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Gamow-Teller strength in $A = 37$ nuclei

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A comparison is made of the recent β^+-decay measurements of García *et al.* and the 1981 ^{37}Cl$(p,n)^{37}$Ar measurements of Rapaport *et al.* in order to see whether the experiments are consistent. García *et al.* were not able to determine the amount of Gamow-Teller strength in the isobaric analog state (IAS) or the Coulomb mixing of Fermi strength to a state near the IAS, and were forced to determine the amount of Gamow-Teller strength in the daughter 1371 keV state by making the branching ratios sum to unity. This approach leaves a range of possible values open for these quantities which we investigate. We find that the experiments could be consistent within a theoretically unlikely part of this range. We also examine the effect of this range of allowed values on solar and supernova neutrino cross sections of ^{37}Cl and find weak dependence on the uncertainties described here. We discuss how the consistency of the two experiments can be tested more conclusively.

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I. INTRODUCTION

A recent beta-decay experiment performed by García *et al.* has extended the measured Gamow-Tellor strength function in the decay of ^{37}Ca up to 8 MeV excitation in the daughter nucleus [1,2]. This represents a considerably larger portion of the Gamow-Teller (GT) strength function than has been accessible to previous beta-decay measurements. Because ^{37}Ca is the isospin mirror of ^{37}Cl, it is possible to compare the strength seen in this decay with that measured in the ^{37}Cl$(p,n)^{37}$Ar experiment of Rapaport *et al.* [3]. García *et al.* made such a comparison and concluded that the β^+ decay contains 50% more GT strength in this energy region than the (p,n) experiment. This conclusion, in conjunction with a comparison with a theoretical spectrum based on the Wildenthal's *sd*-shell interaction (hereafter USD) [4], led Adelberger, García, and Wells [5] to question whether the GT strength is quenched and to question the ability of (p,n) experiments to accurately measure GT strength. Their conclusions are very significant since they impinge on the whole question of GT quenching, which, aside from its general interest, is of considerable interest in astrophysics, affecting the role of ^{37}Cl in the Davis solar neutrino experiment [6], and estimates of electron-capture rates needed for supernovae calculations [7,8]. The purpose of our analysis is to examine both sets of data and, with minimal appeal to theory, to determine whether they are compatible within experimental error.

To compare the data of García *et al.* and Rapaport *et al.*, we must note that the definitions of B(GT) are not the same in the two papers. In this paper we will use the definition employed by García *et al.* An essential point to bear in mind is that García *et al.* measured β-delayed protons, whose intensities are assumed to be proportional to $g_V^2|M(\mathrm{F})|^2+g_A^2|M(\mathrm{GT})|^2$, while Rapaport *et al.* measured reaction cross sections, which in the approximations stated in Goodman *et al.* [9] are assumed to be proportional to $J_\tau^2|M(\mathrm{F})|^2+J_{\sigma\tau}^2|M(\mathrm{GT})|^2$. The two sets must be compared using the deduced matrix elements, and the question at hand is whether the two probes determine GT matrix elements consistently. A value for $|g_A/g_V|^2$ must be assumed to extract GT matrix elements from β-decay rates. To determine matrix elements from (p,n) cross sections, one must use values for J_τ^2 and $J_{\sigma\tau}^2$, which are determined [10] by comparing measured cross sections with matrix elements derived from β-decay rates. Evidence for the proportionality between 0° (p,n) cross sections and β^--decay matrix elements is given in Ref. [10].

The GT sum rule [11] tells us that the summed GT matrix elements squared for a nuclear decay must be at least $3(N-Z)$ in units where $|M(\mathrm{GT})|^2=3$ for the free neutron. This rule is model independent. To compare with the sum rule, we use the value $|g_A/g_V|^2=(1.262)^2$ determined from the decay of the neutron [12], and we divide the B(GT) values given by García *et al.* by this factor to get them into matrix element units. García *et al.* observed a summed GT strength of 3.11 up to $E_x=8.0$ MeV. This translates to a matrix-element sum of 1.95 units, compared to the sum-rule requirement of at least 9 units. Thus at least 7 units remain unobserved by the β-decay experiment.

In Fig. 1 we have plotted the cumulative histogram of strength for the experiments. As was noted by García *et al.*, much of the difference between the data can be explained if one concedes that the (p,n) experiment drastically underestimated the amount of GT strength near the isobaric analog state (IAS). Such a mistake is not surprising, given the relatively poor resolution and the inability of nonpolarized charge-exchange experiments to distinguish between Fermi and GT strengths. It can be seen in

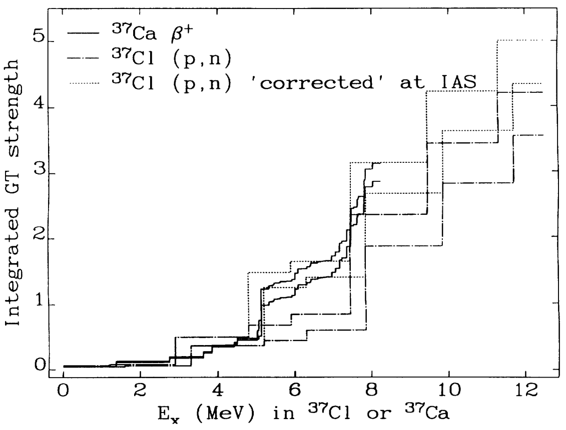

FIG. 1. Cumulative GT strength (in García's units) as a function of daughter excitation energy for the two experiments. A "corrected" (p,n) strength histogram is also shown, in which GT strength is arbitrarily added to the IAS region until the new (p,n) and β^+-decay data at high energies match.

