Gamma emission in precompound reactions. I. Statistical model and collective gamma decay

A. Höring[1,2] and H. A. Weidenmüller[1]
[1]*Max-Planck-Institut für Kernphysik, Heidelberg, Germany*
[2]*National Institute for Nuclear Theory, Seattle, Washington 98195*
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We extend the theory of particle-induced precompound reactions by including gamma decay. We use the Brink-Axel hypothesis and consider the gamma emission of giant-dipole resonances built on the ground state and on the excited states of the composite system. The latter are modeled as multiparticle multihole excitations. In this way, we combine the statistical ansatz and the chaining hypothesis typical for precompound reaction theories, with the collective aspects of gamma decay. Formulas for average S matrix and average cross section are derived in this framework.

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I. INTRODUCTION

The theory of preequilibrium *nucleon-induced* nuclear reactions has been widely discussed and well established, both within classical models and in quantum-statistical formulations. On the other hand, the emission of *photons* in a preequilibrium nuclear reaction has found attention only in the last decade. Models for preequilibrium nuclear reactions are statistical in nature, while gamma emission mainly originates from the decay of the electric dipole giant resonance, i.e., from a highly collective and nonstatistical mode of nuclear excitation. The theoretical modeling of gamma emission in precompound reactions thus requires a suitable combination of statistical and collective aspects of excited nuclear states. This is the problem which we address in this paper.

We focus attention on the gamma decay of the $E1$ giant resonance(s) and neglect all other sources of gamma radiation. We make use of the Brink-Axel hypothesis [1]: Each nuclear state (ground or excited state) serves as the ground state of an $E1$ giant resonance excitation. The gamma emission in a precompound reaction is due to the combined action of all these giant resonance states. Furthermore, we carefully distinguish in the very ansatz of our theory between the purely statistical n-particle n-hole states and the collective states carrying the dipole transition strength. This allows us for the first time to account for the interdependence between deexcitation amplitudes involving different giant resonance states as well as for the interdependence between these GDR states and noncollective states. In this way, we arrive at a formulation which encompasses the model of the direct-semidirect capture process, the exciton model, etc., as special cases. We believe that this contributes to a better conceptual understanding of both the reaction itself, and of various ways of modeling it.

The aim of the present investigation, however, goes beyond establishing a formal framework which contains several models as special cases. The expressions for energy-averaged cross sections for gamma emission derived below can be used for a direct numerical calculation. It turns out that once the multistep compound nucleon-induced reaction (without gamma emission) is properly accounted for in terms of the purely statistical part of the precompound theory, and the $E1$ absorption cross section is taken from experiment, the present formulation allows for a parameter-free calculation of gamma emission in precompound reactions. This is shown in a paper by one of the present authors (A. H.), Herman, and Reffo which forms a sequel to the present publication, and in which a comparison with experimental results is made.

In our statistical modeling, we follow the theory of multistep-compound processes as formulated by Nishioka, Verbaarschot, Weidenmüller, and Yoshida (NVWY) [2]. We refer to this work as the NVWY model. We also use the techniques applied in this paper for the calculation of the average S matrix and the average cross section.

We limit ourselves to nucleon-induced reactions. We could apply our model also to other light-ion-induced reactions. However, both deuterons and alpha particles are isoscalar and are thus not suited for the excitation of dipole states. Moreover, we focus attention on the energy region between 10 and 20 MeV incident energy where preequilibrium decay is dominant.

We discuss our ansatz in Sec. II, introduce the generating functional in Sec. III, present the expressions for average S-matrix elements and average cross sections in Sec. IV, give a physical discussion of our results in Sec. V, compare with other models in Sec. VI, and close with a brief summary in Sec. VII. In writing this paper, we have faced the difficulty that the formal manipulations which lead from the ansatz to the expressions for average S matrix and average cross section are rather complex. We wanted to present our work in such a way that the interested readers should be able, from the information supplied, to check our calculations, and that other readers not interested in the formalism should nonetheless be able to understand both our ansatz and our results. We have attempted to meet this dual requirement by relegating all technical details to a series of appendixes. Readers not interested in the formalism are advised to go from Sec. II straight to Sec. IV.

II. ANSATZ

The starting point for any theory of precompound reactions is a division of the Hilbert space into classes of states of different complexity. To take into account the pairing interaction we use a quasiparticle basis, so that a particle-hole pair corresponds to a two-quasiparticle state. States in a given class then have the same number of quasiparticles (excitons); neighboring classes differ in their exciton number by two. The classes are labeled by capital letters M,N, where M,N stand for the numbers of exciton pairs; the states within a class are labeled by small letters μ,ν.

In the NVWY approach, it is assumed that the subblock $H_{M\mu,M\nu}$ of the Hamiltonian matrix referring to a fixed class M can be modeled as a member of the Gaussian orthogonal ensemble (GOE). This is the essential statistical input for the theory. In order to account for the existence of giant dipole resonances built on the ground and on the excited states of the composite nucleus, we have to modify this assumption. We do this by dividing the states in each class M into those which are dipole excitations on lower-lying states, and into others which are not.

For this purpose, two assumptions are used. (i) Brink-Axel hypothesis [1]: Each nuclear state (ground state or excited state) serves as the basis of a giant-dipole resonance (GDR) excitation with identical properties (excitation energy, spreading width). (ii) Tamm-Dancoff approximation (TDA) (cf., e.g., [3]): The giant resonance built upon a state with $2(N-1)$ excitons is a linear superposition of $2N$ exciton states.

The first assumption is good if the GDR is built on relatively low-lying states for which the mean field is essentially the same as for the ground state. In heavy-ion reactions, the composite nucleus may be excited in such a way that the mean field is changed. The GDR built on such excited states may have properties which differ from those of the GDR built on the ground state, and the Brink-Axel hypothesis is not applicable. For nucleon-induced reactions and in the energy region of interest, the Brink-Axel hypothesis is a good assumption.

The TDA is not as good a description of giant resonances as the random-phase approximation (RPA). In allowing for particle-hole pairs in the nuclear ground state, the RPA includes ground-state correlations and divides the $E1$ strength among several excited states, thereby yielding a realistic description. Unfortunately, we cannot accommodate ground-state correlations in the exciton picture and therefore make the simple second assumption listed above. We make up for this deficiency by allowing the GDR in exciton class M to couple with the statistical states in class $(M+1)$. This mechanism provides a spreading width for the GDR. In this way, a reasonably realistic description is obtained in spite of the simplifications inherent in assumption (ii).

To incorporate these assumptions into our precompound scheme, we label the states in class N with the quantum numbers (N,J,π,μ) where J^π indicate spin and parity. We divide the class (N,J,π) into four subclasses labeled $N1,\ldots,N4$. The states in subclasses Nn, $n=1,2,3$, are GDR excitations on states in class $(N-1)$ with spins $J-1$, J, and $J+1$, respectively, and opposite parity. This is sketched in Fig. 1 where the possible dipole transitions are indicated by arrows, and E_0 is the excitation energy. The states in class $N4$ (not shown in Fig. 1) are statistical states with zero dipole transition strength to states in class $(N-1)$. Since we assume that the GDR is an eigenstate of the nuclear Hamiltonian, the matrix $H_{N\mu,N\nu}$ becomes block diagonal in the labels Nn, $n=1,2,3,4$:

$$H_{NN}=\begin{pmatrix} H_{N1,N1} & 0 & 0 & 0 \\ 0 & H_{N2,N2} & 0 & 0 \\ 0 & 0 & H_{N3,N3} & 0 \\ 0 & 0 & 0 & H_{N4,N4} \end{pmatrix}. \tag{2.1}$$

The density of the states in subclass Nn $(n=1,2,3)$ at exciton energy E is given by the density of states in class $(N-1)$ at energy $(E-E_0)$ with the proper spin, while the density of states in class $N4$ makes up the difference to the actual level density at energy E. We observe that the density of states in subclass $N4$ is very much larger than that in any of the other three. It is obvious by construction that only the states in subclasses $N1,N2,N3$ couple to the gamma channels; each eigenstate of H_{NnNn} with $n=1,2,3$ carries the full GDR transition strength.

It remains to specify the properties of the diagonal blocks H_{NnNn} in Eq. (2.1). It goes without saying that we model H_{N4N4} as a member of the GOE. This is consistent with the entire NVWY approach. But what about the states in subclasses Nn, $n=1,2,3$. Figure 1 shows that these states must be modeled in the same fashion as the states of spin $J-1,J,J+1$ at excitation energy $(E-E_0)$ It is not clear *a priori* that for these states a statistical model is adequate.

In this paper, we assume that the GDR built on the ground state (the state with $N=0$) does not have statistical properties, whereas the GDR built on all states with $N\geq 1$ do have statistical properties and obey GOE statistics, so that for these states the diagonal blocks H_{NnNn} with $n=1,2,3$ are also members of a GOE. This division into nonstatistical and statistical states is admittedly arbitrary. Given sufficient information on spectroscopic

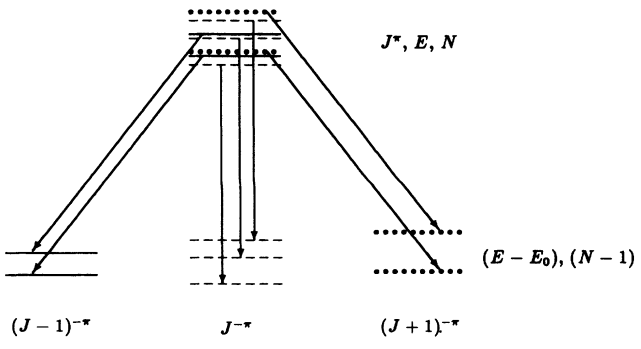

FIG. 1. Construction of a basis.

properties of low-lying $N=1$ states in the composite system, one could single out some GDR states in classes Nn ($n=1,2,3$) with $N=2$ and treat them individually and nonstatistically, too. Our formalism allows for this possibility. We doubt, however, whether this effort would be worthwhile because the remaining statistical features of the system are probably so strong as to wash out all traces of this individual treatment. The results in the following paper demonstrate this feature clearly for the GDR built on the ground state. We have isolated this state, and treated it separately, for two reasons: (i) to demonstrate the feasibility of such a procedure within an otherwise statistical approach and (ii) to separate in the final expressions clearly the direct-semidirect process from the precompound reaction.

