Relativistic effects in proton-proton bremsstrahlung

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The $pp\gamma$ reaction at intermediate energies is investigated using a modern relativistic meson-exchange potential model. Both the single-scattering and rescattering terms are treated including relativistic corrections. It is shown that for energetic photons in the region near the end-point energy the relativistic spin correction is very large and reduces the $pp\gamma$ cross section by a factor of ~ 2. This large correction is dynamical in origin and can be traced in a very simple way to the strong energy dependence of the $^1S_0$ state contribution. It is also shown that, under these kinematical conditions, the rescattering term *reduces* the cross section, which amplifies the relativistic effects on the $pp\gamma$ reaction. For the kinematical conditions of the recent TRIUMF experiment at $T_{\rm lab}=280$ MeV the rescattering contribution *enhances* the cross section and reduces the discrepancy between the data and the calculation.

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I. INTRODUCTION

Interest in the nucleon-nucleon bremsstrahlung ($NN\gamma$) reaction as a means of testing the off-energy-shell behavior of the NN interaction has been rejuvenated considerably, both experimentally and theoretically, in the past few years [1–8]. In fact, it has been demonstrated for the first time by the TRIUMF group [1–4] that $NN\gamma$ reactions, or more specifically the analyzing power in the proton-proton bremsstrahlung ($pp\gamma$) process at an incident energy near the pion production threshold, shows a clear signature of off-shell effects. Also the spin correlation coefficients are shown to be very sensitive to the off-shell behavior of the NN interaction [7]. However, a rather disturbing disagreement has been observed between recent $pp\gamma$ cross-section data [5] at an incident energy of $T_{\rm lab}=280$ MeV and calculations [2,3] using a modern potential model. In order to remove this disagreement, the *data* of Ref. [5] were reduced by a factor of $\frac{2}{3}$; however, no obvious reason for an error in the absolute normalization has been found. A large part of the disagreement between the calculated and the nonrenormalized data may be attributed to the relativistic spin correction (RSC) [9,10] which reduces the calculated cross section by about 20–30 % with respect to the nonrelativistic results [11] (our present calculation which includes the RSC confirms this discrepancy). Therefore, if the unmodified TRIUMF data are correct, the potential model may be unable to describe $NN\gamma$ processes. The calculation [2,3] accompanying the experiment reported in Ref. [5], however, does not include the rescattering (RES) contribution which, in the nonrelativistic approach, enhances the cross section by about 20% [8]. Significant RES corrections have also been found in nonrelativistic calculations made for different kinematical conditions [12–14]. Given the comparable sizes of the RSC and RES contributions it is, therefore, crucial that both corrections are included simultaneously for a proper assessment of the current status of agreement between theory and experiment. This is the major purpose of the present work.

In connection with the problem mentioned above, we also investigate relativistic effects with an emphasis on the RSC, in an effort to better understand its role in $pp\gamma$ reactions. Relativistic effects have been investigated by a number of authors in the past [2,15,16] who have shown that these effects reduce the $pp\gamma$ cross section with respect to nonrelativistic results and that the dominant relativistic correction arises from the RSC. However, the physical basis for this reduction has received limited attention; we focus on identifying the physics underlying this reduction. In particular, we demonstrate that, for energetic photons, the effect is largely dynamical in origin and arises predominantly from the strong energy dependence of the $^1S_0$ state. We show that this can be understood in a very simple way.

The present paper is organized as follows. In Sec. II we outline the formalism used. In Sec. III the RSC is analyzed. The effect of the RES term is discussed in Sec. IV together with a comparison with selected data. The conclusions are discussed in Sec. V. Appendix A contains some details of the formalism. In Appendix B, a procedure for constructing a Lorentz invariant transition amplitude from a nonrelativistic T matrix is presented.

II. FORMALISM

In the present work, the pp bremsstrahlung transition amplitude is calculated in momentum space within a framework of potential models. The calculation includes the single-scattering contribution [second and third terms in Eq. (2.1b)] as well as the double-scattering, or rescattering, contribution [last term in Eq. (2.1b)]. We write the invariant transition amplitude $\tilde{M}$ for producing a photon of momentum $\mathbf{k}$ and polarization $\boldsymbol{\epsilon}$ in an NN collision as

$$\tilde{M}=\sqrt{\varepsilon_1'\varepsilon_2'\omega}\,M\sqrt{\varepsilon_1\varepsilon_2}\ , \tag{2.1a}$$

where ω denotes the energy of the emitted photon and $\varepsilon_1',\varepsilon_2'$ $(\varepsilon_1,\varepsilon_2)$ are the energies of the two interacting nucleons, 1 and 2, in the final (initial) state; they are defined as $\varepsilon_i\equiv\varepsilon_{p_i}\equiv\sqrt{\mathbf{p}_i^2+m^2}$ with $\mathbf{p}_i$ being the momentum of ith nucleon and m the nucleon mass. In the above equation M denotes the bremsstrahlung transition amplitude.

In order to facilitate a close comparison with the bremsstrahlung transition operator obtained within the entirely nonrelativistic approach, we express M as

$$\begin{aligned}M=&\langle\boldsymbol{\epsilon},\mathbf{k};\phi_f|V_{\rm em}|0;\phi_i\rangle+\langle\boldsymbol{\epsilon},\mathbf{k};\phi_f|(T^-)^\dagger\mathcal{G}_fV_{\rm em}|0;\phi_i\rangle\\&+\langle\boldsymbol{\epsilon},\mathbf{k};\phi_f|V_{\rm em}\mathcal{G}_iT^+|0;\phi_i\rangle\\&+\langle\boldsymbol{\epsilon},\mathbf{k};\phi_f|(T^-)^\dagger\mathcal{G}_fV_{\rm em}\mathcal{G}_iT^+|0;\phi_i\rangle\ ,\end{aligned} \tag{2.1b}$$

where ϕ denotes the two-nucleon nonrelativistic unperturbed wave function; $\mathcal{G}$ is the energy denominator (or nucleon Green's function) consistent with the approximation used for solving the T-matrix integral equation. T^+ and T^- stand for the NN T matrices associated with the outgoing $(+)$ and incoming $(-)$ waves, respectively. The subscript i (f) refers to the initial (final) two-nucleon state.

$V_{\rm em}$ in Eq. (2.1b) denotes the effective electromagnetic transition operator to be used with nonrelativistic two-component wave functions. It is defined as

$$\begin{aligned}&V_{\rm em}(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2)\\&=\left[\frac{m}{\varepsilon_{p_1'}}\right]^{1/2}\langle\bar{u}(\mathbf{p}_1')|\hat{V}_{\rm em}^{(1)}|u(\mathbf{p}_1)\rangle\left[\frac{m}{\varepsilon_{p_1}}\right]^{1/2}\\&\quad+\left[\frac{m}{\varepsilon_{p_2'}}\right]^{1/2}\langle\bar{u}(\mathbf{p}_2')|\hat{V}_{\rm em}^{(2)}|u(\mathbf{p}_2)\rangle\left[\frac{m}{\varepsilon_{p_2}}\right]^{1/2}\ ,\end{aligned} \tag{2.2a}$$

with

$$|u(\mathbf{p}_i)\rangle=\left[\frac{\varepsilon_{p_i}+m}{2m}\right]^{1/2}\begin{bmatrix}1\\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}_i}{\varepsilon_{p_i}+m}\end{bmatrix} \tag{2.2b}$$

denoting the positive energy Dirac spinor without the two-component spin-wave function χ_s. We follow Bjorken and Drell's notation [17].

The relativistic electromagnetic interaction Hamiltonian $\hat{V}_{\rm em}^{(i)}$ for nucleon i $(=1,2)$ is obtained from the Dirac equation describing the interaction of a nucleon with an external electromagnetic field. It is given by

$$\hat{V}_{\rm em}^{(i)}=-e_i\gamma_\mu A^\mu+\frac{(\mu_i-1)e}{4m}\sigma_{\mu\nu}F^{\mu\nu}\ , \tag{2.3}$$

where the last term is the magnetic moment interaction representing the observed anomalous magnetic moment of the nucleon i. Here, we use the usual notation for the four-vector potential of the electromagnetic field, A^μ, and the corresponding field tensor, $F^{\mu\nu}=\partial^\nu A^\mu-\partial^\mu A^\nu$. γ_μ denotes the Dirac γ matrix and $\sigma_{\mu\nu}=i[\gamma_\mu,\gamma_\nu]/2$. e_i denotes the nucleon charge ($e_i=e_p=e$ for protons and $e_i=e_n=0$ for neutrons) and μ_i the anomalous nuclear magnetic moment in units of nuclear magnetons (for protons $\mu_i=\mu_p=2.793$ and for neutrons $\mu_i=\mu_n=-0.913$).