Fig. 1 that if roughly 0.78 unit of GT strength are added to the (p,n) data near the IAS, the histograms follow almost in lockstep thereafter.

If this were the only difference between the two experiments, their consistency in measuring GT strength would not be in much doubt. However, Fig. 2, which shows the GT strength measured by each experiment, exhibits some quite obvious differences. We have labeled regions of particular interest. Region *a* is the ground state. Its strength was not directly measured by García *et al.* The same is true of region *b*, the state of 1371 keV. Both these regions will be discussed below. Region *c* exhibits 0.36 unit of GT strength in the (p,n) data, while the four peaks seen by the β^+ decay have 0.254 unit of strength. One possibility is a source of additional strength in the (p,n) measurement. If there is Na contamination in the target, the transition ^{23}Na$(p,n)^{23}$Mg(IAS) would contribute at 3244 keV. Such a reaction would explain the extra strength seen in the charge-exchange experiment. However, recent unpublished results from a repeat (p,n) experiment at the Indiana University Cyclotron Facility [13] make it doubtful that this is ^{23}Na contamination. For the moment, therefore, we must leave this discrepancy for future experiment to resolve. Region *d* is the $A=37$ isobaric analog state region and will be discussed in detail below, as will be regions *e* and *f*. Region *g* is not seen in the β^+ decay because its phase-space integrals are roughly 100 times weaker than those in region *f*. Region *h* is not accessible to the ^{37}Ca β^+ decay because of energetics.

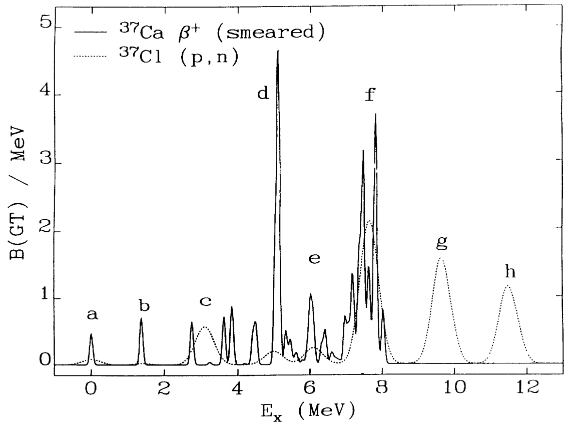

FIG. 2. GT strength (in García's units) for both experiments. The Rapaport *et al.* resonances are given a full width at half maximum (FWHM) of 0.6 MeV, as suggested in their paper, while the García *et al.* data are given an arbitrary (FWHM) of 0.1 MeV. The actual resolution of the latter data set is 16 keV, but this would result in a set of delta functions being plotted and would hinder the ability to read the plot.

Figure 2 exhibits glaring differences at regions *b*, *c*, *d*, and *e*. The difference at region *d* again shows the problems near the IAS. Region *c* has been discussed above, and we can say little more about it until the experimental situation is clarified. This region has little bearing on the main points of the paper. Region *e* is weaker in the (p,n) measurement, and this also may result from ascribing too much Fermi strength to the IAS region. Region *b* is more problematic. The experiments clearly disagree there, whereas near region *f* the agreement is striking. This comparison could indicate either that the intensity of the β-delayed proton groups are not always proportional to the GT strength or that the sensitivity of the (p,n) experiment may vary over the 12-MeV region measured. The second possibility amounts to saying that J_τ and $J_{\sigma\tau}$ can have abrupt and unpredictable changes over relatively small regions of energy. If this is true, it does severely challenge the ability of charge-exchange experiments to extract GT strength.

Because of the importance of region *b* and what it implies, in the next section we examine in depth how the strength at this region was determine by García *et al.* We will then see whether the experiments are consistent with their estimates of GT matrix elements. How well this consistency can be established will reflect how reliably GT strength can be extracted from the experiments. In Sec. III we examine the overall normalization of the (p,n) experiment by comparing the strength measured near region *f* by each experiment, and we attempt to use the García *et al.* data to estimate what $|J_{\sigma\tau}/J_\tau|^2$ at the IAS was for the (p,n) experiment. In Sec. IV we examine how these results affect the sensitivity of the ^{37}Cl experiment to solar and supernova neutrinos. In the last section, we conclude by discussing how the factors which are still unknown in the $A=37$ system can be determined. We discuss how a spin-polarized charge-exchange experiment could disentangle the Fermi and GT strengths in the IAS region.