With these approximations, the Hamiltonian H in the space of exciton configurations attains the following matrix structure:

$$\hat{H}=\begin{pmatrix} E_0 & 0 & V^T_{10,21} & V^T_{10,22} & V^T_{10,23} & V^T_{10,24} & V^T_{10,31} & \cdots \\ 0 & H_{14,14} & H_{14,21} & H_{14,22} & H_{14,23} & H_{14,24} & H_{14,31} & \cdots \\ V_{10,21} & H^T_{14,21} & H_{21,21} & 0 & 0 & 0 & H_{21,31} & \cdots \\ V_{10,22} & H^T_{14,22} & 0 & H_{22,22} & 0 & 0 & H_{22,31} & \cdots \\ V_{10,23} & H^T_{14,23} & 0 & 0 & H_{23,23} & 0 & H_{23,31} & \cdots \\ V_{10,24} & H^T_{14,24} & 0 & 0 & 0 & H_{24,24} & H_{24,31} & \cdots \\ V_{10,31} & H^T_{14,31} & H^T_{21,31} & H^T_{22,31} & H^T_{23,31} & H^T_{24,31} & H_{31,31} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} . \tag{2.2}$$

The entries all stand for block matrices except for the first diagonal element which stems from the single GDR built on the ground state. All other diagonal blocks stand for independent GOE's, with a variance determined from the appropriate level density as discussed above. Matrix elements coupling classes with different exciton numbers N and M, where $N\neq M$, are random Gaussian variables with zero mean and the same variance as used in applications of NVWY theory,

$$\overline{H_{Mm\mu,Nn\nu}}=\delta_{MN}\delta_{mn}\delta_{\mu\nu}h_{Mm} , \tag{2.3}$$

$$H_{Mn\mu,Nn\nu}=\delta_{MN}\delta_{mn}\delta_{\mu\nu}h_{Mm}+\delta H_{Mm\mu,Nn\nu} , \tag{2.4}$$

$$\overline{\delta H_{Mm\mu,Nn\nu}\delta H_{M'm'\mu',N'n'\nu'}}=C_{MmNn}(\delta_{MM'}\delta_{NN'}\delta_{mm'}\delta_{nn'}\delta_{\mu\mu'}\delta_{\nu\nu'}+\delta_{MN'}\delta_{NM'}\delta_{mn'}\delta_{nm'}\delta_{\mu\nu'}\delta_{\nu\mu'}) . \tag{2.5}$$

For the matrix elements labeled V in Eq. (2.2), it suffices to assume the orthogonality relations in Appendix B. This analytical investigation is kept general in the sense that we do not restrict ourselves to a two-body interaction. This restriction will be used, however, in our numerical application (cf. [8]).

With H defined in Eq. (2.2), we write the scattering matrix $S_{ab}(E)$ in the form

$$S_{ab}=\delta_{ab}-2i\pi\sum_{M\mu N\nu} W_{a,M\mu}(D^{-1})_{M\mu,N\nu}W_{b,N\nu} , \tag{2.6}$$

where

$$D_{M\mu,N\nu}=E\delta_{MN}\delta_{\mu\nu}-H_{M\mu,N\nu}+i\pi\sum_c W_{c,M\mu}W_{c,N\nu} . \tag{2.7}$$

For our purpose, we identify the channels a and b with the nucleon and the gamma channel, respectively. It turns out not to be necessary to define the coupling matrix elements $W_{a,M\mu}$ connecting channels and exciton states explicitly. Our results in Sec. III depend only on certain functions of these matrix elements which are related to the transmission coefficients. For the nucleon channels, the latter are determined as in Ref. [4] while for the gamma channels, the transmission coefficients are determined from the gamma-absorption cross section on the ground state of the composite system. Our definition of the model is therefore complete.

This model is peculiar in that it contains only the multistep-compound (MSC) process, and not the multistep-direct (MSD) one. Modeling the states of each subclass Mm by a GOE automatically implies a restriction on quasibound configurations and thus on the MSC process. For this theory to be consistent with the MSC process for particle emission it is inevitable to model the states in subclass $M4$, i.e., the noncollective states in exciton class M, as quasibound states. Our justification for using a MSC description of the states in subclasses Mm, $m=1,2,3$, for $M\geq 2$ lies in their construction. These states are GDR excitations on exciton states with excitation energy $(E-E_0)$; cf. Fig. 1. For excitation energies E of 20 MeV and a giant-dipole resonance energy E_0 of 15 MeV, these exciton states are quasibound configurations and must be modeled in the framework of the MSC process. However, the GDR state, built on these states, may be a quasibound or a continuum state. Our formulation allows for both possibilities. The only factor of importance is that the states which the GDR is built on and de-

cays into in the case of a gamma decay are quasibound configurations.

III. S MATRIX AND GENERATING FUNCTIONAL

In this section, we write the S-matrix element S_{ab} and a pair of S-matrix elements $S_{ab}S_{cd}^*$ as derivatives of a suitably chosen generating functional. This is the first step in a procedure which permits the calculation of the ensemble average over these quantities. We do this in such a way that we emphasize the additional complications of the present problem compared with the NVWY approach. Our notation is largely the same as that used by NVWY.

We start from the representation of the S matrix as introduced in Eqs. (2.6) and (2.7) with the additional subdivision into the subclasses just introduced:

$$S_{ab}(E)=\delta_{ab}-2i\pi(W_{a0},W_a^T)\hat{D}\begin{pmatrix}W_{0b}\\W_b\end{pmatrix} \tag{3.1}$$

with

$$\hat{D}=\begin{pmatrix}E&0\\0&E\end{pmatrix}-\hat{H}+i\pi\sum_c\begin{pmatrix}W_{0c}\\W_c\end{pmatrix}(W_{0c},W_c^T)\ , \tag{3.2}$$

where

$$\hat{H}=\begin{pmatrix}E_0&V^T\\V&H\end{pmatrix}\ , \tag{3.3}$$

$$W_a^T=\{W_{a,Mm\mu}\}\ , \tag{3.4}$$

$$V^T=\{V_{0,Mm\mu}\}\ , \tag{3.5}$$

$$H=\{H_{Mm\mu,Nn\nu}\}\ . \tag{3.6}$$

The "zeroth" state, describing the GDR built on the ground state and forming the only nonstatistical state, has been split off. In the following, we will refer to this state as the doorway state. Without this state the NVWY calculations remain unchanged except for the additional index labeling the four subclasses. Thus, the block on the right bottom of Eq. (3.1) together with (3.2) describes the MSC process which takes into account the GDR built on excited states. The block on the left top of Eqs. (3.1) and (3.2) describes the decay of the GDR built on the ground state. The nondiagonal elements of this equation cause the mixing and are hence responsible for the interference of these two processes. This is possible by means of internal mixing via the $V_{0,Mm\mu}$ and by external mixing via $i\pi\sum_c W_{0c}W_{c,Mm\mu}$.

The formal problem is due to the doorway state and its coupling to the statistical states. To handle this problem, the doorway state is split off. Equation (3.2) can be rewritten with the help of (F2)

$$S_{ab}(E)-\delta_{ab}=-2i\pi\left[W_{a0}G_0W_{0b}+u_a^T\frac{1}{E-H+i\pi\sum_c W_cW_c^T-vG_0v^T}u_b\right]\ , \tag{3.7}$$

where

$$G_0=\frac{1}{E-E_0+i\pi\sum_c W_{0c}^2}\ , \tag{3.8}$$

$$v^T=-V^T+i\pi\sum_c W_{0c}W_c^T\ , \tag{3.9}$$

$$u_a^T=W_{a0}G_0v^T-W_a^T\ . \tag{3.10}$$

The generating functional for the second term of Eq. (3.7) is

$$\begin{aligned}&Z_{ab}(E,j_1)=\int d[\phi]\exp\left[\frac{i}{2}\phi^\dagger D_{ab}(j_1)\phi\right]\ ,\\&D_{ab}(j_1)=1_4\otimes\left[E-H+i\pi\sum_c W_cW_c^T-vG_0v^T\right]+j_1I_4\otimes(u_bu_a^T+u_au_b^T)\ ,\end{aligned} \tag{3.11}$$

where

$$\phi^T=(\{S_{Mm\mu}^1\},\{S_{Mm\mu}^2\},\{\chi_{Mm\mu}\},\{\chi_{Mm\mu}^*\})\ . \tag{3.12}$$

This yields

$$u_a^TD^{-1}u_b=\frac{1}{4}\frac{\partial}{\partial j_1}Z_{ab}(E,j_1)\Big|_{j_1=0}=\frac{1}{4}\frac{\partial}{\partial j_1}Z_{abcd}(E,j_1,j_2)\Big|_{j_1=j_2=0}\ , \tag{3.13}$$

where

$$D=E-H+i\pi\sum_c W_cW_c^T+vG_0v^T\ , \tag{3.14}$$

$$Z_{abcd}(E,j_1,j_2):=Z_{ab}(E,j_1)Z_{cd}^*(E,j_2)\ . \tag{3.15}$$

Using the definitions (A9) the two-point function is given by

$$(u_a^T D^{-1} u_b)(u_c^T D^{-1} u_d)^* = \frac{1}{16} \frac{\partial^2}{\partial j_1 \partial j_2} Z_{abcd}(E, j_1, j_2) \Big|_{j_1 = j_2 = 0} . \tag{3.16}$$

It is our goal to evaluate the average S matrix and the average cross section. For this purpose, the generating functional is averaged over the ensemble of Hamiltonians. This averaging is performed in Appendix A. As a result one obtains

$$\lim_{N_{Mm} \to \infty} \overline{Z}_{abcd} = \int d[\sigma_G] \exp \left\{ \frac{1}{4} \sum_{\substack{M,N \\ m,n}} \begin{bmatrix} M \\ m \end{bmatrix} \neq \begin{bmatrix} N \\ n \end{bmatrix} \frac{\lambda^2_{MmNn}}{\lambda^2_{Mm} \lambda^2_{Nn}} \operatorname{trg}\{\sigma_G^{Mm} \sigma_G^{Nn}\} - \frac{1}{2} \operatorname{trg}\{\ln(\mathbf{E} - \boldsymbol{\Sigma}_G + i\mathbf{W} - v \hat{\mathbf{G}}_0 v^T + \hat{j}_{abcd})\} \right\} , \tag{3.17}$$

where the index G stands for Goldstone modes and

$$\boldsymbol{\Sigma}_G = \delta_{MN} \delta_{mn} \delta_{\mu\nu} \sigma_G^{Mm} . \tag{3.18}$$

