $\Lambda = 1.05$ GeV, given the different methods used in [14] and in this work. [Note that the values for $J_3 + J_4$ in Table II are equal to B and that $Z = (1-B)^{-1} \simeq 1 + B$.]

In this work we have studied the modification of the properties of a constituent quark of the NJL model, if pion dressing of the quark is taken into account a perturbative scheme. We have studied how the quark propagator is modified and have investigated corrections to the model of Ref. [4] for the evaluation of σ_N. The values obtained at the largest Λ considered here are at the upper end of the range determined from experimental data in other theoretical work, $\sigma_N = 45 \pm 8$ MeV [6]. To the extent that the uncertainty in that quantity can be reduced, we will be able to improve our understanding of the relation between the scalar fields of relativistic nuclear physics and the behavior of the quark condensate in matter.

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