II. RELATION BETWEEN THE STATES AT 1371 AND 5051 keV

Although the ^{37}Ca β^+ decay allowed an unprecedented measurement of the transitions, three factors are discussed in García *et al.*, which this experiment could not address. These factors are the amount of GT strength at the 5051 keV state, the amount of Coulomb mixing of the

Fermi strength, and the amount of strength at the 1371 keV state. These factors were also discussed in García's thesis, and we repeat some of this discussion because it is central to what follows.

The amount of GT strength in the $\frac{3}{2}^+$ state at 5051 keV, the isobaric analog state, could not be distinguished from the large Fermi strength there. This GT strength will change the normalization of the whole experiment since it enhances the absolute size of the transition to this state, and all other transitions were measured relative to it. García *et al.* assigned a value of 0.1 unit of GT strength based on the prediction of the USD interaction [4]. Because the strength in this state has not been experimentally measured, we treat $B_{5051}(\mathrm{GT})$ as a free parameter for now. In the conclusion we review the arguments supporting the USD estimate.

The second unknown in the ^{37}Ca β^+-decay experiment is also a consequence of the inability to distinguish between Fermi and Gamow-Teller strengths. In ^{37}K there is a $\frac{3}{2}^+$ state of 5016 keV, which is close enough to the IAS so that Coulomb mixing of the two states could occur. This mixing will also affect the overall normalization, since it steals Fermi strength from the superallowed transition as well as transferring some GT strength from the 5016 keV state to the IAS. Thus it affects the total amount of GT strength inferred by the experiment because it replaces possible GT strength at 5016 keV with Fermi strength. García *et al.* examined this effect by considering the two most extreme limits: no mixing, in which all of the Fermi strength is at the 5051 keV state and all of the strength of 5016 keV is GT, and the full mixing case, where all of the strength at 5016 keV is Fermi and the rest of the Fermi strength lies in the 5051 keV state. These cases provide a range within which the experiment must lie, and the range is not large. We will use the same approach here.

The third unknown in the β^+-decay experiment is the amount of GT strength in the $\frac{1}{2}^+$ state at 1371 keV. Because this state and the ground state of ^{37}K are below the proton threshold, it was not possible for García *et al.* to measure decays to these states directly. The transition to the ground state has been measured for $^{37}\mathrm{Ar}_{\mathrm{g.s.}}(e^-,\nu)^{37}\mathrm{Cl}_{\mathrm{g.s.}}$, and it was assumed, using isospin invariance, that the $^{37}\mathrm{Ca}_{\mathrm{g.s.}}(e^+,\nu)^{37}\mathrm{K}_{\mathrm{g.s.}}$ decay had the same matrix element to within 3%. This leaves the transition to the 1371-keV state to be found by satisfying the branching ratio

$$\frac{K}{t_{1/2}}=\sum_j f(E_{0j})[B_j(\mathrm{F})+B_j(\mathrm{GT})]\ , \tag{1}$$

where j denotes the jth daughter state, $K=6170\pm4$ s, $t_{1/2}=0.175\pm0.003$ s, $f(E_{0j})$ is the phase-space integral for the positron decay determined by Wilkinson and Macefield [14], and E_{0j} is the energy of the decay to the jth daughter state. $B_j(\mathrm{F})$ can be nonzero for only the two states discussed above.

The strength at the 1371 keV state can be determined only after one has made assumptions about the two uncertainties discussed above. Once an assumption is made about the Coulomb mixing to the 5016 keV state, there will be a linear relation between $B_{1371}(\mathrm{GT})$ and $B_{5051}(\mathrm{GT})$:

$$B_{1371}(\mathrm{GT})=-\frac{f(E_{5051})}{f(E_{1371})}B_{5051}(\mathrm{GT})+\frac{K}{t_{1/2}}-\left\{\sum_{\substack{j\\ \mathrm{not}\,\{1371\\5051}} f(E_{0j})[B_j(\mathrm{F})+B_j(\mathrm{GT})]\right\}\ . \tag{2}$$

Figure 3 shows this relation between the GT strength in the two states for the two extremes of Coulomb mixing. The real amount of mixing will lie in the band between these extremes. The uncertainty in $B_{1371}(\mathrm{GT})$ is also shown for each case. This uncertainty comes from the uncertainties in K, $t_{1/2}$, and the measurements of the other transitions. More than half of this uncertainty comes from the uncertainty of the $t_{1/2}$. There is no longer any uncertainty included for the GT strength at the 5051 keV state because its value is assumed. Table I lists the parameters of the lines shown in this and all subsequent figures. In preparing these relations, one must not use the table of transition strengths given in the García *et al.* paper because that table is the average of the two mixing cases for $B_{5051}(\mathrm{GT})=0.1$. It is necessary to start with one of the Coulomb cases listed in García's thesis.