Then, working in the Coulomb gauge, the effective electromagnetic transition operator in Eq. (2.2a) can be expressed as the sum of four terms

$$V_{\rm em}=V_{\rm conv}+V_{\rm mag}+V_{\rm rsc}+V_{\rm rem}\ , \tag{2.4a}$$

where, in the NN center-of-mass (c.m.) frame,

$$V_{\rm conv}=-\left[\frac{2\pi}{k}\right]^{1/2}\frac{e}{2m}\boldsymbol{\epsilon}\cdot(\mathbf{p}+\mathbf{p}')\{\tilde{e}_1^-\,\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k}/2)-\tilde{e}_2^+\,\delta(\mathbf{p}-\mathbf{p}'+\mathbf{k}/2)\}\ , \tag{2.4b}$$

$$V_{\rm mag}=-i\left[\frac{2\pi}{k}\right]^{1/2}\frac{e}{2m}\{\tilde{\mu}_1^-\boldsymbol{\epsilon}\cdot(\mathbf{k}\wedge\boldsymbol{\sigma}_1)\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k}/2)+\tilde{\mu}_2^+\boldsymbol{\epsilon}\cdot(\mathbf{k}\wedge\boldsymbol{\sigma}_2)\delta(\mathbf{p}-\mathbf{p}'+\mathbf{k}/2)\}\ , \tag{2.4c}$$

$$V_{\rm rsc}=i\left[\frac{2\pi}{k}\right]^{1/2}\frac{e}{2m}\{\tilde{\nu}_1^-\boldsymbol{\epsilon}\cdot(\mathbf{p}'\wedge\boldsymbol{\sigma}_1)\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k}/2)-\tilde{\nu}_2^+\boldsymbol{\epsilon}\cdot(\mathbf{p}'\wedge\boldsymbol{\sigma}_2)\delta(\mathbf{p}-\mathbf{p}'+\mathbf{k}/2)\}\ , \tag{2.4d}$$

and

$$V_{\rm rem}=-i\left[\frac{2\pi}{k}\right]^{1/2}\frac{e}{2m}\{\tilde{\eta}_1^-\boldsymbol{\epsilon}\cdot(\mathbf{p}'\wedge\mathbf{k})\boldsymbol{\sigma}_1\cdot\mathbf{p}\,\delta(\mathbf{p}-\mathbf{p}'-\mathbf{k}/2)+\tilde{\eta}_2^+\boldsymbol{\epsilon}\cdot(\mathbf{p}'\wedge\mathbf{k})\boldsymbol{\sigma}_2\cdot\mathbf{p}\,\delta(\mathbf{p}-\mathbf{p}'+\mathbf{k}/2)\}\ . \tag{2.4e}$$

In the above equations, $\mathbf{p}$ and $\mathbf{p}'$ denote the relative momenta of the two interacting nucleons before and after the emission of a photon with momentum $\mathbf{k}$ and polarization $\boldsymbol{\epsilon}$. $\boldsymbol{\sigma}_i$ stands for the Pauli spin matrix for nucleon i ($=1,2$). The factors $\tilde{e}_i^{\pm}$, $\tilde{\mu}_i^{\pm}$, $\tilde{\nu}_i^{\pm}$, and $\tilde{\eta}_i^{\pm}$ are functions of nucleon and photon momenta; they are given explicitly in Appendix A. The leading terms in $\tilde{e}_i^{\pm}$ and $\tilde{\mu}_i^{\pm}$ are given by $\tilde{e}_i^{\pm}=\delta_{e,e_i}$ and $\tilde{\mu}_i^{\pm}=\mu_i-\delta_{0,e_i}$ which give rise to the convection and magnetization current operators of the conventional nonrelativistic approach. $\tilde{\nu}_i^{\pm}=(\mu_i-1)k/m$ and $\tilde{\eta}_i^{\pm}=(\mu_i-1)/2m^2$ to leading orders.

In Eq. (2.4), the dominant RSC is denoted by V_{rsc}. It results from taking the matrix element of the relativistic four-component electromagnetic transition operator between the upper and lower components of the Dirac spinors. Inclusion of V_{rsc} introduces a "composite" current (convection $\otimes$ spin) term into V_{em}. The composite currents also appears in other reactions such as (p,p') when the spin-dependent part of the NN coupling is momentum dependent [18], as arises from an analogous two-component reduction [19] of the Dirac spinors, for example. We note that in Eq. (2.4d) we have made use of the fact that in the NN c.m. frame $\mathbf{p}_1+\mathbf{p}_1'=\mathbf{p}+\mathbf{p}'-\mathbf{k}/2$ in the first term and $\mathbf{p}_2+\mathbf{p}_2'=-(\mathbf{p}+\mathbf{p}'+\mathbf{k}/2)$ in the second term in brackets. Together with the restrictions imposed by momentum conservation these terms reduce to $\mathbf{p}_1+\mathbf{p}_1'=2\mathbf{p}'$ and $\mathbf{p}_2+\mathbf{p}_2'=-2\mathbf{p}'$. The remaining relativistic correction, V_{rem} in Eq. (2.4), has a different operator structure than the first three terms. Since $\tilde{\eta}_i^{\pm}=(\mu_i-1)/2m^2$ to leading order, it is easily seen that this term is of order $\mathcal{O}((p/m)^2)$ while the dominant RSC, V_{rsc} is of order $\mathcal{O}(p/m)$. At intermediate nucleon incident energies the contribution from V_{rem} to the bremsstrahlung reaction is negligible compared to those from the other terms.

We note that in the present approach we do not introduce an additional term due to the Lorentz boosting effect [20–25] in the electromagnetic transition operator V_{em} given by Eq. (2.4a), since this operator is fully covariant apart from the factors $\sqrt{m/\varepsilon}$. In Ref. [26] we expanded the functions $\tilde{e}_i^{\pm}$, $\tilde{\mu}_i^{\pm}$, $\tilde{\nu}_i^{\pm}$, and $\tilde{\eta}_i^{\pm}$ of Eq. (2.4) in powers of m^{-1} keeping terms through m^{-2} and therefore an additional term, ΔV_{em}, due to Lorentz boosting was, in principle, necessary. In the present work we use the full expression for V_{em}.

In a consistent approach, the T matrix which describes the interaction between the two nucleons should be obtained from a relativistic integral equation, such as the Bethe-Salpeter equation or some approximation to it, so that the resulting Lorentz invariant T matrix $\tilde{T}$ (see Appendix B) and the interaction T entering Eq. (2.1b) should be related to each other by

$$\tilde{T}(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2)=\sqrt{\varepsilon_1'\varepsilon_2'}\,T(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2)\sqrt{\varepsilon_1\varepsilon_2} \tag{2.5}$$

in order to yield a Lorentz invariant transition amplitude $\tilde{M}$ [Eq. (2.1a)]. Conversely, T should have a Lorentz transformation property such that $\tilde{T}$ given by Eq. (2.5) be a Lorentz invariant. Unlike the description of the NN elastic scattering process, this issue becomes important in the description of a process, such as NN bremsstrahlung at high incident energies, where the calculations are based on NN potential models and one makes use of the Lorentz invariant nature of the transition amplitude. The approximations one makes to the relativistic integral equation for obtaining the T matrix $\tilde{T}$ are usually consistent with the transformation property that T must obey. A problem arises, however, when one uses a T matrix obtained nonrelativistically in the relativistic scheme, since in this case, the T matrix obeys the Lippmann-Schwinger equation which preserves the Galilean invariance and not the Lorentz invariance of the NN potential. This is the case of T matrices based on phenomenological NN potentials, such as the Hamada-Johnston [27], Reid [28], or the Yukawa parametrized version of the Paris potential [29]. Although in the present work we use a relativistic T-matrix interaction and, therefore, do not face the problem mentioned above, we discuss for completeness in Appendix B a procedure for constructing a Lorentz invariant transition amplitude from such a nonrelativistic T matrix.