IV. AVERAGE S MATRIX AND AVERAGE CROSS SECTION

The next step is to evaluate the average S matrix and average cross section. These quantities are essentially obtained by partial differentiation of the average generating functional with respect to j_1, and j_1 and j_2, respectively, and then setting the source term equal to zero. The calculation is carried out in Appendixes C and D, respectively. As a result one obtains for the average S matrix

$$\overline{S_{ab}(E)} = -2i\pi \frac{W_{a0}}{(1+X_a)} \frac{1}{(E-E_0) + \frac{i}{2} \left[\sum_c [2\pi W_{0c}^2/(1+X_c)] + \sum_{Mm} 2\pi v_{0,Mm}^2 \rho_{Mm} \right]} \frac{W_{0b}}{(1+X_b)} + \delta_{ab} \frac{(1-X_a)}{(1+X_a)} . \tag{4.1}$$

The first term describes the so-called "semidirect" process (SD): "direct"—since the average S matrix is no longer diagonal in the channel indices—and "semi"—because the decay goes via a resonance. The SD term factorizes in three parts: $\sqrt{2\pi W_{a0}^2}/(1+X_a)$ describing the coupling of channel a to the GDR built on the ground state, the propagator

$$\left[(E-E_0) + \frac{i}{2} \left[\sum_c \frac{2\pi W_{0c}^2}{(1+X_c)} + \sum_{Mm} 2\pi v_{0,Mm}^2 \rho_{Mm} \right] \right]^{-1}$$

and $\sqrt{2\pi W_{0b}^2}/(1+X_b)$ describing the decay of the GDR into channel b. $2\pi \sum_{Mm} v_{0,Mm}^2 \rho_{Mm}$ is the internal spreading width and $2\pi \sum_c W_{0c}^2$ is the decay width of the GDR. $\sum_c 2\pi W_{0c}^2/(1+X_c)$ can be rewritten as a sum of decay and external spreading width, yielding

$$\sum_c \frac{2\pi W_{0c}^2}{(1+X_c)} = \sum_c \left[2\pi W_{0c}^2 - \frac{X_c}{1+X_c} 2\pi W_{0c}^2 \right] . \tag{4.2}$$

Such a resonant form of the average S matrix is characteristic for the existence of a doorway state, e.g., the isobaric analogue state. Neglecting the division into subclasses, Eq. (4.1) is equivalent to Eq. (3.1) in Ref. [5].

As will be discussed in greater detail in Sec. VI C, this term differs formally from the term usually appearing in DSD calculations by the X_a [cf. Eq. (C7)]. These terms stem from the fact that the channels may also couple to statistical states. Thus, the flux that couples to the GDR built on the ground state is reduced. The correction factors X_a play a role similar to that of the optical model in a DWBA calculation. They are directly related to the transmission coefficients T_c for channel c; see Eq. (E5).

For the cross section, the average generating functional is partially differentiated with respect to j_1 and j_2 setting j equal to zero after that. It is then expanded to first order in $(\sum_c T_c)^{-1}$. In the energy region of interest the average level spacing, d, is much smaller than the average width of the resonances, Γ. With the help of the Weisskopf equation

$$\Gamma = \frac{d}{2\pi} \sum_c T_c \tag{4.3}$$

the width is given as the product of the recurrence frequency of a wave packet, $d/2\pi$, and the probability for the system to decay into a channel, $\sum_c T_c$. Thus for $\Gamma \gg d$ the relation $\sum_c T_c \gg 1$ holds and $(\sum_c T_c)^{-1}$ is a good expansion parameter.

The evaluation is carried out in Appendix D. As in the NVWY model, the remaining integration can be reduced to Gaussian integrals. It is shown in Appendix D that only two terms have a nonvanishing contribution. The term coming from the disconnected part yields the term depending on the average S matrix,

$$\overline{\sigma_{ab}^{\mathrm{SD}}}=\left|\delta_{ab}-\overline{S_{ab}}\right|^2 . \tag{4.4}$$

The connected part yields the precompound part of the cross section:

$$\overline{\sigma_{ab}^{pc}}=\sum_{MmNn}(T_{aa}^{Mm}T_{bb}^{Nn}+T_{ab}^{Mm}T_{ba}^{Nn})\Pi_{MmNn} , \tag{4.5}$$

where the matrix Π is given by its inverse

$$\begin{aligned}(\Pi^{-1})_{MmNn}=&4\sum_{Kk}\frac{\lambda_{MmKk}^2}{\lambda_{Mm}\lambda_{Kk}}\Delta_{Kk}\Delta_{Mm}\delta_{MN}\delta_{mn}+4\sum_c\frac{\hat{X}_{Mmc}}{(1+X_c)^2}(1+X_c)\left[1-\frac{\tilde{\Gamma}_0/2}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2}\frac{2\pi W_{0c}^2}{1+X_c}\right]\delta_{MN}\delta_{mn}\\&+4\frac{\tilde{\Gamma}_0/2}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2}\left[\sum_c\frac{\pi W_{0c}^2}{1+X_c}\frac{\hat{X}_{Mmc}}{1+X_c}+\hat{X}_{0,Mm}\right]\delta_{MN}\delta_{mn}-4\frac{\lambda_{MmNn}^2}{\lambda_{Mm}\lambda_{Nn}}\Delta_{Mm}\Delta_{Nn}(1-\delta_{MN}\delta_{mn})\\&-4\sum_c\frac{\hat{X}_{Mmc}}{1+X_c}\left[1-\frac{\tilde{\Gamma}_0/2}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2}\frac{2\pi W_{0c}^2}{1+X_c}\right]\frac{\hat{X}_{Nnc}}{1+X_c}\\&-4\left[\sum_c\frac{\pi W_{0c}^2}{1+X_c}\frac{\hat{X}_{Mmc}}{1+X_c}+\hat{X}_{0,Mm}\right]\frac{1}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2}\left[\sum_c\frac{\pi W_{0c}^2}{1+X_c}\frac{\hat{X}_{Nnc}}{1+X_c}+\hat{X}_{0,Nn}\right] ,\end{aligned} \tag{4.6}$$

$T_{ab}=\sum_{Mm}T_{ab}^{Mm}$ are the matrix elements of the Satchler T matrix [6]. They are a generalization of the transmission coefficients for the case the average S matrix is not diagonal in the channel indices. They are defined as the unitarity deficit of the average S matrix:

$$T_{ab}=\delta_{ab}-\sum_c\bar{S}_{ac}\overline{S_{cb}^*} . \tag{4.7}$$

In the case where the average S matrix is diagonal, the T matrix also becomes diagonal with the transmission coefficients as its elements. The matrix elements are given by

$$\begin{aligned}T_{aa}^{Mm}T_{bb}^{Nn}=&\left[T_{aMm}\left[1-\frac{\tilde{\Gamma}_0/2}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2}\frac{2\pi W_{a0}^2}{(1+X_a)}\right]+\frac{2\pi W_{a0}^2}{(1+X_a)^2}\frac{1}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2}\left[\sum_c\frac{2\pi W_{0c}^2}{(1+X_c)}\frac{\hat{X}_{Mmc}}{(1+X_c)}+\hat{X}_{0Mm}\right]\right]\\&\times\left[T_{Nnb}\left[1-\frac{\tilde{\Gamma}_0/2}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2}\frac{2\pi W_{0b}^2}{(1+X_b)}\right]+\frac{2\pi W_{0b}^2}{(1+X_b)^2}\frac{1}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2}\left[\sum_c\frac{2\pi W_{0c}^2}{(1+X_c)}\frac{\hat{X}_{Nnc}}{(1+X_c)}+\hat{X}_{0Nn}\right]\right] ,\end{aligned} \tag{4.8}$$

$$\begin{aligned}T_{ab}^{Mm}T_{ba}^{Nn}=&\frac{2\pi W_{a0}^2}{(1+X_a)^2}\frac{1}{[(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2]^2}\frac{2\pi W_{0b}^2}{(1+X_b)^2}\left\{\left[\sum_c\frac{\pi W_{0c}^2}{(1+X_c)^2}\hat{X}_{Mmc}+\hat{X}_{0,Mm}+\frac{\tilde{\Gamma}_0}{2}\left[\frac{\hat{X}_{Mma}}{1+X_a}+\frac{\hat{X}_{Mmb}}{1+X_b}\right]\right]\right.\\&\times\left[\sum_c\frac{\pi W_{0c}^2}{(1+X_c)^2}\hat{X}_{Nnc}+\hat{X}_{0,Nn}+\frac{\tilde{\Gamma}_0}{2}\left[\frac{\hat{X}_{Nna}}{1+X_a}+\frac{\hat{X}_{Nnb}}{1+X_b}\right]\right]\\&\left.+(E-E_0)^2\left[\frac{\hat{X}_{Mma}}{1+X_a}-\frac{\hat{X}_{Mmb}}{1+X_b}\right]\left[\frac{\hat{X}_{Nna}}{1+X_a}-\frac{\hat{X}_{Nnb}}{1+X_b}\right]\right\} .\end{aligned} \tag{4.8}$$

V. DISCUSSION OF THE RESULTS

Aside from kinematical and geometrical factors, the total average cross section is given by

$$\overline{\sigma_{ab}}=|\delta_{ab}-\overline{S_{ab}}|^2+\sum_{MmNn}(T_{aa}^{Mm}T_{bb}^{Nn}+T_{ab}^{Mm}T_{ba}^{Nn})\Pi_{MmNn} . \tag{5.1}$$

Together with Eqs. (4.1), (4.6), and (4.8) this constitutes the central result of this paper.

In Sec. 7 of Ref. [7] compound nucleus reactions in the presence of a doorway state are discussed. We have checked that neglecting the division into classes our result is consistent with the result there. We have also checked that our result fulfills the unitarity relation.