As noted, $B_{5051}(\mathrm{GT})$ has not been measured. The β^+-decay experiment can only specify a range for $B_{1371}(\mathrm{GT})$, as a function of what $B_{5051}(\mathrm{GT})$ might be, where we have assumed that the intensity of the β-delayed protons is directly proportional to the Fermi and GT strengths. Because $B(\mathrm{GT})\geq0.0$, the ends of this range are clearly determined. All that the ^{37}Ca β^+-decay experiment allows us to say is $0.0\leq B_{1371}(\mathrm{GT})\leq0.11$ and

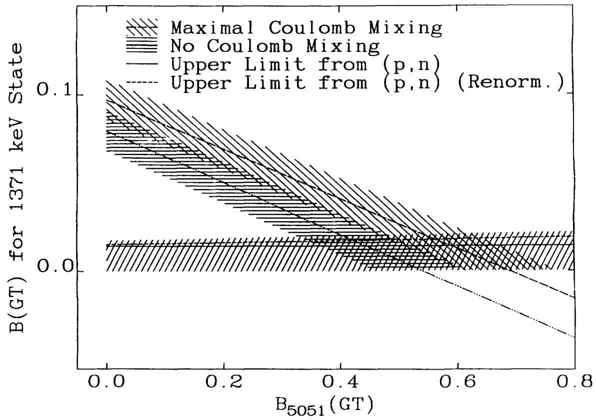

FIG. 3. $B_{1371}(\mathrm{GT})$ as a function of $B_{5051}(\mathrm{GT})$. The extremes of Coulomb mixing are plotted. The crosshatching around each case indicates the range of uncertainty due to the experimental uncertainties. The lower, almost horizontal crosshatched region shows the value of $B_{1371}(\mathrm{GT})$ which is allowed by the (p,n) experiment. If both experiments are consistent, the actual values of $B_{1371}(\mathrm{GT})$ and $B_{5051}(\mathrm{GT})$ must lie within the intersection of these crosshatched regions.

TABLE I. Equations of lines calculated in paper. The top entry for each quantity is the case with no Coulomb mixing. The lower entry is the case with maximal Coulomb mixing. B_{5051}(GT) is denoted by B.

Quantity	Parameters of Line
B_{1371}(GT)	$(-0.1472\pm0.004)B+(0.079\pm0.012)$
	$(-0.1412\pm0.004)B+(0.097\pm0.011)$
Total B(GT)	$(1.788\pm0.024)B+(2.934\pm0.074)$
	$(1.716\pm0.023)B+(2.718\pm0.071)$
N	$(0.3569\pm0.0556)B+(1.071\pm0.166)$
	$(0.3426\pm0.0534)B+(1.028\pm0.160)$
$B_{1371}(\mathrm{GT})_{(p,n)}$	$(0.005\,020\pm0.000\,782)$ $B+(0.015\,06\pm0.002\,35)$
Renormalized	$(0.004\,818\pm0.000\,751)$ $B+(0.014\,46\pm0.002\,25)$
B_{IAS}	$(1.234\pm0.010)B+(0.7032\pm0.031)$
	$(1.185\pm0.010)B+(0.5542\pm0.030)$
$\sigma_{8\mathrm{B}}$	$(0.1952\pm0.0003)B+(1.076\pm0.002)$
	$(0.1873\pm0.0002)B+(1.052\pm0.002)$
σ_{HEP}	$(1.035\pm0.003)B+(4.232\pm0.010)$
	$(0.9930\pm0.0024)B+(4.107\pm0.009)$
$\langle\sigma\rangle$	$(5.449\pm0.008)B+(19.48\pm0.03)$
	$(5.229\pm0.008)B+(18.83\pm0.03)$
$\sigma(\langle E_\nu\rangle)$	$(2.792\pm0.030)B+(10.66\pm0.11)$
	$(2.680\pm0.028)B+(10.33\pm0.10)$

$0.78 \geq B_{5051}(\mathrm{GT}) \geq 0.0$. The discussion here differs from that of García *et al.* because we have allowed B_{5051}(GT) to vary. The (p,n) experiment was only able to place an upper limit on the amount of strength in the 1371-keV state: $B_{1371}(\mathrm{GT}) \leq 0.0141$. This limit is shown in Fig. 3. If the experiments are consistent, the allowed range is much smaller and B_{5051}(GT) must be very large.

The upper limit from the (p,n) experiment constrains B_{5051}(GT) to lie within a fraction [i.e., $0.37 \leq B_{5051}(\mathrm{GT}) \leq 0.78$] of the range left open by the García *et al.* experiment. As noted, the USD interaction leads to the expectation that the GT strength in the IAS would not be as strong as this limit implies. The strength at the 1371-keV state is thus a severe test of each experiment's ability to measure GT strength. If the values of the GT strength in the 1371 and 5051 keV states are actually in a part of the range not allowed by the (p,n) limit, the experiments are not consistent. In the last section, we will discuss how these questions could be resolved.