When the electromagnetic coupling to a nucleon is taken to first order the NN bremsstrahlung process involves extra intermediate states [described by the propagators $\mathcal{G}_{i,f}$ in Eq. (2.1b)] compared to the NN elastic scattering process. Most of the existing NN bremsstrahlung calculations, within either the relativistic or nonrelativistic approaches, assume an expression of the form (in the NN c.m. frame) [2,12,30]

$$\mathcal{G}_{i,f}=\frac{1}{E_{i,f}-2\varepsilon_{p''}} \tag{2.6}$$

for the energy denominators irrespective of the type of propagator entering the T-matrix integral equation. In the above equation, $E_i=2\varepsilon_p$ and $E_f=2\varepsilon_{p'}$; the double prime denotes intermediate states. In a consistent approach, however, the above propagator $\mathcal{G}$ should be of the same type as that used for solving the T-matrix integral equation [31]. For example, for the bremsstrahlung transition amplitude based on the one-boson-exchange potential (OBEPQ) T matrix used in the next two sections, we have

$$\mathcal{G}_{i,f}=\left[\frac{\varepsilon_{p''}}{m}\right]\frac{4m}{E_{i,f}^2-(2\varepsilon_{p''})^2} \tag{2.7}$$

which is consistent with the Blankenbecler-Sugar (BbS) choice of the two-nucleon propagator (see Appendix B) [32]; an analogous observation holds for transition amplitudes based on nonrelativistic T matrices as discussed in Appendix B. The NN bremsstrahlung reaction, therefore, probes not the differences in the T-matrix interactions used, but the composite differences in the T-matrix interactions and the corresponding propagators $\mathcal{G}$ in Eq. (2.1b).

We should stress, however, that within a complete relativistic formulation it is difficult to use consistently T-matrix interactions which are obtained by treating the two nucleons symmetrically, such as in the three-dimensional BbS reduction of the Bethe-Salpeter equation. The reason is that in the NN bremsstrahlung process one of the nucleons is necessarily off-shell while the other one is on-shell and, therefore, the nucleons cannot be treated symmetrically.

III. RELATIVISTIC EFFECTS

To our knowledge, only the work by Liou and Sobel [15] examines the physics behind the RSC to the $pp\gamma$ process. These authors found that for proton incident energies up to $T_{\rm lab}=160$ MeV and for proton scattering angles around 30° relativistic effects are very small, contrary to their expectation [15] that the RSC would be large since, on kinematical grounds, one might expect it to reduce the cross section by a factor of $(1-[1-\frac{1}{2}\mu_p]p/m)^2$. By more closely examining the various spin matrix elements of the RSC term, the authors of Ref. [15] found that these matrix elements are approximately proportional to the sum of the momenta of the interacting nucleons, $\mathbf{p}_1+\mathbf{p}_2$, and so are small in the NN c.m. frame. The fact that this is not the complete basis for the RSC effect becomes clear by considering the case of two protons scattering at the smaller angles of $\theta_1=15°$ and $\theta_2\sim 20°$ at an incident energy of $T_{\rm lab}=156$ MeV. In this kinematical regime relativistic corrections have been shown [16] to be very important; in particular, they reduce the cross section by $\sim 30\%$ with respect to the nonrelativistic results which is surprising in view of the findings reported in Ref. [15]. One might argue that this is due to much higher energy photons being produced in the geometry considered in Ref. [16] than in the case of Ref. [15]; when the proton scattering angles are decreased the photon energy increases. However, as shown below, the dominant RSC term ($V_{\rm rsc}$), unlike the magnetization current $V_{\rm mag}$, varies slowly with photon momentum, so that the large reduction of the cross section observed in the calculation of Ref. [16] relative to that of Ref. [15] is due to more than just the increase in the photon energy. Also, in calculations accompanying the recent coplanar geometry experiment by the TRIUMF group [5] at an incident energy of $T_{\rm lab}=280$ MeV and proton scattering angles of $\theta_1\sim\theta_2\sim 12°$, relativistic effects have been shown to reduce the cross section by about 20–30 % [11]. In these calculations the incident energy is much higher and the proton scattering angles are smaller than in the calculations of Ref. [16]. Consequently, based on kinematical factors alone, one would expect a larger reduction of the cross section than was found in Ref. [16]. Therefore, the behavior of the RSC to $pp\gamma$ reactions cannot be explained exclusively in terms of the kinematics involved.

Before discussing the major results of the present section, we note that a simple comparison of Eq. (2.4c) and Eq. (2.4d), together with the fact that $\tilde{\mu}_1^-=\tilde{\mu}_2^+=\mu_p$ and $\tilde{\nu}_1^-=\tilde{\nu}_2^+=(\mu_p-1)k/m$ to leading order, reveal that it is not sufficient to analyze the effect of the RSC in terms of an overall reduction factor of the cross section. We also note that while $V_{\rm mag}$ in Eq. (2.4c) contains an overall factor of k, $V_{\rm rsc}$ in Eq. (2.4d) contains an overall factor of p'. Therefore, together with the leading term in $\tilde{\nu}_1^-$ and $\tilde{\nu}_2^+$, $V_{\rm rsc}$ is proportional to the product $p'k/m$ while $V_{\rm mag}$ is proportional to k. Due to energy-momentum conservation, when k increases (decreases) p' decreases (increases); consequently, $V_{\rm rsc}$ varies slowly as the energy of the emitted photon is varied.

As has been shown elsewhere [14], for energetic photons produced in the $pp\gamma$ process, the dominant current is the magnetization current, $V_{\rm mag}$. Moreover, among the various ways in which a photon may be created in a $NN\gamma$ reaction, the one in which the photon is emitted *before* the strong interaction takes place is favored due to the larger NN coupling at lower NN c.m. energy. The spin matrix element of $V_{\rm mag}$ is, then, proportional to [6]

$$\langle S'|V_{\rm mag}|S\rangle \propto (1-\delta_{S',0}\delta_{S,0})[(-1)^S\tilde{\mu}_1^- f(\mathbf{p}',\mathbf{p}-\mathbf{k}/2) + (-1)^{S'}\tilde{\mu}_2^+ f(\mathbf{p}',\mathbf{p}+\mathbf{k}/2)]\ , \qquad (3.1)$$

where S (S') denotes the NN total spin in the initial (final) state and f is a function common to $\tilde{\mu}_1^-$ and $\tilde{\mu}_2^+$. The first factor reflects the fact that the spin operator cannot connect spin-singlet states. Moreover, for the range of photon momentum $\mathbf{k}$ involved in most of the existing $NN\gamma$ experiments, both $f(\mathbf{p}',\mathbf{p}-\mathbf{k}/2)$ and $f(\mathbf{p}',\mathbf{p}+\mathbf{k}/2)$ have the same sign. Also, to leading order $\tilde{\mu}_1^-=\tilde{\mu}_2^+$ for identical particles so that the above matrix element will be suppressed to a large extent if a spin-singlet state is present in either the initial or final state as has been pointed out in Ref. [6]. In particular, the strong $^1S_0$ state only contributes significantly to the $pp\gamma$ cross section for those kinematical conditions where the cancellation in Eq. (3.1) is minimized.

It is now easy to understand the effect of the RSC. Given the form of the spin matrix element of $V_{\rm mag}$ [Eq. (3.1)] and comparing Eq. (2.4c) and Eq. (2.4d), one finds that the spin matrix element of $V_{\rm rsc}$ is proportional to

$$\langle S'|V_{\rm rsc}|S\rangle \propto -(1-\delta_{S',0}\delta_{S,0})[(-1)^S\tilde{\nu}_1^- g(\mathbf{p}',\mathbf{p}-\mathbf{k}/2) - (-1)^{S'}\tilde{\nu}_2^+ g(\mathbf{p}',\mathbf{p}+\mathbf{k}/2)]\ , \qquad (3.2)$$

where g is a function common to $\tilde{\nu}_1^-$ and $\tilde{\nu}_2^+$. To leading order, $\tilde{\nu}_1^-=\tilde{\nu}_2^+=(\mu_p-1)k/m$ as mentioned before. The two terms in brackets in the above matrix element [or alternatively the transition operator $V_{\rm rsc}$ in Eq. (2.4d)] differ by a minus sign relative to the magnetization current contribution. These two terms correspond to contributions from the interacting nucleons 1 and 2, respectively. The origin of this relative sign is due to the fact that $V_{\rm rsc}$ is proportional to terms involving the cross product of the Pauli spin matrix with the sum of the nucleon momenta in the initial and final states, i.e., $\mathbf{p}_1+\mathbf{p}_1'$ in the first term and $\mathbf{p}_2+\mathbf{p}_2'$ in the second term. In the NN c.m. frame the two interacting nucleons have momenta which differ by a minus sign. This relative minus sign between the two terms does not occur in the case of the magnetization current contribution, where the photon momentum appears in place of the nucleon momentum in both terms (1 and 2). Due to this relative minus sign, the contribution from singlet states in general, and the strong $^1S_0$ state in particular, will not be suppressed as in the magnetization current term. Therefore, when the two nucleons interact *after* losing energy by emitting energetic photons, the ${}^1S_0$ state contribution becomes significant, even though the matrix element contains the relatively small factors $p'\tilde{v}_1^-$ and $p'\tilde{v}_2^+$ compared to $k\tilde{\mu}_1^-$ and $k\tilde{\mu}_2^+$ in the magnetization current matrix element. This occurs because the final state c.m. energy is small in this case which favors scattering in the ${}^1S_0$ state where the T-matrix element is very large. We emphasize that the contribution from $V_{\rm rsc}$ itself is much smaller than that from $V_{\rm mag}$ but what makes the RSC term, $V_{\rm rsc}$, so important is the interference between $V_{\rm rsc}$ and the dominant magnetic current term $V_{\rm mag}$. Whether the interference is destructive or constructive depends on the relative size of the first and second terms in Eq. (3.2). For most of the kinematical conditions considered the interference between $V_{\rm rsc}$ and $V_{\rm mag}$ is destructive.