The first term on the right-hand side of Eq. (5.1) is the SD part of the average cross section. This part has been discussed in Sec. IV. The second term coming from the fluctuating part of the S matrix again consists of two terms. The

first term can be written as in the case of the NVWY model as a product of three factors: the probability that the entrance channel couples to a state of class Mm, the probability that this class couples to class Nn, and the probability for the latter to decay into channel b, summed over all exciton classes. The second term is less intuitive. It stems from the fact that the average S matrix is no longer diagonal, and it is needed for flux conservation. As can be seen, T_{ab}^{Mm} consists only of terms that include the doorway state. Formally this is due to the fact that u_a and u_b are no longer orthogonal. Physically this means that, due to the existence of the doorway state, not only probabilities but also amplitudes and therefore quantum mechanics "survive."

Next, the inverse probability transport matrix is investigated in detail. For a better understanding, it is first transcribed into physical quantities using Appendix E. It is then discussed term by term:

$$
\begin{aligned}
(\Pi^{-1})_{MmNn} = & -2\pi\rho_{Mm} v^2_{Mm,Nn} 2\pi\rho_{Nn} (1-\delta_{MN}\delta_{mn}) \quad \text{(a)} \\
& -4\sum_c \frac{2\pi\rho_{Mm} v^2_{Mmc}}{1+X_c} \left[1 - \frac{\tilde{\Gamma}_0/2}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2} \frac{2\pi W^2_{0c}}{1+X_c} \right] \frac{2\pi\rho_{Nn} v^2_{Nnc}}{1+X_c} \quad \text{(b)} \\
& - \left[\sum_c \frac{2\pi W^2_{0c}}{1+X_c} \frac{2\pi\rho_{Mm} v^2_{Mmc}}{1+X_c} + 2\pi\rho_{Mm} v^2_{Mm0} \right] \frac{1}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2} \\
& \times \left[\sum_c \frac{2\pi W^2_{0c}}{1+X_c} \frac{2\pi\rho_{Nn} v^2_{Nnc}}{1+X_c} + 2\pi\rho_{Nn} v^2_{Nn0} \right] \quad \text{(c)} \\
& + 2\pi\rho_{Mm} \Gamma^{\downarrow}_{Mm} \delta_{MN}\delta_{mn} \quad \text{(d)} \\
& + \sum_c T_{Mmc}(1+X_c) \left[1 - \frac{\tilde{\Gamma}_0/2}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2} \frac{2\pi W^2_{0c}}{1+X_c} \right] \delta_{MN}\delta_{mn} \quad \text{(e)} \\
& + \left[\sum_c \frac{\pi W^2_{0c}}{1+X_c} \frac{2\pi\rho_{Mm} v^2_{Mmc}}{1+X_c} + 2\pi\rho_{Mm} v^2_{Mm0} \right] \frac{1}{(E-E_0)^2+\frac{1}{4}\tilde{\Gamma}_0^2} \tilde{\Gamma}_0 \delta_{MN}\delta_{mn} \quad \text{(f)} \, . \qquad (5.2)
\end{aligned}
$$

Aside from the quantities defined previously, we have introduced the mean level densities ρ_{Mm} in subclasses Mm, the mean square matrix element $v^2_{Mm,Nn}$ connecting subclasses Mm and Nn, and we have written the mean square coupling elements connecting class Mm and channel c as v^2_{Mmc}. For illustration, the off-diagonal elements are sketched in Fig. 2. Each term (a), (b), or (c) describes the average coupling of a state in class Mm to a state in class Nn. There are three possibilities: There is the possibility of a direct coupling described by term (a). Term (b) describes the possibility of coupling these classes via a channel. A third possibility given by term (c) is the coupling via the doorway state. This coupling can again take place directly or via a channel state.

Figure 3 shows the diagonal terms of the matrix Π^{-1}. The diagonal elements describe the decay of class Mm. There are again three possibilities: the system can decay to a state of different complexity described by its spreading width [cf. (d)], or it may decay into a channel [cf. term (e)]. The third possibility is given by the coupling to the doorway state which itself may decay into a channel or a class of different complexity.

T_{aa}^{Mm} gives the probability for channel a to couple to a state in class Mm; see Eq. (4.8). Again, this coupling takes place directly or via the doorway state. The flux for the direct coupling is reduced by the fact that channel a can couple directly to the doorway state which again can decay into a channel. This is described formally by $\overline{S_{ab}}$.

When we specialize this general result to the special case of gamma emission, a crucial rule is played by the

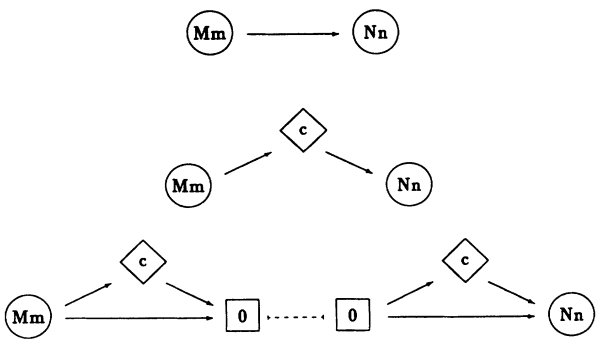

FIG. 2. Off-diagonal elements of the matrix Π^{-1} (5.2): ◇ stands for the space of channels, ○ for the space of bound states, □ for the doorway state, and — — — for the propagator.

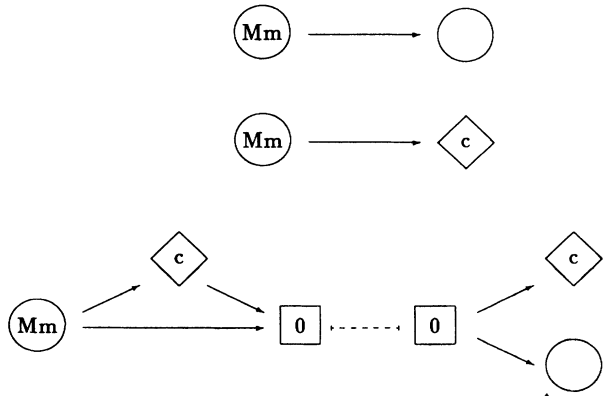

FIG. 3. Diagonal elements of the matrix Π^{-1} (5.2): ◇ stands for the space of channels, ○ for the space of bound states, □ for the doorway state, and — — — for the propagator.

division into subclasses. Only states of subclasses 1, 2, and 3 are GDR states and only these may decay directly into the gamma channel. This is due to the way our model is formulated giving the GDR 100% dipole strength. The noncollective states in subclass 4 obtain dipole strength only through coupling to these GDR states. Thus, they take part in the gamma decay only indirectly. This distinction between collective and noncollective states is a major strength of our formulation. As will become clearer in the next section, this is a major difference to the other models. This distinction also has an effect on the individual matrix elements of the matrix Π^{-1} as it allows to carefully distinguish the different physical processes, like creation or decay of a giant resonance or transitions to noncollective states. This point is discussed in detail in the numerical application of our result [8].

VI. COMPARISON TO OTHER MODELS

To embed our calculations into a larger context, our model is compared to other theories. There are mainly two types of models to theoretically describe gamma emission in preequilibrium nuclear reactions. One type is like the NVWY model based on the exciton picture. In this framework, the master equation formulation and the MSC model as formulated by Feshbach, Kerman, and Koonin (FKK) are discussed. The other type of model that describes capture gamma-ray reactions is the DSD model.

A. Exciton model in the framework of the master equation

One of the most commonly used theories to describe preequilibrium nuclear reactions is the exciton model. It has its origin in the work of Griffin [9] and was formulated in the framework of a master equation by Cline and Blann [10]. Generally, a master equation describes the time evolution of a Markovian process, i.e., a process in which the transition probability does not depend on the previous history of the system. The Pauli master equation for excitons is given by[1]

$$\dot{P}_M(t)=-\frac{\Gamma_M}{\hbar}P_M(t)+\sum_N P_N(t)W_{N\to M}\ , \tag{6.1}$$

where $P_M(t)$ is the occupation probability of class M at time t, $W_{N\to M}$ is the transition probability of class N to M per unit time, and $\Gamma_M/\hbar$ is the probability for class M to decay. In this model, the average cross section is given by

$$\bar{\sigma}_{ab}=\sigma_a^c\sum_N W_b(E,N)\tau(N)\ , \tag{6.2}$$

where σ_a^c is the compound nucleus formation cross section from channel a, $\tau(N)$ is the average lifetime of a state in exciton class N, and $W_b(E,N)$ is the probability for the nucleus to decay from class N into channel b:

$$\tau(N)=\int_0^\infty P_N(t)dt\ , \tag{6.3}$$

$$W_b(E,E_b,N)=T_b\frac{\rho_f(E-E_b,N-N_b)}{\rho_i(E,N)}\ . \tag{6.4}$$

The exciton model has been successfully applied in recent years to different preequilibrium nuclear reactions. This model has a few drawbacks [11]: Not being derived directly from the Schrödinger equation it has to be classified as phenomenological. Furthermore, it contains one free parameter, i.e., the average transition probability between two exciton classes. Thus, only relative cross sections can be calculated.[2] Often times, other simplifying assumptions are used such as the calculation of average lifetimes as the exact calculation is numerically quite complicated.

It can be shown (cf. [7]) that inversion of the matrix Π^{-1} is equivalent to solving the master equation. Thus, the NVWY model can be viewed as a quantum mechanical justification of the exciton model. One major difference, however, is that in the quantum mechanical formulation, bound states and continuum states are treated separately as MSC and MCD processes, respectively. In the next section the reason why the MSC contribution is of major importance in the energy region of 10 to 20 MeV will be discussed. Also, within the NVWY model the average transition matrix element is not a free parameter but is determined from the imaginary part of the optical model potential [4].

The exciton model was generalized to account for gamma decay mainly by Plyuyko and Prokopets [14], by Běták and Dobes [15], and by Akkermans and Gruppelaar [16]. In [17] it was applied to a number of nuclei throughout the periodic table.