In Fig. 4 we show the total B(GT) for $E_x < 8$ MeV measured by García *et al.* as a function of B_{5051} (GT). As with the amount of strength in the 1371 keV state, the total GT strength falls within a range, constrained by the positivity of B_{1371}(GT) and B_{5051}(GT). It can be seen that $2.65 \leq B_{<8\ \mathrm{MeV}}(\mathrm{GT}) \leq 3.81$. This result will be used below.

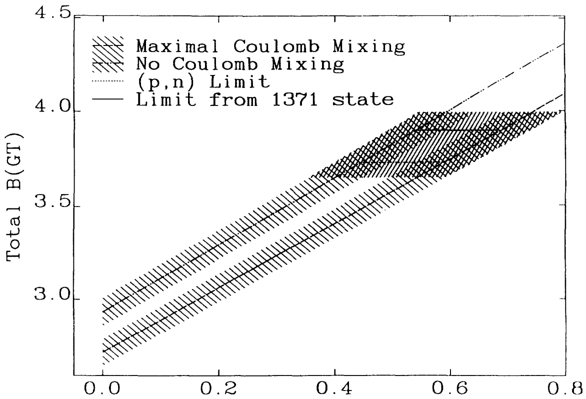

FIG. 4. Total GT strength below 8.0 MeV measured by the ^{37}Ca β^+-decay experiment as a function of B_{5051}(GT). Again, the experimental uncertainties for each case are shown by crosshatching. Intermediate amounts of Coulomb mixing would lie between these cases. The region which is doubly crosshatched is the region within the β^+ decay, and the (p,n) experiments are consistent.

III. SYNTHESIZING THE TWO EXPERIMENTS

It was seen in the previous section that, for some values of B_{1371}(GT) and B_{5051}(GT), the experiments of García *et al.* and Rapaport *et al.* could be consistent. In this section we assume that they are consistent and see what can be learned about $J^2_{\sigma\tau}/J^2_{\tau}$ in this system. In order to investigate these topics, we must ensure that the overall normalization of the (p,n) experiment is correct. At the time the (p,n) measurement was reported, extensive systematics of (p,n) data were not yet available and the overall normalization that was used to relate the (p,n) cross sections to GT transition probabilities could have been in error. The normalization was done with the combined use of distorted-wave impulse approximation (DWIA) calculations [9] and the cross sections for some other (p,n) transitions. In any case, the hope is that the new beta-decay data should provide a more secure reference for normalizing the (p,n) data, and the (p,n) data can then provide an extension of the measured GT-strength function beyond the limit of the beta-decay measurement, if we assume that the experiments are consistent.

In region f the experiments see similar strength distributions. Sextro, Gough, and Cerny [15] saw nothing in this region, and one could argue that the García *et al.* detection is at least a partial confirmation of the (p,n) experiment. Because the agreement is so promising in this region, we will obtain an estimate of the overall normalization N:

$$N = \frac{B_{\mathrm{peak}\ f}(\mathrm{GT})_{\beta^+}}{B_{\mathrm{peak}\ f}(\mathrm{GT})_{(p,n)}} . \tag{3}$$

We define region f to extend from $E_x = 6.9$ MeV upward in the García *et al.* data. In Fig. 5 we plot N as a function of B_{5051}(GT) for the two cases of Coulomb mixing. It can be seen that $0.87 \leq N \leq 1.56$. The case without

Coulomb mixing has been used to correct the (p,n) limit in Fig. 3, and it can be seen that this renormalization does slightly increase the range allowed by the limit [$B_{5051}(\mathrm{GT})\geq 0.33$ now], but not significantly.

If we assume that J_τ^2 and $J_{\sigma\tau}^2$ are well understood and that the Rapaport *et al.* data are reliable above 8.1 MeV, we can use this normalization factor to combine the two experiments and build a total GT distribution for the $A=37$ system. This distribution will be composed of the García *et al.* data below 8.1 MeV and the renormalized Rapaport *et al.* data above 8.1 MeV (peaks at 9.65 and 11.6 MeV). This synthesis will be a function of $B_{5051}(\mathrm{GT})$ and assumptions about Coulomb mixing since it builds upon all of our previous results.

It is interesting to try to estimate the ratio $J_{\sigma\tau}^2/J_\tau^2$ for the (p,n) experiment, using the normalization factor calculated above. In order to make this estimate, we must again assume that $J_{\sigma\tau}^2$ and J_τ^2 do not vary over relatively small ranges of energy. For the region of the IAS the Rapaport *et al.* experiment measured

$$J_\tau^2|M(\mathrm{F})|^2+J_{\sigma\tau}^2|M(\mathrm{GT})|^2=4.8\pm0.7 \ . \tag{4}$$