The features just discussed above are illustrated in Fig. 1 where the $pp\gamma$ cross sections in the coplanar geometry are shown at incident energies of $T_{\rm lab}=280$ and 100 MeV for proton scattering angles of $\theta_1=12.4°$ and $\theta_2=12.0°$ [Figs. 1(a) and 1(b)]. We calculate all bremsstrahlung amplitudes in the appropriate NN c.m. frame. In this work the relativistic T-matrix interaction based on the one-boson-exchange potential [33] (OBEPQ version) is used and partial-wave states through total angular momentum $J=11$ are included. The data are from Ref. [5]. First, we note that in the calculation which includes the RSC (solid curves) the cross sections for forward and backward photon emission angles are reduced by $\sim 25\%$ at $T_{\rm lab}=280$ MeV and by $\sim 50\%$ at $T_{\rm lab}=100$ MeV with respect to the corresponding nonrelativistic results (dash-dotted curves). The reason for relatively larger RSC effect at $T_{\rm lab}=100$ MeV than at $T_{\rm lab}=280$ MeV is due to the relatively larger influence of the ${}^1S_0$ state at lower incident energy. The dashed curves are the results when the ${}^1S_0$ state contribution from $V_{\rm rsc}$ (and only from $V_{\rm rsc}$) is switched off. As discussed above, the RSC is dominated by scattering in the ${}^1S_0$ state; this is particularly clear at $T_{\rm lab}=100$ MeV where the RSC is almost entirely due to the strong ${}^1S_0$ state contribution. Figure 1(c) also illustrates the role of RSC for the inclusive cross section at an incident energy of $T_{\rm lab}=280$ MeV and for a fixed photon energy of $\omega=125$ MeV which is only about 5 MeV less than the maximum photon energy one can obtain at this proton incident energy. Effects similar to those observed in Fig. 1(b) at $T_{\rm lab}=100$ MeV can be seen.

In a coplanar geometry experiment, the RSC becomes more pronounced as the proton scattering angles decrease since the energy of the emitted photon increases as these angles decrease so that the available c.m. energy of the scattered protons decreases and the T matrix in the ${}^1S_0$ state becomes very large. This is illustrated in Fig. 2(a) where the $pp\gamma$ cross section at $T_{\rm lab}=280$ MeV is shown as a function of symmetric proton scattering angles, $\theta_1=\theta_2$, and for a fixed photon emission angle of $\theta=175°$. We see that around $\theta_1=\theta_2=5°$ the RSC (solid curve) reduces the cross section by more than a factor of 2 compared to the nonrelativistic result (dash-dotted curve). This is particularly relevant for experiments currently being performed at Indiana University Cyclotron Facility (IUCF) [34], where the $pp\gamma$ cross sections are measured for proton scattering angles down to $\sim 5°$. We observe that most calculations in the past and, in particular, the calculation of Ref. [15], were for proton scattering angles around $\sim 30°$ and, consequently, did not

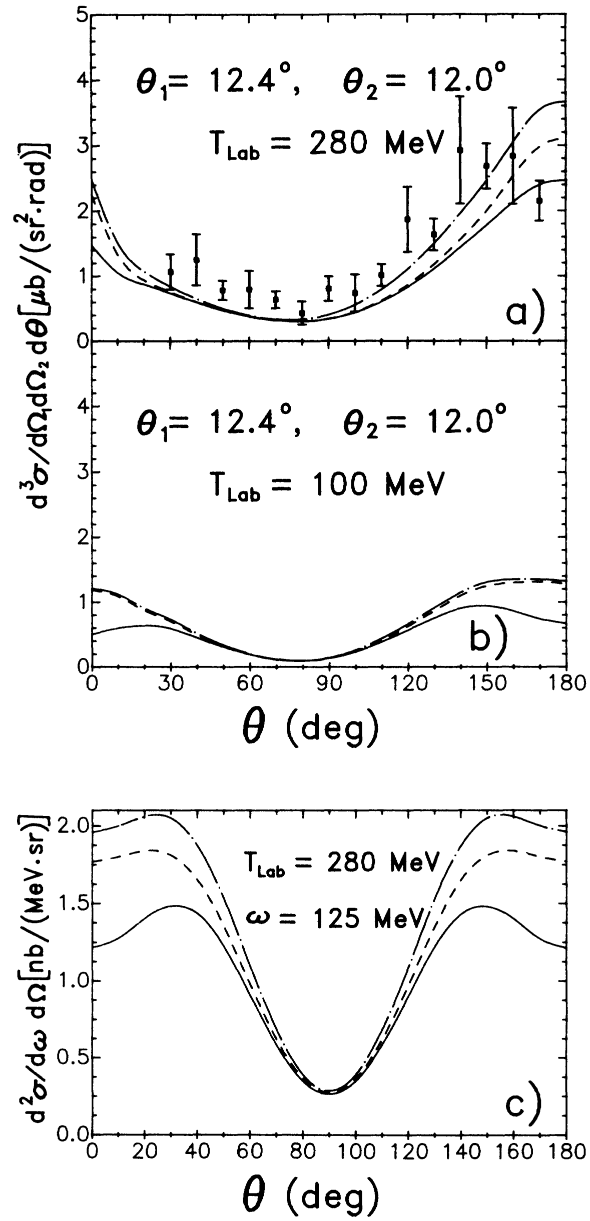

FIG. 1. $pp\gamma$ cross sections with (solid lines) and without (dash-dotted lines) the RSC. Results with RSC when the contribution from the ${}^1S_0$ state in $V_{\rm rsc}$ [see Eq. (2.4)] is switched off are represented by dashed lines. The one-boson-exchange potential of Ref. [33] has been used for generating the NN T matrix. (a) Coplanar geometry cross section in the laboratory frame as a function of photon emission angle at a proton incident energy of $T_{\rm lab}=280$ MeV and for proton scattering angles of $\theta_1=12.4°$ and $\theta_2=12°$. The data are from Ref. [5] and do not contain the arbitrary normalization factor of $\frac{2}{3}$. (b) Same as (a) at $T_{\rm lab}=100$ MeV. (c) Inclusive cross section in the initial NN center-of-mass frame as a function of photon emission angle θ at $T_{\rm lab}=280$ MeV for a fixed photon energy of $\omega=125$ MeV.

strongly sample the $^1S_0$ state contribution at small c.m. energies where it is very large. As we have shown in Fig. 1(b), for sufficiently small forward proton scattering angles, the RSC is large even at $T_{\rm lab}=100$ MeV. In Fig. 2(a), a nearly constant RSC effect for $\theta_1=\theta_2>10°$ where the photon energy is rapidly decreasing is due to the fact that in this region of proton scattering angles the T matrix in the $^1S_0$ state is much smaller and less energy dependent than at smaller proton angles. In addition, the decrease in the photon energy is to a large extent compensated by the increase in the nucleon relative momentum p' as discussed before.

Since the strongest off-energy-shell dependence is in the $^1S_0$ state, off-shell effects on the $pp\gamma$ cross section are large where the RSC is also large; this can be seen in Figs. 2(a) and 2(b), where the on-shell results (dotted curves) are shown for exclusive and inclusive cross sections. However, because of the destructive interference between $V_{\rm rsc}$ and $V_{\rm mag}$, off-shell effects on the $pp\gamma$ cross section become less pronounced in an approach where relativistic corrections to $V_{\rm em}$ are included as compared to an entirely nonrelativistic approach [6]. Nevertheless, the sensitivity to off-shell effects is similar to that of a nonrelativistic calculation and enough to be seen clearly, especially for kinematical conditions where the photon energy is near its end point as occurs in Fig. 2(b) or in Fig. 2(a) for proton angles around $\theta_1=\theta_2\sim5°$.

We mention that the analyzing power is relatively weakly sensitive to the RSC compared to the corresponding cross section. This is not surprising to the extent that the dominant RSC effect is in the $^1S_0$ state and that the analyzing power is relatively insensitive to this state in $NN\gamma$ reactions [6].