To describe the gamma decay within the exciton model, the decay probability for particles is replaced by the probability to emit a photon. The latter is obtained using the Brink-Axel hypothesis and the principle of detailed balance:

$$W_\gamma(N,E_\gamma)=\frac{E^2}{\pi^2\hbar^3c^2}\sum_K\sigma_\gamma(N\to K,E_\gamma)\frac{\rho_K(E-E_\gamma)}{\rho_N(E)}\ , \tag{6.5}$$

where

$$\sigma_\gamma(K\to N,E_\gamma)=\sigma_\gamma^{\rm abs}(E_\gamma)b(K\to N,E_\gamma)\ . \tag{6.6}$$

The coefficients $b(K\to N,E_\gamma)$ describe the splitting of

[1]For simplicity and since it is commonly used in the literature, we will refer to this model as the exciton model. We emphasize, however, that there are other preequilibrium models that are also based on the exciton picture.

[2]This disadvantage inspired Blann [12,13] to formulate the hybrid model. This model forms a unification of the exciton model and the Harp-Miller-Berne model. Therein the transition probability is calculated as a function of the mean free path of a nucleon within nuclear matter. This model has been applied to gamma decay, cf., e.g., [33] and references therein.

the absorption cross section for the decay into the three possible exciton classes. The determination of these coefficients is not without ambiguity [16]. The three references [14–16] differ mainly in determining these coefficients. Furthermore, to account for flux conservation the three coefficients have to fulfill certain normalization conditions. As opposed to particle emission, the average matrix element coupling two exciton classes is not a free parameter but is taken from data on the particle emission [18].

In addition to the differences between the NVWY model and the exciton model already discussed, there are additional differences in the formulation of the gamma decay. The exciton model not being derived microscopically cannot distinguish GDR states from noncollective states. This causes a certain ambiguity: For transitions between two exciton classes the exciton states are treated as statistical states. However, in using the gamma absorption cross section, the exciton states are treated as collective GDR states for the description of gamma decay. In contradistinction, in our model the basis is chosen such that the GDR states and the noncollective states are distinguished. Only the former can decay directly into the gamma channel. The GDR states decay by gamma emission into the states they are built on. Thus, the problem of splitting the absorption cross section into the different possible exciton numbers of the final state does not arise. This difference in the formulation has an additional consequence. As it distinguishes GDR states from noncollective states our model also allows for a careful distinction between matrix elements describing different physical processes, such as creation of a GDR, decay of a GDR, statistical transitions, etc. This also causes a major difference in its numerical application [8].

B. FKK model

The model for preequilibrium nuclear reactions introduced by Feshbach, Kerman, and Koonin [19] is also based on the exciton picture. (For a recent review on all its applications cf. [20].) They were the first authors to distinguish MSC and MSD reactions. We will restrict ourselves to a discussion of the former. As a two-body interaction is used, only neighboring classes couple, which is referred to as "chaining hypothesis" in their paper. The state density in the next higher exciton class is larger than in the preceding class. This causes the probability for a transition to the next higher class to be much larger than the probability for a transition to the next lower class. In the framework of FKK the latter coupling is neglected—"never come back hypothesis." As a consequence the equilibrium limit is not contained automatically as in the NVWY model. It is added separately as the "r stage." The "r stage" is defined as the last class of which the level density is smaller than ten times the density of the next higher class. Emission from this stage is then added in the framework of a Hauser-Feshbach calculation. For practical applications these assumptions are very helpful and within the overall uncertainty of such preequilibrium models. The cross section within this model is given by

$$\bar{\sigma}_{ab}=\sigma_a^c \sum_{N=1}^{r-1} \frac{\langle \Gamma_{Nb}^{\uparrow} \rho_f \rangle}{\langle \Gamma_N \rangle} \prod_{M=1}^{N-1} \frac{\langle \Gamma_M^{\downarrow} \rangle}{\langle \Gamma_N \rangle} \ . \tag{6.7}$$

The cross section is again given as a product of three factors: the compound nucleus formation cross section from channel a, σ_a^c, the sum of the emission probabilities of all exciton classes, $\sum_{N=1}^{r-1}\langle \Gamma_{Nb}^{\uparrow}\rho_f \rangle / \langle \Gamma_N \rangle$, and the probability that the system reaches this exciton class, $\prod_{M=1}^{N-1}\langle \Gamma_M^{\downarrow} \rangle / \langle \Gamma_N \rangle$. The probability of reaching the "r stage" without emitting a nucleon is given in the framework of this model by

$$P_r=\prod_{k=1}^{r} \frac{\langle \Gamma_k^{\downarrow} \rangle}{\langle \Gamma_k \rangle} \ . \tag{6.8}$$

The Hauser-Feshbach term is multiplied by this probability.

This result is very similar to the exciton model. But it is not derived from a time-dependent master equation.

The FKK model was recently generalized by Obložinský and Chadwick to allow for gamma decay [21]. For this purpose the term describing the probability for particle emission was replaced by a term describing the gamma emission. The latter is given by

$$\frac{\langle \Gamma_{N\gamma}^{\uparrow} \rho_f \rangle}{\langle \Gamma_N \rangle} \ , \tag{6.9}$$

where

$$\frac{d\Gamma_{N\gamma J}^{\uparrow}}{dE_\gamma}=\sum_M \sum_{J'} \langle \Gamma_{N\gamma J}^{MJ'}(E-E_\gamma) \rangle \rho_{MJ'}(E-E_\gamma) \ . \tag{6.10}$$

As in the exciton model there is in addition to the sum over the possible angular momenta of the final states J' the summation over the possible exciton classes M of the final states. Thus the problem arises how to split the emission appropriately. The decay width of the gamma channel is again given in terms of the absorption cross section. Like the exciton model, this model is not derived microscopically and the discussion at the end of Sec. VI A is also applicable here.

One might be tempted to conclude that it is this model which is closest to the precompound part of our approach since both approaches are based on the MSC process. Again, due to our microscopic construction of the GDR states and the distinction of the GDR states from the noncollective states, this is, however, not the case. As discussed in the last paragraph of Sec. II all the noncollective states in our approach are modeled as quasibound configurations. However, the GDR state itself is not restricted to be quasibound. In the approach of Obložinský and Chadwick all states are quasibound configurations, and the possibility of the GDR state to be a continuum state is not accounted for. Thus, their total cross section is smaller. Comparing the numerical results [8] it turns out that their cross section is roughly one order of magnitude smaller than ours. This reflects the intuitive expectation that in the energy region of interest it is very likely for the GDR state to lie in the continuum.

This model was also generalized to take into account the angular distribution of emitted γ radiation.

C. DSD model

Historically, the direct-semidirect model (DSD) was the first description for capture gamma-ray processes. It has its origin in the work of Brown [22] and of Clement, Lane, and Rook [23]. It was then applied by Potokar *et al.* [24,25] to neutron-induced reactions and by Dietrich [26] and Snover [27] to proton-induced reactions. It was improved and numerically applied in numerous works, e.g., by Longo and Saporetti (cf., e.g., [28] and references therein).

In the framework of this model two physical processes are distinguished. The projectile can in the first step excite the GDR in the target nucleus. The latter can make a gamma decay. This is described by the "semidirect" or "resonant" term. The projectile can also be captured directly by the nuclear potential in emitting a gamma. This process is referred to as "direct capture." These two processes are added coherently in the average S matrix, allowing for interference effects between them. The cross section is apart from constants given by the square of the following transition matrix element:

$$M_{i\to f}=\langle\psi_f|H_\gamma|\psi_i\rangle+\frac{\langle\psi_f|H|\phi_{\rm int}\rangle\langle\phi_{\rm int}|H_\gamma|\psi_i\rangle}{E-E_r+(i/2)\Gamma}\ . \tag{6.11}$$

Here $\phi_{\rm int}$ denotes the intermediate GDR state, E_r is the resonance energy, and Γ the resonance width of the GDR. The matrix element describing the gamma decay of the GDR is given in terms of its decay width. The latter is determined with the help of the energy weighted sum rule. The other matrix elements are determined explicitly by performing the respective integration and angular momentum coupling. This causes the numerical calculations to be involved.

Comparison of the SD contribution to the corresponding term in our formulation shows great similarity. The essential difference is that our formulation explicitly takes into account the possible coupling to the statistical states. As will be seen in the numerical applications [8], this coupling diminishes the SD cross section significantly. In Eq. (6.11) this coupling is implicitly taken into account by introducing a form factor in the matrix elements consisting of a real and an imaginary part.

A further difference resides in the direct capture which is not taken into account in our model. Clearly, this process cannot originate from our ansatz in which direct reactions are explicitly excluded. Such a term has to be added to the average S-matrix element. However, one can convince oneself that this term is small [29]: It is given by the overlap integral of the wave functions in the initial and final states. Since the energy of the projectile is rather large the associated wave function is oscillating rapidly and the matrix element is small.

VII. SUMMARY

We have extended the theory of the MSC process as formulated in Ref. [2] by including gamma emission from giant-dipole resonances. These were modeled in terms of the TDA as two-quasiparticle excitations on the ground state and —using the Brink-Axel hypothesis—on the excited states of the composite system. The latter were in turn modeled as n-quasiparticle n-quasihole states. In this way, our approach could be fitted into the general framework of precompound reaction theory. Taking into account the conservation of energy, angular momentum, and parity caused each exciton class to be divided into four subclasses. Three of these describe GDR's and couple directly to the gamma channels, whereas the fourth describes noncollective states which can gamma decay only indirectly via their coupling to a GDR. This coupling produces the spreading widths of the GDR's. The lowest excitation class contains only two subclasses, the statistical one plus a single state, the GDR built on the ground state of the composite system.

In the limit of strongly overlapping resonances (or of many open channels), we calculated analytically the ensemble averages of the S matrix and of the cross section. Formally, the NVWY calculations had to be modified mainly because of the presence of the doorway state, i.e., the GDR built on the ground state of the composite system. Independently of our specific application, our calculation is thus of general theoretical interest: It shows how to treat a doorway state in the framework of a statistical theory.

Our central result is embodied in Eqs. (5.1), (4.1), (4.6), and (4.8). The average cross section is a sum of two terms. The term containing the average S-matrix element describes the semidirect process. (We have not taken into account the direct capture.) The other term originates from the fluctuating part of the S matrix and gives the precompound contribution. Both terms differ, however, from the form they would have if we were to consider only SD, or only MSC reactions: Flux conservation reduces the SD, and modifies the expression for the MSC process.

The parameters in our expression for the average cross section can be determined from properties of the particle-induced precompound reaction without gamma emission, on the one hand, and from the giant dipole resonance absorption cross section of the composite nucleus in its ground state, on the other hand. In this sense, our final result allows for a parameter-free calculation of gamma emission in precompound reactions. This is shown in the following paper.