We know that $|M(\mathrm{F})|^2=3$ and $|M(\mathrm{GT})|^2$, a function of $B_{5051}(\mathrm{GT})$, can be obtained from the García *et al.* experiment: $|M(\mathrm{GT})|^2=B_{\mathrm{IAS}}(B_{5051}(\mathrm{GT}))/(1.262)^2$. $B_{\mathrm{IAS}}(B_{5051}(\mathrm{GT}))$ is given in Table I, and it has been defined as the amount of GT strength between 4.9 and 5.7 MeV. $J_\tau^2|M(\mathrm{F})|^2$ and $J_{\sigma\tau}^2|M(\mathrm{GT})|^2$ are still unknown in the above equation. To obtain their ratio, we use the strength seen by Rapaport *et al.* at 7.65 MeV:

$$J_{\sigma\tau}^2|M(\mathrm{GT})|^2=4.7\pm0.7 \ , \tag{5}$$

where $|M(\mathrm{GT})|^2$ is given by $(0.87\pm0.13)\times N(B_{5051}(\mathrm{GT}))$ and N is the normalization factor computed above. Now $J_{\sigma\tau}^2$ can be obtained and used to find J_τ^2. This has been done, and in Fig. 6 we plot the ratio as a function of $B_{5051}(\mathrm{GT})$. We are not able to say very much about the ratio because the uncertainties become quite large with increasing $B_{5051}(\mathrm{GT})$. Recall that below $B_{5051}(\mathrm{GT})\sim 0.33$ the experiments are not consistent and these results are not trustworthy. The value 4.89 based on the DWIA analysis of the (p,n) reaction at this proton energy [9] is within our range of uncertainty, but the range is so very large as to be unenlightening. The reason for this mushrooming of uncertainties is that J_τ^2 is linearly tending to zero near $B_{5051}(\mathrm{GT})=1.0$, enhancing the size of the overall uncertainty. In any case, $J_{\sigma\tau}^2/J_\tau^2$ does not appear to be inconsistent with the expectation of Ref. [10], i.e., $J_{\sigma\tau}^2/J_\tau^2=[E_p/(55\ \mathrm{MeV})]^2=(120/55)^2=4.76$.

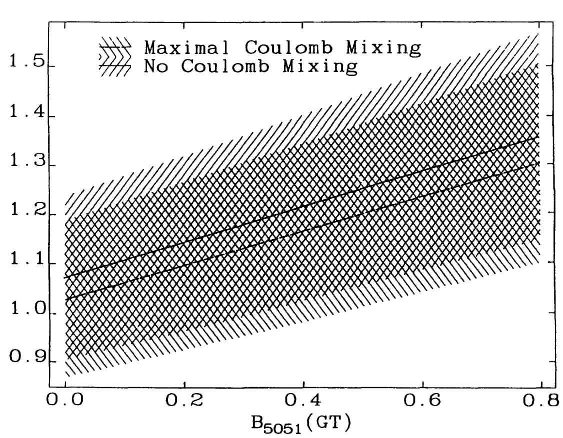

FIG. 5. Normalization of the (p,n) experiment calibrated as a function of $B_{5051}(\mathrm{GT})$. Experimental uncertainties are given by the crosshatching.

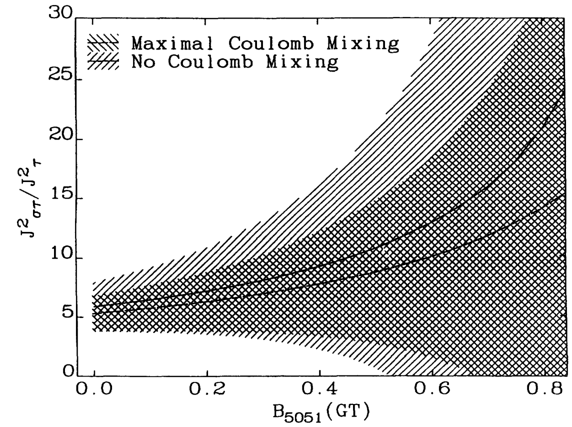

FIG. 6. Ratio $J_{\sigma\tau}^2/J_\tau^2$ as a function of $B_{5051}(\mathrm{GT})$. Experimental uncertainties are given by the crosshatching.

IV. NEUTRINO CROSS SECTION ON ^{37}Cl

Because $B_{<8\ \mathrm{MeV}}(\mathrm{GT})$ can vary by 30%, it is necessary to consider what this range of strength will do to the cross section of solar and supernova neutrinos on ^{37}Cl. For the ^{37}Cl solar neutrino experiment, the peaks seen at 9.65 and 11.5 MeV in the (p,n) experiment are not relevant because such states are unstable to neutron emission, resulting in $^{36}\mathrm{Ar}+n$, which will not be detected. Thus, in what follows, we use the strength distribution of García *et al.* as a function of $B_{5051}(\mathrm{GT})$, which was obtained above.