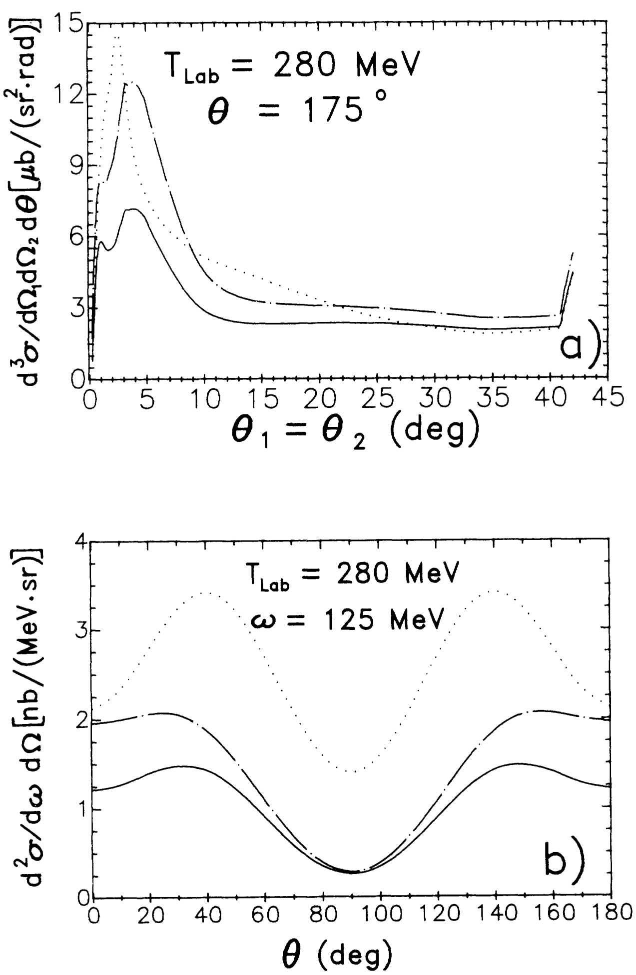

FIG. 2. $pp\gamma$ cross sections with (solid lines) and without (dash-dotted lines) the RSC. On-shell results according to Ref. [6] with RSC are represented by dotted lines. The one-boson-exchange potential of Ref. [33] has been used for generating the NN T-matrix elements. (a) Coplanar geometry cross section in the laboratory frame as a function of symmetric proton scattering angle $\theta_1=\theta_2$ at a proton incident energy of $T_{\rm lab}=280$ MeV and for a fixed photon emission angle of $\theta=175°$. (b) Same as Fig. 1(c).

IV. RESCATTERING CONTRIBUTION AND COMPARISON WITH THE DATA

It has been shown [35,36] that in order to preserve the gauge invariance of the theory, both the one-body RES term and the two-body current term should be included to the same order in the photon momentum. However, two-body currents are expected to contribute very weakly to the $pp\gamma$ reaction. Accordingly, the violation of gauge invariance in this reaction introduced by omitting the two-body current term is not expected to be serious. Moreover, a fully consistent treatment of the two-body current within a realistic potential model calculation is not yet available; see, however, Ref. [37] where consistent two-body currents have been obtained for separable interactions. We note that most modern meson-exchange NN potentials are nonlocal potentials which generate two-body currents. Keeping in mind the present uncertainty in the theory, we investigate in this section the role of the RES contribution in the $pp\gamma$ reaction within the present approach and compare with the recent data of the TRIUMF group [5]. To our knowledge, the present calculation is the first $NN\gamma$ calculation which includes, simultaneously, both the RSC and relativistic (one-body) RES corrections.

The one-body RES bremsstrahlung amplitude is given by the last term on the right-hand side of Eq. (2.1b). A diagrammatic representation of the RES process is given in Fig. 3. We should mention that our numerical calculations for both the single-scattering and RES (or double-scattering) contributions to the $NN\gamma$ observables has been checked in the nonrelativistic limit against existing nonrelativistic calculations by a number of authors [12]. The single-scattering contribution calculated in the present approach has been also compared with the calculation by the TRIUMF group [2,3,5]. Our results are in excellent agreement with those of Ref. [12] and also agree to within $\sim3\%$ with those results obtained by the TRIUMF group [2,3,5] provided we use (apart from using the same NN T matrix) the following: (i) the same propagator $\mathcal{G}$ [which appears in Eq. (2.1b)], and (ii) multiply the bremsstrahlung amplitude of Refs. [2,3,12] by a factor of $\sqrt{m/\varepsilon'}\sqrt{m/\varepsilon}$ which is required to satisfy the relativistic unitarity condition of the nonrelativistically constructed

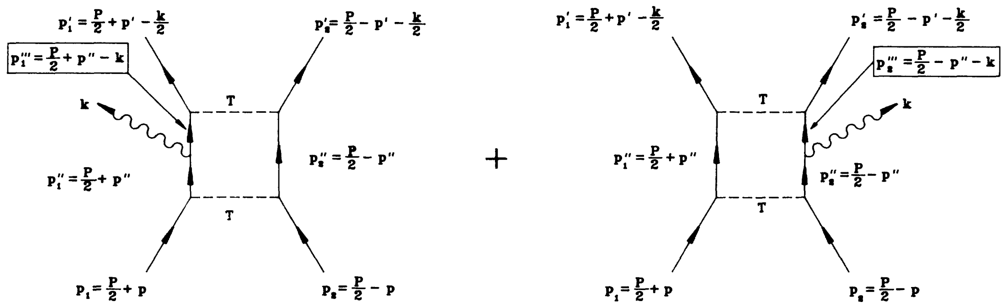

FIG. 3. Diagrammatic representation of the one-body RES process.

S matrix; this factor is also required for constructing a Lorentz invariant transition amplitude from a nonrelativistic T matrix as discussed in Appendix B.

In Fig. 4 we illustrate the effect of the one-body RES term in the coplanar geometry $pp\gamma$ cross section as a function of symmetric proton scattering angles $\theta_1=\theta_2$ at an incident energy of $T_{\rm lab}=280$ MeV and at a fixed photon emission angle of $\theta=175°$. As can be seen, in the present approach, the RES contribution (lower solid curve) enhances the cross section by $\sim 20\%$ with respect to the result without the RES term (lower dashed curve) for proton angles $\theta_1=\theta_2>10°$. However, for proton angles $\theta_1=\theta_2<10°$, the RES contribution reduces the cross section; moreover, its effect is more pronounced than at larger proton scattering angles, especially for $\theta_1=\theta_2\sim 5°$ where the cross section is reduced by $\sim 2\ \mu\text{b/sr}^2\,\text{rad}$ (or $\sim 28\%$). This is due, mainly, to the strong $^1S_0$ state contribution from the RSC term, $V_{\rm rsc}$, as has been pointed out in the previous section. For very small proton scattering angles, the emitted photon carries most of the available kinetic energy and, consequently, the energy in the c.m. of the two scattered protons is very small. The $NN\gamma$ process, then, necessarily samples the $^1S_0$ state which is quite strong for such small energies. In Fig. 4 we also display the results with (upper solid curve) and

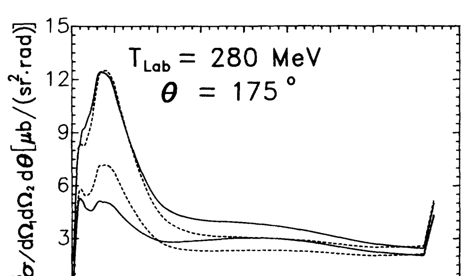

FIG. 4. Coplanar geometry $pp\gamma$ cross sections in the laboratory frame as a function of symmetric proton scattering angles $\theta_1=\theta_2$ at an incident energy of $T_{\rm lab}=280$ MeV and at a fixed photon emission angle of $\theta=175°$. The lower (upper) two curves correspond to the calculation with (without) the RSC. The results which include (do not include) the one-body RES contribution are represented by the solid (dashed) lines.

without (upper dashed curve) the RES contribution when the RSC is omitted. Here, the larger enhancement of the cross section (due to the RES term) compared to the calculation which includes the RSC in the region of $\theta_1=\theta_2>10°$ is due to the absence of the destructive interference between $V_{\rm mag}$ and $V_{\rm rsc}$. For proton scattering angles $\theta_1=\theta_2<10°$, the RES contribution without the RSC is relatively small, in contrast to the case where the RSC is included. From Fig. 4 it is clear that this is due to the absence of $V_{\rm rsc}$ in the entirely nonrelativistic approach. We emphasize that relativistic corrections, other than that due to $V_{\rm rsc}$, are negligible for the kinematical conditions considered here. It is interesting to note that, for proton scattering angles between $\sim 15°$ and $\sim 35°$ where most of the calculations have been made in the past, the calculation with the RES+RSC contribution yields practically the same results as the much simpler calculation without the RES and RSC terms. At proton angles around $\theta_1=\theta_2=5°$, however, inclusion of the RES term further enhances the relativistic effects. An understanding of the effects of different terms on $NN\gamma$ reactions at these small angles, in particular the effect of the RES, is especially relevant for experiments currently being performed at IUCF [34].