We thank the Department of Energy for partial support during the completion of this work.

APPENDIX A: AVERAGE GENERATING FUNCTIONAL

The following definitions have been used for Eq. (3.16):

$$Z_{abcd}(E,j_1,j_2)=\int d[\psi]\exp\left[\frac{i}{2}\psi^\dagger\mathbf{L}^{1/2}\mathbf{D}(\hat{\mathbf{J}})\mathbf{L}^{1/2}\psi\right], \tag{A1}$$

$$\mathbf{D}(\hat{\mathbf{J}})=\mathbf{E}-\mathbf{H}+i\mathbf{W}-v\hat{\mathbf{G}}_0v^T+\hat{\mathbf{j}}_{abcd} \ , \tag{A2}$$

$$\psi^T=(\{S^1_{Mm\mu}(1)\},\{S^2_{Mm\mu}(1)\},\{S^1_{Mm\mu}(2)\},\{S^2_{Mm\mu}(2)\},\{\chi_{Mm\mu}(1)\},\{\chi^*_{Mm\mu}(1)\},\{\chi_{Mm\mu}(2)\},\{\chi^*_{Mm\mu}(2)\}) \ , \tag{A3}$$

$$\mathbf{E}=E\mathbf{1}_8\otimes\mathbf{1}_N-\mathbf{1}_8\otimes h \ , \tag{A4}$$

$$h=\mathrm{diag}(h_{Mm}) \ , \tag{A5}$$

$$\mathbf{H}=\mathbf{1}_8\otimes H \ , \tag{A6}$$

$$\mathbf{W}=\pi L_8\otimes\sum_c W_c W_c^T \ , \tag{A7}$$

$$\hat{\mathbf{G}}_0=\begin{bmatrix} G_0 & 0 \\ 0 & G_0^* \end{bmatrix}\otimes\mathbf{1}_4\otimes\mathbf{1}_N \ , \tag{A8}$$

$$\mathbf{J}=\hat{\mathbf{j}}_{abcd}=\begin{bmatrix} j_1 I_4\otimes(u_b u_a^T+u_a u_b^T) & 0 \\ 0 & j_2 I_4\otimes(u_d u_c^T+u_c u_d^T) \end{bmatrix} . \tag{A9}$$

It is our goal to evaluate the average S matrix and the average cross section. We proceed along the same lines as in Ref. [2]. The ensemble average over the ensemble of Hamiltonians, the Hubbard-Stratonovitch transformation, and the ψ integration remain the same, the only difference being in the additional class indices and the additional term $v\hat{\mathbf{G}}_0v^T$. To avoid unnecessary duplication, we refer to the original reference for more details and only stress the differences to that work. One obtains

$$\overline{Z_{abcd}(E,\hat{J})}=\int d[\sigma]\exp\left\{\frac{1}{2}\sum_{MmNn} g_{MmNn}\,\mathrm{trg}(\sigma^{Mm}\sigma^{Nn})-\frac{1}{2}\,\mathrm{trg}\ln N(\hat{J})\right\} , \tag{A10}$$

$$N(\hat{J})=\mathbf{E}-\Sigma+i\mathbf{W}+v\hat{\mathbf{G}}_0v^T+\hat{\mathbf{J}}+i\eta \ ,$$

where

$$g_{MmNn}=(C^{-1})_{MmNn} \ , \tag{A11}$$

$$\Sigma=\delta_{MN}\delta_{mn}\delta_{\mu\nu}(\sigma^{Mm}) \ . \tag{A12}$$

Before evaluating the integral with the help of the method of steepest descent in the limit $N_{Mm}\to\infty$, where N_{Mm} is the number of states in class Mm, we need to discuss the N_{Mm} dependence of the additional term $v\hat{\mathbf{G}}_0v^T$. $V_{0,Mm\mu}$ are matrix elements of the Hamiltonian $\hat{H}$ and therefore of the order $1/\sqrt{N_{Mm}}$, $\hat{G}_0$ is of zeroth order in N_{Mm}, such that the term $v\hat{\mathbf{G}}_0v^T$ like $\mathbf{W}$ is of the order $1/N_{Mm}$.

Hence, the saddle-point equation remains unchanged except for the additional indices:

$$\sigma_{Mm}(E-h_{Mm}-\sigma_{Mm})=\lambda^2_{Mm} \ . \tag{A13}$$

The integration over the massive modes also remains unchanged. One obtains Eq. (3.17).

APPENDIX B: ORTHOGONALITY RELATION OF THE VECTORS W_a AND V_0

Due to the orthogonal invariance of H_{Mm}, the result can only depend on the orthogonal-invariant terms $\sum_\mu W^2_{Mm\mu,a}$, $\sum_\mu V^2_{Mm\mu,0}$, and $\sum_\mu W_{Mm\mu,a}V_{Mm\mu,0}$. For those the following orthogonality relations

$$\sum_\mu W_{Mm\mu,a}W_{Mm\mu,b}=W^T_{Mm,a}W_{Mm,b}=N_{Mm}v^2_{Mm,a}\delta_{ab} \ , \tag{B1}$$

$$\sum_\mu W_{Mm\mu,a}V_{Mm\mu,0}=W^T_{Mm,a}V_{Mm,0}=0 \ , \tag{B2}$$

$$\sum_\mu V_{Mm\mu,0}V_{Mm\mu,0}=V^T_{Mm,0}V_{Mm,0}=N_{Mm}v^2_{Mm,0} \tag{B3}$$

are used. The physical motivation for these assumptions is the following. As discussed previously, the background S matrix, i.e., the S matrix excluding the doorway state, is a generalization of the MSC process taking into account the GDR built on excited states. Requiring this averaged background S matrix to be diagonal is equivalent to the orthogonality relation of the vectors W_a.

While the matrix elements $W_{a,Mm\mu}$ couple the statistically distributed bound states $Mm\mu$ to the nonstatistical channel state a, the matrix elements $V_{0,Mm\mu}$ couple the statistically distributed bound states to the collective GDR built on

the ground state, the doorway state. These two couplings are comparable in the sense that both couple a nonstatistical state to the statistically distributed bound states. In this sense, the doorway state plays a similar role as a channel. This explains the two other orthogonality relations (B2) and (B3).

APPENDIX C: AVERAGE ONE-POINT FUNCTION

The average one-point function is obtained by partial differentiation with respect to j_1 of the averaged generating function and then setting j equal to zero:

$$\frac{\partial}{\partial j_1}\overline{Z_{ab}}(E,j_1)\bigg|_{j_1=0}=-\frac{1}{2}\frac{\partial}{\partial j_1}\operatorname{trg}\ln\left\{\lambda(\Sigma^0)^{-1}+i\pi\sum_c W_c W_c^T-vG_0v^T+j_1(u_a u_b^T+u_b u_a^T)\right\}\bigg|_{j_1=0} Z_{ab}(E,0)\ , \tag{C1}$$

where

$$\Sigma^0=\operatorname{diag}(\sigma^0_{Mm})\ , \tag{C2}$$

$$\lambda=\operatorname{diag}(\lambda_{Mm})\ . \tag{C3}$$

Thus, only terms of first order in j contribute. Using the normalization $Z_{ab}(E,0)=1$ and the cyclic invariance of the trace one obtains

$$\frac{\partial}{\partial j_1}\overline{Z_{ab}}(E,j_1)\bigg|_{j_1=0}=-\frac{1}{2}\operatorname{trg}\left\{(W_{a0}G_0v^T-W_a^T)\sum_n\left[-i\pi\lambda^{-1}\Sigma^0\sum_c W_c W_c^T+\lambda^{-1}\Sigma^0vG_0v^T\right]^n \times\lambda^{-1}\Sigma^0(vG_0W_{0b}-W_b)+(a\leftrightarrow b)\right\}\ . \tag{C4}$$

Next the geometrical series is expanded and the individual terms are rearranged.

As an illustration, we obtain for one of the four terms in Eq. (C4)

$$\begin{aligned}
&W_a^T\sum_n\left[\lambda^{-1}\Sigma^0\left[-i\pi\sum_c W_c W_c^T-vG_0v^T\right]\right]^n\lambda^{-1}\Sigma^0W_b\\
&\quad=W_a^T\sum_n\left[-\lambda^{-1}\Sigma^0i\pi\sum_c W_c W_c^T\right]^n\lambda^{-1}\Sigma^0W_b+W_a^T\sum_n\left[-\lambda^{-1}\Sigma^0i\pi\sum_c W_c W_c^T\right]^n\lambda^{-1}\Sigma^0vG_0v^T\\
&\qquad\times\sum_n\left[-\lambda^{-1}\Sigma^0i\pi\sum_c W_c W_c^T\right]^n\lambda^{-1}\Sigma^0W_b\\
&\quad+W_a^T\sum_n\left[-\lambda^{-1}\Sigma^0i\pi\sum_c W_c W_c^T\right]^n\lambda^{-1}\Sigma^0vG_0v^T\sum_n\left[-\lambda^{-1}\Sigma^0i\pi\sum_c W_c W_c^T\right]^n\lambda^{-1}\Sigma^0vG_0v^T\\
&\qquad\times\sum_n\left[-\lambda^{-1}\Sigma^0i\pi\sum_c W_c W_c^T\right]^n\lambda^{-1}\Sigma^0W_b+\cdots\\
&\quad=\delta_{ab}\frac{(1/i\pi)X_a}{1+X_a}+\frac{1}{1+X_a}W_a^T\lambda^{-1}\Sigma^0vG_0v^T\lambda^{-1}\Sigma^0W_b\frac{1}{1+X_b}\\
&\quad+\frac{1}{1+X_a}W_a^T\lambda^{-1}\Sigma^0vG_0v^T\frac{1}{1+\lambda^{-1}\Sigma^0i\pi\sum_c W_c W_c^T}\lambda^{-1}\Sigma^0vG_0v^T\lambda^{-1}\Sigma^0W_b\frac{1}{1+X_b}+\cdots\ .
\end{aligned} \tag{C5}$$