The cross section for neutrino-induced β^- decay from the ground state of ^{37}Cl to the jth state of ^{37}Ar is given by the formula [16]

$$\sigma_j(E_\nu)=\frac{1.206\times10^{-42}}{2\pi\alpha}\times\frac{w_e(j,E_\nu)p_e(j,E_\nu)F(Z,w_e(j,E_\nu))}{ft_{1/2}(j)}\ \mathrm{cm}^2 \ , \tag{6}$$

where α is the fine-structure constant, $w_e(j,E_\nu)$ and $p_e(j,E_\nu)$ are the resultant electron energy and momentum in units of electron rest mass, and $ft_{1/2}(j)$ is given in Eq. (1). For ^{37}Cl the electron energy is given by

$$w_e(j,E_\nu)=\frac{E_\nu-(E_j+0.303\ \mathrm{MeV})}{0.511\ \mathrm{MeV}} \ . \tag{7}$$

For a given source of the neutrinos, the total cross section of ^{37}Cl for neutrinos from that source is given by

$$\sigma=\sum_j \int dE_\nu \sigma_j(E_\nu)\varphi(E_\nu) , \tag{8}$$

where $\varphi(E_\nu)$ is the normalized energy distribution of neutrinos with energy. This distribution will vary with source.

Figure 7 shows the total cross sections calculated for the ^{37}Cl experiment for solar neutrinos from ^{8}B β^+ decay and ^{3}He+p (Hep). The neutrino-energy distributions have been taken from Bahcall [16]. The cross section for ^{8}B neutrinos increases by only 15% over the range of B_{5051}(GT), while the Hep cross section increases by 20%. The larger difference between the two extremes of Coulomb mixing in the Hep cross section is a result of the higher-energy neutrinos from this source, which make the total cross section larger. For the sun the flux of Hep neutrinos is 700 times less than the ^{8}B flux and thus only the latter will be seen by the ^{37}Cl experiment. The variation in B_{5051}(GT) makes the solar neutrino problem worse with increasing B_{5051}(GT), but only by 15%.

The variation of total cross section with B_{5051}(GT) is more pronounced for supernova neutrinos, because the neutrinos have much higher energies. Figure 8 shows the total cross section, assuming that the electron neutrinos emitted by the supernova have a Fermi-Dirac distribution with zero chemical potential and temperature of 4.5 MeV (we have chosen these parameters for ease of comparison with García's thesis). The average of the cross section over the Fermi-Dirac spectrum given by $\langle\sigma\rangle$ and the cross section at the average neutrino energy (approximately $3.15T_\nu$) denoted by $\sigma(\langle E_\nu\rangle)$ are plotted as functions of B_{5051}(GT). $\langle\sigma\rangle$ and $\sigma(\langle E_\nu\rangle)$ increase by 24% and 22%, respectively, over this range of B_{5051}(GT). In estimating detector response to supernova neutrinos, only $\langle\sigma\rangle$ should be used.

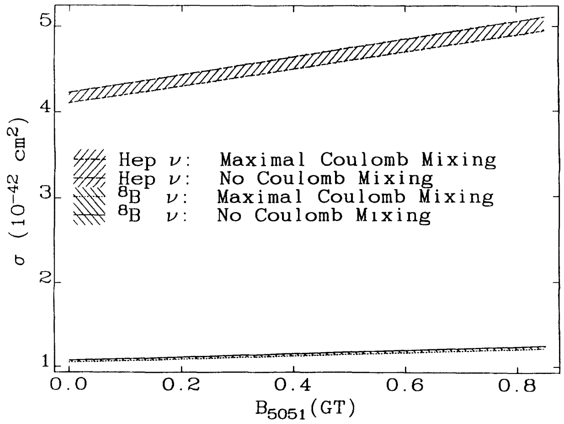

FIG. 7. Total cross section of solar neutrinos on ^{37}Cl for the two most important sources as a function of B_{5051}(GT). In each case the crosshatching covers the region allowed by any amount of Coulomb mixing, including experimental uncertainties. In these curves the uncertainties are so small that they are not visible.

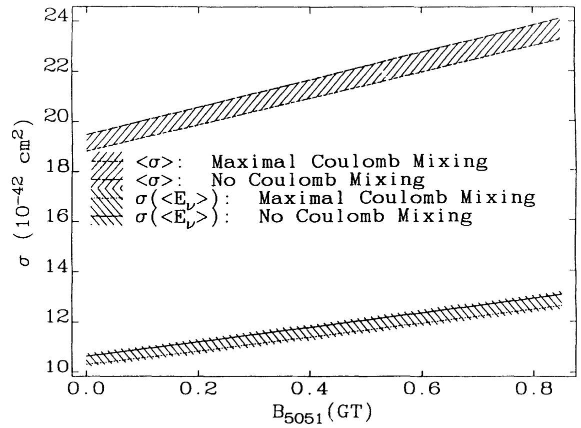

FIG. 8. Total cross section of supernova neutrinos on ^{37}Cl plotted as a function of B_{5051}(GT). In each case the crosshatching covers the region allowed by any amount of Coulomb mixing, including experimental uncertainties.

V. CONCLUSION

As has been seen above, there is a range within which the β^+ decay and charge-exchange experiments deduce GT matrix elements consistently. This argument has been made by comparing the experiments in two regions: 1317 keV and near 7650 keV. These regions are not obscured by the strong Fermi transition at 5051 keV. The lack of knowledge of how much GT strength is in this state is what causes there to be a range of uncertainty in the β^+-decay experiment. In order to determine whether the experiments are actually consistent, the GT strength either at 1371 or 5051 keV will have to be measured accurately, so that we can see where in this range the actual value is. Such a measurement of one of these quantities will drastically reduce the allowed range and should show whether the two experiments are consistent.