Figure 5 shows a comparison of the present cross-section calculations and the data of Ref. [5] in the coplanar geometry as a function of photon emission angle θ at an incident energy of $T_{\rm lab}=280$ MeV and for various asymmetric proton scattering angles. The solid and dashed curves correspond to the calculations including the RSC with and without the RES contribution, respectively. The results without the RSC and RES terms are also shown (dotted curves). As we expect from the discussion of Fig. 4, the RES term enhances the cross section for these proton scattering angles and for $\theta_1\sim\theta_2$ reduces the discrepancy between the theory and experiment significantly. However, the enhancement is not large enough to remove the discrepancy completely. As the proton angles increase, the results with the RES+RSC contribution become very close to the results without the RES and RSC terms as discussed before.

In Fig. 6 a comparison is made between the present results and the analyzing power data of Ref. [5] in the coplanar geometry. Both calculations including the RSC, with (solid) and without (dashed) the RES contribution, and the calculation excluding the RSC (dotted) yield re-

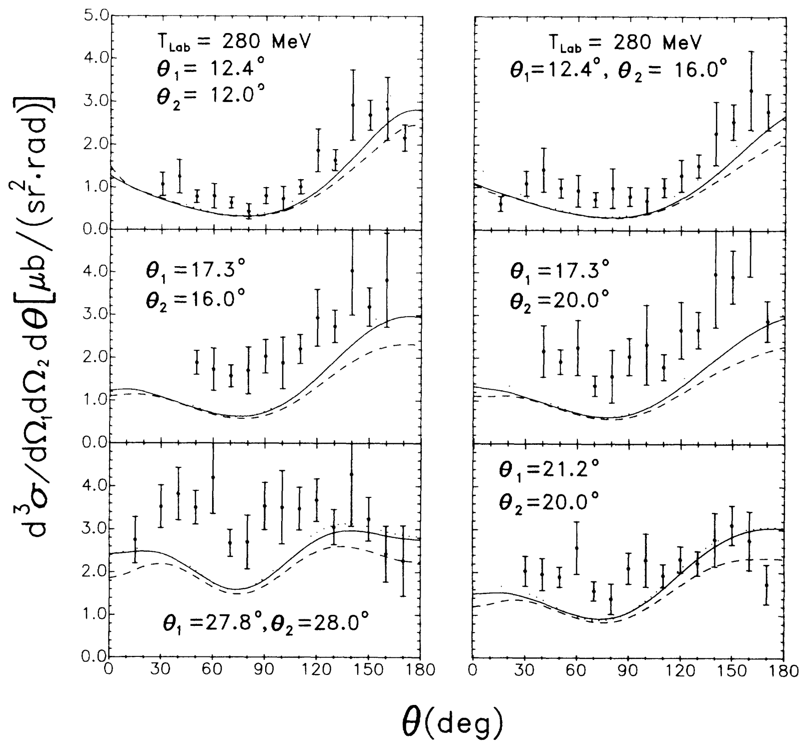

FIG. 5. Coplanar geometry $pp\gamma$ cross sections in the laboratory frame as a function of photon emission angle θ at an incident energy of $T_{\rm lab}=280$ MeV for various asymmetric proton scattering angles θ_1 and θ_2. The solid and dashed curves correspond to the calculation including the RSC with and without the one-body RES contribution, respectively, while the dotted curves correspond to the calculation without both the RSC and RES terms. The data are from Ref. 5 but do *not* contain the arbitrary *normalization* factor of $\frac{2}{3}$.

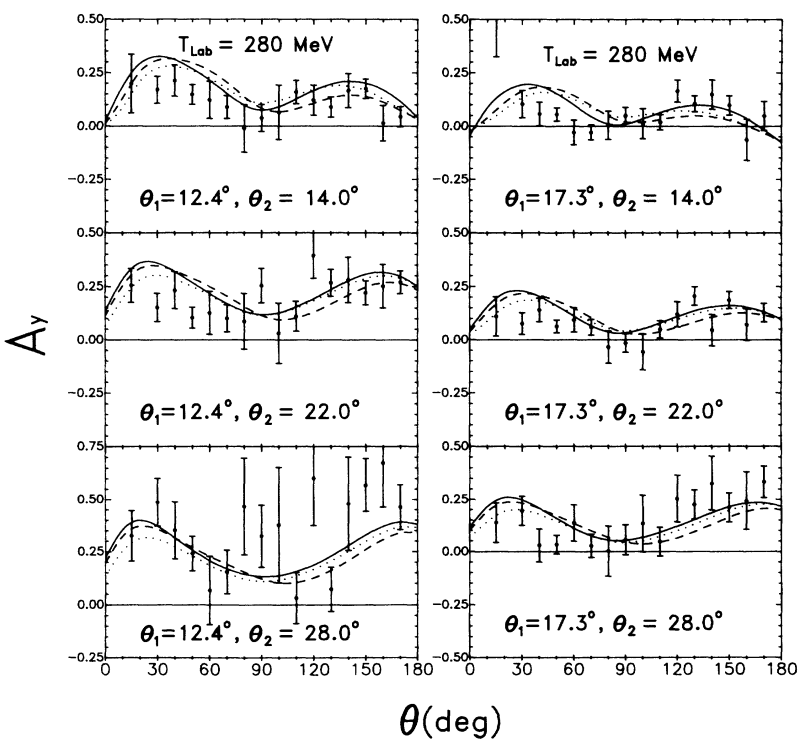

FIG. 6. Same as Fig. 5 for the analyzing power. The data are from Ref. [5] and have been multiplied by a factor of -1.

sults which show non-negligible differences from each other. However, these differences are not large enough to be disentangled by the data; i.e., they all reproduce the data equally well.

V. CONCLUSIONS

Using the basic structure of the $NN\gamma$ amplitude, we have shown that the RSC in $pp\gamma$ reactions is large when the emitted photon has an energy in the region near the maximum value allowed kinematically, even when the incident energy is relatively low. This is due, primarily, to the ${}^1S_0$ state contribution which is large and strongly energy dependent compared to other states. By considering this energy dependence we have shown how and under what kinematic conditions this state gives rise to such a strong RSC. Another aspect of the relativistic correction is that, due to the destructive interference between the RSC and magnetization current contributions, off-shell effects on the $pp\gamma$ cross section are less striking in the present approach than in the completely nonrelativistic approach. The analyzing power is less affected by relativistic corrections than the corresponding cross section since this spin observable is not very sensitive to the ${}^1S_0$ state [6] where the dominant (and large) RSC occurs in the cross section.

We have also investigated the role of the one-body RES contribution to the $pp\gamma$ reaction within the present approach. A significant difference in its role relative to that in the completely nonrelativistic approach arises for kinematical conditions where the photon energy is close to its end-point energy. Under these conditions the ${}^1S_0$ state contribution from $V_{\rm rsc}$ becomes very important; we note that $V_{\rm rsc}$ is absent in the nonrelativistic approach. For these kinematical conditions the inclusion of the RES term in the present approach further amplifies the relativistic effects on the $pp\gamma$ cross section.

For the coplanar geometry experiment of the TRIUMF group [5] at $T_{\rm lab}=280$ MeV and proton scattering angles larger than 10°, the inclusion of the RES term enhances the cross section. Although this enhancement significantly reduces the difference observed between the data and theoretical calculations which only include the external current (or single scattering), the RES correction is not large enough to completely resolve the discrepancy. Therefore, if the normalization of the TRIUMF cross-section data is correct, the present finding strongly indicates that modern potential models (at least the standard models) are unable to describe $pp\gamma$ reactions in which energetic photons are produced. It should be mentioned that the Coulomb effect which has been ignored in the present calculation is known to reduce the cross section [13], so that its inclusion will further increase the disagreement with the data. It is of crucial importance to have more cross-section data *with accurate normalization* under these kinematical conditions where potential model calculations can be better tested. In this connection, $pp\gamma$ experiments being performed at IUCF [34] for very small proton scattering angles are of special interest. The RES contribution to the analyzing power is not negligible; however, the data are not sufficiently accurate to disentangle such an effect.

Finally, no two-body current contribution has been included in this work. A fully consistent treatment of the two-body current contribution within a potential model calculation is not yet available and is beyond the scope of the present work.