The orthogonality relations of the W's have been taken into account. Additionally, the definitions

$$X_{Mm,a}=\lambda_{Mm}^{-1}\pi N_{Mm}v^2_{Mm,a}\ , \tag{C6}$$

$$X_a=\sum_{Mm}X_{Mm,a}i\sigma^0_{Mm} \tag{C7}$$

were used. Equation (C5) can be rewritten yielding

$$\begin{aligned}
&W_a^T\sum_n\left[\lambda^{-1}\Sigma^0\left[-i\pi\sum_c W_c W_c^T-vG_0v^T\right]\right]^n\lambda^{-1}\Sigma^0W_b\\
&\quad=\delta_{ab}\frac{(1/i\pi)X_a}{1+X_a}+\frac{\lambda^{-1}W_a^Tv}{1+X_a}G_0\frac{1}{1-v^T\dfrac{1}{1+i\pi\lambda^{-1}\Sigma^0\sum_c W_c W_c^T}\lambda^{-1}\Sigma^0vG_0}\frac{\lambda^{-1}\Sigma^0v^TW_b}{1+X_b}\ .
\end{aligned} \tag{C8}$$

The other terms can be evaluated with the same technique. The result is

$$\frac{\partial}{\partial j_1}\overline{Z_{ab}}(E,j_1)\bigg|_{j_1=0} = -\frac{1}{2}\,\mathrm{trg}\left\{\delta_{ab}\frac{(1/i\pi)X_a}{1+X_a} - W_{a0}G_0W_{0b} + \left[W_{a0}-\frac{\lambda^{-1}W_a^T v}{1+X_a}\right]\frac{D_0}{1-v^T\dfrac{1}{1+i\pi\lambda^{-1}\Sigma^0\sum_c W_c W_c^T}\lambda^{-1}\Sigma^0 v} \times\left[W_{b0}-\frac{\lambda^{-1}\Sigma^0 v^T W_b}{1+X_b}\right]+(a\leftrightarrow b)\right\}. \tag{C9}$$

Using the orthogonality relations of the vectors V_0 and W_a, one obtains

$$\lambda^{-1}\Sigma^0 W_a^T v = i\pi W_{a0}\sum_{Mm}\sigma^0_{Mm}X_{Mm,a}\ , \tag{C10}$$

$$v^T\lambda^{-1}\Sigma^0 v = \sum_{Mm}\sigma^0_{Mm}X^0_{Mm} - \pi^2\sum_c W^2_{0c}\sum_{Mm}\sigma^0_{Mm}X_{Mm,c}\ , \tag{C11}$$

where

$$X_{Mm,0} = \lambda^{-1}_{Mm}\pi N_{Mm}v^2_{Mm,0}\ . \tag{C12}$$

Collecting everything yields the average S-matrix element, which takes the form

$$\overline{S_{ab}(E)} = -2i\pi\frac{W_{a0}}{(1+X_a)}\frac{1}{E-E_0+i\sum_c\dfrac{\pi W^2_{0c}}{(1+X_c)}-\sum_{Mm}\sigma^0_{Mm}X^0_{Mm}}\frac{W_{0b}}{(1+X_b)}+\delta_{ab}\frac{(1-X_a)}{(1+X_a)}\ . \tag{C13}$$

APPENDIX D: AVERAGE TWO-POINT FUNCTION

1. Partial differentiation of the generating functional

Expanding the averaged generating functional to second order in $\hat{\mathbf{J}}$ yields

$$\overline{Z_{abcd}}(E,j_1,j_2) = \lim_{\hat\alpha\to 0} C\int d[\sigma_G]\{\tfrac{1}{4}\,\mathrm{trg}[(r\lambda^{-2}\boldsymbol{\Sigma}_{\mathbf{G}}\mathbf{j}_{abcd})^2]+\tfrac{1}{8}[\,\mathrm{trg}(r\lambda^{-2}\boldsymbol{\Sigma}_{\mathbf{G}}\mathbf{j}_{abcd})]^2\} \times\exp\left[\frac{1}{4}\sum_{\substack{M,N\\ m,n}}{}'\frac{\lambda^2_{MmNn}}{\lambda^2_{Mm}\lambda^2_{Nn}}\,\mathrm{trg}\{\sigma_G^{Mm}\sigma_G^{Nn}\}-\hat\alpha\sum_{Mm}\mathrm{trg}(t_{12}^{Mm}t_{21}^{Mm}) -\frac{1}{2}\,\mathrm{trg}\ln[1+\lambda^{-2}\boldsymbol{\Sigma}_{\mathbf{G}}(i\mathbf{W}-v\mathbf{G}_0v^T)]\right], \tag{D1}$$

where $r=[1+\lambda^{-2}\boldsymbol{\Sigma}_{\mathbf{G}}(i\mathbf{W}-v\mathbf{G}_0v^T)]^{-1}$. Partial differentiation of the averaged generating functional with respect to j_1 and j_2 and afterwards setting j_1 and j_2 equal to zero yields

$$\frac{\partial^2}{\partial j_1\partial j_2}\overline{Z_{abcd}}(E,j_1,j_2)\bigg|_{j_1=j_2=0} = \lim_{\hat\alpha\to 0} C\int d[\sigma_G]\{\tfrac{1}{2}\,\mathrm{trg}[r\lambda^{-2}\boldsymbol{\Sigma}_{\mathbf{G}}I(1)(u_bu_a^T+u_au_b^T)r\lambda^{-2}\boldsymbol{\Sigma}_{\mathbf{G}}I(2)(u_du_c^T+u_cu_d^T)] +\tfrac{1}{4}\,\mathrm{trg}[r\lambda^{-2}\boldsymbol{\Sigma}_{\mathbf{G}}I(1)(u_bu_a^T+u_au_b^T)]\,\mathrm{trg}[r\lambda^{-2}\boldsymbol{\Sigma}_{\mathbf{G}}I(2)(u_du_c^T+u_cu_d^T)]\} \times\exp\left[\frac{1}{4}\sum_{\substack{M,N\\ m,n}}{}'\frac{\lambda^2_{MmNn}}{\lambda^2_{Mm}\lambda^2_{Nn}}\,\mathrm{trg}\{\sigma_G^{Mm}\sigma_G^{Nn}\}-\hat\alpha\sum_{Mm}\mathrm{trg}(t_{12}^{Mm}t_{21}^{Mm}) -\frac{1}{2}\,\mathrm{trg}\ln[1+\lambda^{-2}\boldsymbol{\Sigma}_{\mathbf{G}}(i\mathbf{W}-v\mathbf{G}_0v^T)]\right]. \tag{D2}$$

2. Asymptotic expansion

After using the orthogonality relations of the vectors V_0 and W_a we make an asymptotic expansion in Λ^{-1}. This corresponds to expanding the generating functional to second order in the t matrices. Then the remaining integral is solved.

First the "trg ln" term in the exponent of Eq. (D2) is considered. In the case of the NVWY model, this term together with the term quadratic in σ_G yields the inverse of the probability transport matrix Π. Using Eq. (F5) this term can be rewritten in the following way yielding

$$\begin{aligned}
&\exp\left\{\operatorname{trg}\ln\left[1+\lambda^{-2}\boldsymbol{\Sigma}_G\left[i\pi\sum_c W_c W_c^T L - v\mathbf{G}_0 v^T\right]\right]\right\} \\
&\quad=\exp\left\{\operatorname{trg}\ln\begin{bmatrix}1+i\pi\lambda^{-2}\boldsymbol{\Sigma}_G\sum_c W_c W_c^T L & \lambda^{-2}\boldsymbol{\Sigma}_G v\\ v^T & \mathbf{D}_0\end{bmatrix}\right\}\exp\left\{\operatorname{trg}\ln(-\mathbf{D}_0)\right\} \\
&\quad=\exp\left[\operatorname{trg}\ln\left[1+i\pi\lambda^{-2}\boldsymbol{\Sigma}_G L\sum_c W_c W_c^T\right]+\operatorname{trg}\ln\left[\mathbf{D}_0 - v^T\frac{1}{1+i\pi\lambda^{-2}\boldsymbol{\Sigma}_G L\sum_c W_c W_c^T}\lambda^{-2}\boldsymbol{\Sigma}_G v\right]-\operatorname{trg}\ln(\mathbf{D}_0)\right],
\end{aligned} \tag{D3}$$

where $\mathbf{D}_0=\mathbf{G}_0^{-1}$. As in the case of the one-point function the geometrical series can be expanded and rearranged. Using the orthogonality relations (cf. Appendix B) yields

$$\begin{aligned}
&v^T\sum_n\left[-i\pi\lambda^{-2}\boldsymbol{\Sigma}_G L\sum_c W_c W_c^T\right]^n\lambda^{-2}\boldsymbol{\Sigma}_G v \\
&\qquad=\sum_{Mm} X_{0,Mm}\lambda_{Mm}^{-1}\sigma_G^{Mm}+i\pi L\sum_c W_{0c}^2\frac{1}{1+i\pi\sum_{Mm}X_{Mm,c}\lambda_{Mm}^{-1}\sigma_G^{Mm}L}i\pi\sum_{Mm}X_{Mm,c}\lambda_{Mm}^{-1}\sigma_G^{Mm}L
\end{aligned} \tag{D4}$$

and

$$\begin{aligned}
&\mathbf{D}_0 - v^T\sum_n\left[-i\pi\lambda^{-2}\boldsymbol{\Sigma}_G L\sum_c W_c W_c^T\right]^n\lambda^{-2}\boldsymbol{\Sigma}_G v \\
&\qquad=E-E_0+i\pi L\sum_c W_{0c}^2\frac{1}{1+i\pi\sum_{Mm}X_{Mm,c}\lambda_{Mm}^{-1}\sigma_G^{Mm}L}-\sum_{Mm}X_{0,Mm}\lambda_{Mm}^{-1}\sigma_G^{Mm}.
\end{aligned} \tag{D5}$$

This term as well as the other terms in the exponent are expanded to second order in the t_{12}. As in Ref. [2] we first expand the saddle-point matrices to second order in t_{12},

$$\sigma_G^{Mm}=\lambda_{Mm}\sigma_D^{Mm}+2i\lambda_{Mm}\delta_{Mm}\begin{bmatrix}t_{12}^{Mm}t_{21}^{Mm} & it_{12}^{Mm}\\ it_{21}^{Mm} & t_{21}^{Mm}t_{12}^{Mm}\end{bmatrix}. \tag{D6}$$

A lengthy but straightforward calculation shows that the exponent can be written in the form

$$-\frac{1}{2}\sum_{MmNn}(\Pi^{-1})_{Mm,Nn}\operatorname{trg}\{t_{12}^{Mm}t_{21}^{Nn}\} \tag{D7}$$

with the matrix (Π^{-1}) given by Eq. (4.6) where $\hat{X}_{Mmc}=X_{Mmc}\Delta_{Mm}$.