If the two experiments are consistent, a relatively large amount of GT strength, ≈ 0.5 unit, must reside in the IAS. This does not agree with the USD estimate of 0.1. The USD interaction has been optimized to accurately reproduce $M1$ transitions and magnetic moments for *sd*-shell nuclei [17]. For ^{37}Cl the effective magnetic-moment operator of Ref. [17] predicts a magnetic moment of 0.657, while the experimentally measured moment is 0.684, accurate to within 4%. The isovector-spin contribution to the magnetic moment is exactly the Gamow-Teller operator. Thus, because the interaction successfully reproduces the magnetic moments and $M1$ transition strengths of many *sd*-shell nuclei, it is unlikely that the USD estimate of GT strength in the 5051-keV state is very inaccurate because it is the IAS and identical with the ^{37}Cl ground state. For example, the USD GT strength for the ^{39}Ca to ^{39}K(IAS) transition is within 20% of the experimentally measured value [18]. Even if we still assume a factor of 2 uncertainty in B_{5051}(GT) (which is already a large range, given the arguments just made), it can be seen from Fig. 3 that the experiments are not consistent in their measurement of GT strength. This inconsistency will have to be resolved in future experiments.

A possible difficulty in the interpretation of the β^+ experiments is the assumption that the Fermi and GT strengths are strictly proportional to the proton-emission strengths [19]. It has already been noted that gamma decay could compete effectively with proton emission for at least some of the ^{37}K levels [20]. As a consequence, the Fermi and GT strengths as measured in the β^+ emission would, in some cases, be underpredicted. A good candidate for this situation is the ^{37}K level at $E_x = 3110$ keV, since, as seen in Fig. 2, the β^+ result is very much weaker than the (p,n) result (actually about 50 times weaker). This would be consistent with the fact that this state was not observed in the high-resolution (p,γ) reaction on ^{36}Ar [21]. An obvious explanation for this is that the width for proton emission is much less than the width for γ emission. If gamma decay does compete with proton emission for some of these states, then the intensities of the β-delayed protons will not be proportional to the Fermi and GT strengths and the inconsistency seen in this paper is resolved. Clearly, this question requires more experimental work.

Currently, the (p,n) experiment is being redone [13]. Its measurement of the 1371 keV state will be extremely important. If the new experiment does confirm the upper limit of the old one, we will be better able to test the ability of charge-exchange reactions to measure GT strength. With the new experiment's better resolution and the resolution of the questions about region c, it will be possible to make a more detailed comparison of the charge-exchange and β^+ data.

It would be valuable to measure the GT strength at 1371 and 5051 keV independently. At 5051 keV this could perhaps be done by the spin-polarization experiment $^{37}\mathrm{Cl}(\vec{p},n)^{37}\mathrm{Ar}$. Such an experiment could distinguish the Fermi and GT strengths in the IAS region. The resolution would be poor, but the García *et al.* data give all the detail needed to pick out B_{5051}(GT) from the rest of the GT strength which would be observed there, particularly if there is as much strength there as the Rapaport *et al.* data require. The experiment would be most successful if its resolution were good enough to resolve region e from the IAS.

The analysis of the experiment would do the following. The cross section for GT transitions into region d would be measured. The total GT strength in region d, B_d(GT), could be determined by dividing the measured cross section by $J^2_{\sigma\tau}$ determined from the new $^{37}\mathrm{Cl}(p,n)^{37}\mathrm{Ar}$ experiment. We assume that $J^2_{\sigma\tau}$ near the IAS will not vary largely from the fiducial value measured nearby. From the García *et al.* experiment, we know that

$$B_d(\mathrm{GT}) = (1.21 \pm 0.04) B_{5051}(\mathrm{GT}) + 0.63 \pm 0.10 , \qquad (9)$$

where we have summed over all states for which $4.9 \le E_x \le 5.7$ MeV. The uncertainties include the full range of Coulomb mixing. It is then possible to use the spin-polarized result to invert this and to determine B_{5051}(GT) unambiguously.

The region allowed for B_{1371}(GT) and B_{5051}(GT) could be narrowed by a more accurate measurement of the strength in either state. In the case of the strength at 1371 keV, it is possible, although very difficult, to make a direct measurement of the beta decay from ^{37}Ca to this level. This would be a direct and totally independent measurement of the GT strength in this state. It also does not require any assumptions about the variation of $J^2_{\sigma\tau}$.

Thus we find that the two experiments are not consistent in their extraction of GT strength, if one believes that B_{5051}(GT) is 0.1 within even a factor of 2, as predicted by the USD interaction. This single theoretical restriction leads to the conclusion that the quantities in one or both of the experiments are not always proportional to the GT matrix elements squared. Resolving this inconsistency will require more study of each experiment on both experimental and theoretical fronts.

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