ACKNOWLEDGMENTS

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APPENDIX A

In Eq. (2.4), the quantities $\tilde{e}_i^{\pm}$, $\tilde{\mu}_i^{\pm}$, $\tilde{\nu}_i^{\pm}$, and $\tilde{\eta}_i^{\pm}$ are given by

$$\tilde{e}_i^{\pm} \equiv \frac{1}{2}\left[\frac{mm}{\varepsilon_p \varepsilon_{q^\pm}}\right]^{1/2} \{\delta_{e,e_i}\mathcal{J}^+(\mathbf{q}^\pm,\mathbf{p}) - \kappa_i[\mathcal{P}(\mathbf{q}^\pm,\mathbf{p}) - \mathcal{J}^-(\mathbf{q}^\pm,\mathbf{p})]\} , \tag{A1}$$

$$\tilde{\mu}_i^{\pm} \equiv \frac{1}{2}\left[\frac{mm}{\varepsilon_p \varepsilon_{q^\pm}}\right]^{1/2} \left\{\delta_{e,e_i}\mathcal{J}^+(\mathbf{q}^\pm,\mathbf{p}) - \kappa_i[\mathcal{P}(\mathbf{q}^\pm,\mathbf{p}) - \mathcal{J}^-(\mathbf{q}^\pm,\mathbf{p})] + (\mu_i - 1)\left[\frac{\varepsilon_{q^\pm}+\varepsilon_p}{m}\right]\left[\frac{\varepsilon_p+m}{\varepsilon_{q^\pm}+m}\right]^{1/2}\right\} , \tag{A2}$$

$$\tilde{\nu}_i^{\pm} \equiv \frac{1}{2}\left[\frac{mm}{\varepsilon_p \varepsilon_{q^\pm}}\right]^{1/2} \{\mathcal{J}^-(\mathbf{q}^\pm,\mathbf{p}) - \kappa_i[\mathcal{P}(\mathbf{q}^\pm,\mathbf{p}) + \mathcal{J}^+(\mathbf{q}^\pm,\mathbf{p})]\} , \tag{A3}$$

and

$$\tilde{\eta}_i^{\pm} \equiv \left[\frac{mm}{\varepsilon_p \varepsilon_{q^\pm}} \right]^{1/2} \frac{1}{m} \frac{\mu_i - 1}{\sqrt{\varepsilon_{q^\pm}+m}\sqrt{\varepsilon_p+m}} , \qquad \text{(A4)}$$

where the auxiliary quantities are defined as

$$\mathcal{J}^{\pm}(\mathbf{q}^{\pm},\mathbf{p}) = \left[\frac{\varepsilon_p+m}{\varepsilon_{q^\pm}+m} \right]^{1/2} \pm \left[\frac{\varepsilon_{q^\pm}+m}{\varepsilon_p+m} \right]^{1/2} , \qquad \text{(A5)}$$

$$\mathcal{P}(\mathbf{q}^{\pm},\mathbf{p}) = \frac{k}{\sqrt{\varepsilon_{q^\pm}+m}\sqrt{\varepsilon_p+m}} , \qquad \text{(A6)}$$

and

$$\kappa_i = (\mu_i - 1)k/2m . \qquad \text{(A7)}$$

The vector $\mathbf{q}^{\pm}$ is defined as

$$\mathbf{q}^{\pm} = \mathbf{p}' \pm \mathbf{k}/2 . \qquad \text{(A8)}$$

APPENDIX B

In this appendix we discuss a procedure for constructing a Lorentz invariant transition amplitude from a Galilean invariant T matrix. Since it has been shown in Ref. [14] that the NN bremsstrahlung amplitude will be Lorentz invariant if the corresponding NN amplitude is invariant, we will restrict our consideration to the simpler NN scattering process.

Consider the NN cross section which can be expressed in terms of invariant quantities as [38]

$$d\sigma = \left\{ \frac{1}{\varepsilon_1\varepsilon_2[(\boldsymbol{\beta}_1-\boldsymbol{\beta}_2)^2-(\boldsymbol{\beta}_1\times\boldsymbol{\beta}_2)^2]^{1/2}} \right\} \int \frac{d^3p_1'}{\varepsilon_1'(2\pi)^3} \int \frac{d^3p_2'}{\varepsilon_2'(2\pi)^3} |\sqrt{\varepsilon_1'\varepsilon_2'}T(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2)\sqrt{\varepsilon_1\varepsilon_2}|^2 \times\{(2\pi)^4\delta(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1'-\mathbf{p}_2')\delta(\varepsilon_1+\varepsilon_2-\varepsilon_1'-\varepsilon_2')\} . \qquad \text{(B1)}$$

In the above equation, $\mathbf{p}_i$ and $\varepsilon_i = \varepsilon_{p_i} = \sqrt{\mathbf{p}_i^2+m^2}$ denote, respectively, the momentum and energy (in an arbitrary frame) of nucleon i in the initial state; $\boldsymbol{\beta}_i = \mathbf{p}_i/\varepsilon_i$. The primed quantities refer to the final state. The NN transition amplitude $T(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2)$ should be constructed such that the quantity

$$\tilde{T}(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2) = \sqrt{\varepsilon_1'\varepsilon_2'}T(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2)\sqrt{\varepsilon_1\varepsilon_2} \qquad \text{(B2)}$$

is a Lorentz invariant since each term in curly brackets and each integral in Eq. (B1) is an invariant.

The NN differential cross section in the NN c.m. system is, then, given by

$$\frac{d\sigma}{d\Omega} = \left[\frac{\varepsilon_p}{4\pi} \right]^2 |T(\mathbf{p}',\mathbf{p})|^2 , \qquad \text{(B3)}$$

with the final and initial relative momenta, $\mathbf{p}'$ and $\mathbf{p}$, obeying $|\mathbf{p}'|=|\mathbf{p}|$.

Equations (B1) and (B3) are obtained only from kinematical considerations for computing cross sections. In this sense, we can regard Eq. (B1) as being the definition of the transition amplitude T. Now, one has to compute either exact values or reliable approximations for the necessary T from dynamical considerations. The transition amplitude T is generally obtained from an integral equation of the form

$$\mathcal{T} = \mathcal{K} + \mathcal{K}\mathcal{G}\mathcal{T} . \qquad \text{(B4)}$$

In a relativistic approach, for example, the above equation may represent the Blankenbecler-Sugar (BbS) equation with $\mathcal{K}$ denoting some NN potential and $\mathcal{G}$ the two-nucleon BbS propagator. The T-matrix interaction based on the OBEPQ potential [33] used in the present work, for example, is obtained from such an equation with the NN potential given by a one-boson-exchange (OBE) model for the nuclear force [33]. An important condition to be satisfied here, for whatever approximation one uses, either to the NN potential or to the integral equation [Eq. (B4)] itself, is that the resulting transition amplitude T to be used in Eq. (B1) should have the Lorentz transformation property such that $\tilde{T}$ as defined by Eq. (B2) be Lorentz invariant.

The nonrelativistic approximation to the NN cross section can be obtained by expanding ε_i and keeping only the lowest-order contribution in Eqs. (B1)–(B3). Of course, in this case $\tilde{T}$ in Eq. (B2) (and consequently T) should accordingly be invariant under the Galilean transformation. Within the nonrelativistic approach, Eq. (B4) represents the Lippmann-Schwinger (LS) equation. In this case, one usually assumes a phenomenological (or a semiphenomenological) NN potential parametrized in terms of simple functions such as the Yukawa parametrized version of the Paris potential [29]. Again, whatever approximation one makes for the NN potential, that approximation should be consistent with the Galilean invariant nature of the transition amplitude given by Eq. (B1) *in the nonrelativistic limit.*

The approximations one makes to obtain the transition amplitude T are usually consistent with the transformation property that T must obey within either the relativistic or the nonrelativistic approach. A problem arises, however, when one uses a transition amplitude obtained nonrelativistically in the relativistic scheme, since, in this case, the corresponding $\tilde{T}$ given by Eq. (B2) should be a Lorentz invariant and not a Galilean invariant amplitude. The question, therefore, is how can one use nonrelativistic transition amplitudes consistently in a relativistic approach. This appendix addresses this question.

Before discussing this problem, we first consider a simple example of a transition amplitude which results from two nucleons interacting by exchanging a meson and include terms only up to second order in the NN-meson coupling constant. This example will illustrate explicitly the Lorentz structure of the resulting transition amplitude and its relationship to the invariant amplitude $\tilde{T}$. We, then, discuss how the T-matrix interaction based on the OBEPQ version of the Bonn potential [33] is related to the transition amplitude T in Eq. (B1). These examples will suggest how one can make nonrelativistic T matrices consistent with relativistic kinematics.