The integration measure in second order in t transforms as in [2]:

$$d[\sigma_G]\rightarrow\prod_{Mm}d(t_{12}^{Mm}). \tag{D8}$$

Hence, one obtains again Gaussian integrals.

As in [2] the preexponential term has a connected and a disconnected part. The former yields the average fluctuating part of the cross section whereas the latter yields the part being a function of the average S matrix. Using the cyclic invariance of the trace, the preexponential terms are functions of the following term:

$$u_a^T r\lambda^{-2}\Sigma_G I u_b = \begin{bmatrix} W_{a0}G_0 v^T - W_a^T & 0 \\ 0 & W_{a0}G_0^* v^{*T} - W_a^T \end{bmatrix} \times \sum_n \left[-\lambda^{-2}\Sigma_G \begin{bmatrix} i\pi\sum_c W_c W_c^T - vG_0 v^T & 0 \\ 0 & -i\pi\sum_c W_c W_c^T - v^* G_0^* v^{*T} \end{bmatrix} \right]^n \times \lambda^{-2}\Sigma_G \begin{bmatrix} I(1) & 0 \\ 0 & I(2) \end{bmatrix} \begin{bmatrix} vG_0 W_{0b} - W_b & 0 \\ 0 & v^* G_0^* W_{0b} - W_b \end{bmatrix} . \tag{D9}$$

The terms within each block are similar to the ones of the one-point function. Using the orthogonality relations of the V's and W's and rearranging the geometric series yields

$$u_a^T r\lambda^{-2}\Sigma_G I(p) u_b = \delta_{ab} \frac{\sum_{Mm} X_{Mm,a}\lambda_{Mm}^{-1}\sigma_G^{Mm}}{1+i\sum_{Mm} X_{Mm,a}\lambda_{Mm}^{-1}\sigma_G^{Mm}L} I(p) - W_{a0}\hat{G}_0 W_{0b} I(p) + \frac{W_{a0}}{1+i\sum_{Mm} X_{Mm,a}\lambda_{Mm}^{-1}\sigma_G^{Mm}L} \frac{1}{E-E_0+i\pi L\sum_c \frac{W_{0c}^2}{1+i\sum_{Mm} X_{Mm,c}\lambda_{Mm}^{-1}\sigma_G^{Mm}} - \sum_{Mm} X_{0Mm}\sigma_G^{Mm}} \times \frac{W_{0b}}{1+iL\sum_{Mm} X_{Mm,b}\lambda_{Mm}^{-1}\sigma_G^{Mm}} I(p) , \tag{D10}$$

where $p=1,2$. Next, the preexponential terms are expanded to second order in t_{12} using Eq. (D6). Without performing a detailed calculation it is seen that the disconnected term in second order in the t_{12} consists of terms of the form $\mathrm{trg}\{I(1)\}\,\mathrm{trg}\{I(2)\}$, $\mathrm{trg}\{t_{12}^{Mm}t_{21}^{Nn}I(1)\}\,\mathrm{trg}\{I(2)\}$, and $\mathrm{trg}\{(1)\}\,\mathrm{trg}\{t_{21}^{Mm}t_{12}^{Nn}I(2)\}$. The Gaussian integration of the last two terms vanishes (cf. Appendix A of Ref. [30]), such that only the zeroth-order term in t_{12} has a nonvanishing contribution. Likewise, for the connected term one obtains in second order in the t_{12} terms of the form $\mathrm{trg}\{I(1)I(2)\}$, $\mathrm{trg}\{I(1)t_{12}^{Mm}I(2)\}$, $\mathrm{trg}\{t_{21}^{Nn}I(1)I(2)\}$, and $\mathrm{trg}\{t_{12}^{Mm}I(2)t_{21}^{Nn}I(1)\}$. Performing the Gaussian integration only the term quadratic in t_{12} contributes. The expansion itself is lengthy but straightforward and only the result is given here.

The disconnected term yields

$$16\left[\delta_{ab}\frac{-i\hat{X}_a}{1+\hat{X}_a} - W_{a0}G_0 W_{0b} + \frac{W_{a0}}{1+\hat{X}_a} g_0 \frac{W_{0b}}{1+\hat{X}_b}\right]\left[\delta_{ab}\frac{i\hat{X}_a}{1+\hat{X}_a} - W_{a0}G_0^* W_{0b} + \frac{W_{a0}}{1+\hat{X}_a} g_0^* \frac{W_{0b}}{1+\hat{X}_b}\right] , \tag{D11}$$

with

$$g_0 = \frac{1}{(E-E_0)+\frac{i}{2}(\sum_c [2\pi W_{0c}^2/(1+X_c)]+2X_0)} . \tag{D12}$$

Our generating functional does not yield the total two-point function [cf. Eq. (3.16)]. Adding the additional terms and using

$$\int \prod_{M'm'} d[t_{12}^{M'm'}] \exp\left\{-\frac{1}{2}\sum_{MmNn}(\Pi^{-1})_{MmNn}\,\mathrm{trg}(t_{12}^{Mm}t_{21}^{Nn})\right\} = 1 \tag{D13}$$

yields the SD contribution to the average cross section [cf. Eq. (4.4)]. Performing the asymptotic expansion for the connected part and using

$$\int \prod_{M'm'} d[t_{12}^{M'm'}] \exp\left\{-\frac{1}{2}\sum_{MmNn}(\Pi^{-1})_{MmNn}\,\mathrm{trg}(t_{12}^{Mm}t_{21}^{Nn})\right\} \mathrm{trg}[I(1)t_{12}^{Kk}I(2)t_{21}^{Ll}] = 16\Pi_{KkLl} \tag{D14}$$

yields the fluctuating part of the average cross section. This part can be written in the form of Eq. (4.5).

APPENDIX E: TRANSCRIPTION OF THE MICROSCOPIC TERMS IN PHYSICAL TERMS

For illustration, we have transcribed our results in physical terms. The relevant relations are derived here.

In the framework of GOE's the average level density is given by the Wigner semicircle law (cf., e.g., [31]):

$$\rho_{Mm} = N_{Mm}\frac{1}{2\pi N_{Mm}C_{MmMm}^2} \times \sqrt{4N_{Mm}C_{MmMm} - (E-h_{Mm})^2} . \tag{E1}$$

With $C_{MmMm} = \lambda_{Mm}^2/N_{Mm}$ one obtains

$$\rho_{Mm} = \frac{N_{Mm}}{\pi\lambda_{Mm}}\left[1 - \frac{(E-h_{Mm})^2}{4\lambda_{Mm}^2}\right]^{1/2} , \tag{E2}$$

yielding

$$\mathrm{Im}(\sigma_{Mm})=\Delta_{Mm}=\frac{\pi\lambda_{Mm}}{N_{Mm}}\rho_{Mm} \ . \tag{E3}$$

Furthermore, we use

$$C_{Mm,Nn}=:v^2_{MmNn} \tag{E4}$$

for the average transition matrix element coupling states of classes Mm and Nn where $Mm\neq Nn$. The optical-model-transmission coefficient is given in this theory by

$$T_a=\frac{4X_a}{|1+X_a|^2} \tag{E5}$$

and

$$T_{Mma}=\frac{4X_{Mma}}{|1+X_a|^2} \ . \tag{E6}$$

APPENDIX F: A FEW RELEVANT MATHEMATICAL RELATIONS

Let A and D be block diagonal matrices. For the inverse of the matrix

$$\begin{bmatrix}\alpha & \beta\\ \gamma & \delta\end{bmatrix}:=\begin{bmatrix}A & B\\ C & D\end{bmatrix}^{-1} \tag{F1}$$

the relations for its submatrices

$$\begin{aligned}\alpha&=(A-BD^{-1}C)^{-1} \ ,\\ \beta&=-A^{-1}B(D-CA^{-1}B)^{-1}\\ &=-(A-BD^{-1}C)^{-1}BD^{-1} \ ,\\ \gamma&=-D^{-1}C(A-BD^{-1}C)^{-1} \ ,\\ \delta&=(D-CA^{-1}B)=D^{-1}+D^{-1}C(A-BD^{-1}C)^{-1}BD^{-1}\end{aligned} \tag{F2}$$

are valid. These equations are derived by solving

$$\begin{bmatrix}\alpha & \beta\\ \gamma & \delta\end{bmatrix}\begin{bmatrix}A & B\\ C & D\end{bmatrix}=1 \ . \tag{F3}$$

The second relations for β and δ are received by expanding the geometric series and rearranging the individual terms.

Let again A and D be block diagonal matrices. The determinant of the matrix

$$\begin{bmatrix}A & B\\ C & D\end{bmatrix} \tag{F4}$$

can be rewritten adding a multiple of the lower or upper row, respectively, yielding

$$\begin{aligned}\det\begin{bmatrix}A & B\\ C & D\end{bmatrix}&=\det(A)\det(D-CA^{-1}B)\\ &=\det(A-BD^{-1}C)\det(D) \ .\end{aligned} \tag{F5}$$

The determinant of graded matrices is also invariant under adding a multiple of a row or column as long as the graded symmetry is not broken. Thus, these relations also hold, if A, B, C, and D are graded matrices. Using

$$\det g\begin{bmatrix}A & B\\ C & D\end{bmatrix}=\exp\mathrm{trg}\ln\begin{bmatrix}A & B\\ C & D\end{bmatrix} \tag{F6}$$

respective relations for the "trg ln" terms can be derived:

$$\begin{aligned}\mathrm{trg}\ln\begin{bmatrix}A & B\\ C & D\end{bmatrix}&=\mathrm{trg}\ln(A)+\mathrm{trg}\ln(D-CA^{-1}B)\\ &=\mathrm{trg}\ln(A-BD^{-1}C)+\mathrm{trg}\ln(D) \ .\end{aligned} \tag{F7}$$

These equations can also be derived directly in expanding the logarithm (cf., e.g., (7.16) in [32]).

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