NN cross section from a relativistic one-meson-exchange interaction

Let us consider a simple case of two nucleons interacting through the exchange of a meson up to order g^2 in the NN-meson coupling strength. The scattering matrix S for such a process, then, can be written [17] (apart from the nonscattering term)

$$S=g_\alpha^2\left[\frac{m}{\varepsilon_1'}\right]^{1/2}\left[\frac{m}{\varepsilon_2'}\right]^{1/2}\left[\frac{m}{\varepsilon_1}\right]^{1/2}\left[\frac{m}{\varepsilon_2}\right]^{1/2}(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')$$
$$\times\left\{[\bar{u}(\mathbf{p}_1',s_1')\Gamma_\alpha(1)u(\mathbf{p}_1,s_1)]\frac{i}{(p_1'-p_1)^2-\mu_\alpha^2}\cdot[\bar{u}(\mathbf{p}_2',s_2')\Gamma_\alpha(2)u(\mathbf{p}_2,s_2)]\right.$$
$$\left.-[\bar{u}(\mathbf{p}_2',s_2')\Gamma_\alpha(1)u(\mathbf{p}_1,s_1)]\frac{i}{(p_2'-p_1)^2-\mu_\alpha^2}\cdot[\bar{u}(\mathbf{p}_1',s_1')\Gamma_\alpha(2)u(\mathbf{p}_2,s_2)]\right\}, \tag{B5}$$

where $p_i=((p_0)_i,\mathbf{p}_i)$ and s_i denote the four-momentum and spin of the ith nucleon, respectively, in the initial state. The primed quantities refer to the final state. $u(\mathbf{p}_i,s_i)$ denotes the positive energy Dirac spinor which is normalized to $\bar{u}(\mathbf{p}_i,s_i)u(\mathbf{p}_i,s_i)=1$. The Bjorken and Drell [17] notation is adopted. $\Gamma_\alpha(i)$ denotes the NN-meson coupling at the vertex i; the subscript α stands for the type of meson exchanged (scalar, pseudoscalar, vector, etc.). g_α denotes the corresponding NN-meson coupling strength and μ_α the mass of the exchanged meson. In Eq. (B5), the two terms in curly brackets correspond to the direct and exchange terms, respectively.

We now define the NN potential V_α as

$$V_\alpha(p_1',p_2';p_1,p_2)=\left[\frac{m}{\varepsilon_1'}\right]^{1/2}\left[\frac{m}{\varepsilon_2'}\right]^{1/2}\tilde{V}_\alpha(p_1',p_2';p_1,p_2)\left[\frac{m}{\varepsilon_1}\right]^{1/2}\left[\frac{m}{\varepsilon_2}\right]^{1/2}, \tag{B6}$$

with

$$\tilde{V}_\alpha(p_1',p_2';p_1,p_2)=g_\alpha^2\left\{[\bar{u}(\mathbf{p}_1',s_1')\Gamma_\alpha(1)u(\mathbf{p}_1,s_1)]\frac{i}{(p_1'-p_1)^2-\mu_\alpha^2}\cdot[\bar{u}(\mathbf{p}_2',s_2')\Gamma_\alpha(2)u(\mathbf{p}_2,s_2)]\right.$$
$$\left.-[\bar{u}(\mathbf{p}_2',s_2')\Gamma_\alpha(1)u(\mathbf{p}_1,s_1)]\frac{i}{(p_2'-p_1)^2-\mu_\alpha^2}\cdot[\bar{u}(\mathbf{p}_1',s_1')\Gamma_\alpha(2)u(\mathbf{p}_2,s_2)]\right\}. \tag{B7}$$

We note that $\tilde{V}_\alpha$ is a Lorentz invariant.

In terms of the NN potential defined by Eq. (B6), the scattering matrix S becomes

$$S=(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')V_\alpha(p_1',p_2';p_1,p_2). \tag{B8}$$

It is easily shown that the NN cross section is given by Eq. (B1) with

$$T(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2)=V_\alpha(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2). \tag{B9}$$

Here, V_α depends only on the three-vector $\mathbf{p}_i$ since the nucleons are on the mass shell and consequently $(p_0)_i=\varepsilon_i=\sqrt{\mathbf{p}_i^2+m^2}$.

In the above example, by substituting Eqs. (B6) and (B9) into Eq. (B2) we can see explicitly that

$$\tilde{T}(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2)=m^2\tilde{V}_\alpha(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2) \tag{B10}$$

is a Lorentz invariant as it should be.

NN cross section from a relativistic T matrix

We now turn to the consideration of how the T matrix based on the OBEPQ version of the OBE model developed by the Bonn group [33] is related to the transition amplitude T in Eq. (B1). The OBEPQ potential is obtained from a relativistic meson exchange theory and is given by

$$\tilde{V}_{\mathrm{OB}}=\sum_\alpha \tilde{V}_\alpha. \tag{B11}$$

Actually, each $\tilde{V}_\alpha$ (α is a scalar, pseudoscalar, etc.) is multiplied by a form factor which also has an invariant structure [33]. We redefine $\tilde{V}_\alpha$ to incorporate this factor. The T-matrix interaction based on the OBEPQ potential obeys the relativistic BbS equation which is obtained from the Bethe-Salpeter equation through a three-dimensional reduction by putting the nucleons in the intermediate states also on the mass shell, i.e., $(p_0)_i=\varepsilon_i=\sqrt{\mathbf{p}_i^2+m^2}$. The resulting equation retains the covariant form of the Bethe-Salpeter equation and the corresponding T matrix satisfies the relativistic elastic unitarity relation. In the NN c.m. system, the equation reads [39]

$$\tilde{T}_{\rm OB}(\mathbf{p}',\mathbf{p})=\tilde{V}_{\rm OB}(\mathbf{p}',\mathbf{p})+\int\frac{d^3p''}{\varepsilon_{p''}}\tilde{V}_{\rm OB}(\mathbf{p}',\mathbf{p}'')\times\frac{m}{\mathbf{p}^2/m-\mathbf{p}''^2/m+i\eta}\tilde{T}_{\rm OB}(\mathbf{p}'',\mathbf{p})\ . \tag{B12}$$

The amplitude $\tilde{T}_{\rm OB}$ in the above equation is a Lorentz invariant because $\tilde{V}_{\rm OB}$ is an invariant and the BbS equation preserves the Lorentz transformation property of $\tilde{V}_{\rm OB}$.

It is easy to see that the relationship between T in Eq. (B1) and $\tilde{T}_{\rm OB}$ in Eq. (B12) is

$$T(\mathbf{p}',\mathbf{p})=\left[\frac{m}{\varepsilon_{p'}}\right]^{1/2}\left[\frac{m}{\varepsilon_{p'}}\right]^{1/2}\tilde{T}_{\rm OB}(\mathbf{p}',\mathbf{p})\left[\frac{m}{\varepsilon_p}\right]^{1/2}\left[\frac{m}{\varepsilon_p}\right]^{1/2}, \tag{B13}$$

since then we have

$$\tilde{T}(\mathbf{p}',\mathbf{p})=m^2\tilde{T}_{\rm OB}(\mathbf{p}',\mathbf{p})\ , \tag{B14}$$

as it should be.

The consistency of Eqs. (B13) and (B14) with the simple example of the Born approximation as discussed in the previous section is evident if we consider only the first term on the right-hand side of Eq. (B12).

If we now multiply both sides of Eq. (B12) by the factor $\sqrt{m/\varepsilon_{p'}}\sqrt{m/\varepsilon_p}$ and define the quantities

$$\begin{aligned}V_{\rm OB}(\mathbf{p}',\mathbf{p})&=\left[\frac{m}{\varepsilon_{p'}}\right]^{1/2}\tilde{V}_{\rm OB}(\mathbf{p}',\mathbf{p})\left[\frac{m}{\varepsilon_p}\right]^{1/2},\\ T_{\rm OB}(\mathbf{p}',\mathbf{p})&=\left[\frac{m}{\varepsilon_{p'}}\right]^{1/2}\tilde{T}_{\rm OB}(\mathbf{p}',\mathbf{p})\left[\frac{m}{\varepsilon_p}\right]^{1/2},\end{aligned} \tag{B15}$$

we get

$$T_{\rm OB}(\mathbf{p}',\mathbf{p})=V_{\rm OB}(\mathbf{p}',\mathbf{p})+\int d^3p''V_{\rm OB}(\mathbf{p}',\mathbf{p}'')\times\frac{1}{\mathbf{p}^2/m-\mathbf{p}''^2/m+i\eta}T_{\rm OB}(\mathbf{p}'',\mathbf{p})\ , \tag{B16}$$

which is formally identical to the nonrelativistic LS equation. It is, actually, this equation that we solve to obtain the T matrix for the OBEPQ potential.

In terms of $T_{\rm OB}$, Eq. (B13) becomes

$$T(\mathbf{p}',\mathbf{p})=\left[\frac{m}{\varepsilon_{p'}}\right]^{1/2}T_{\rm OB}(\mathbf{p}',\mathbf{p})\left[\frac{m}{\varepsilon_p}\right]^{1/2}, \tag{B17}$$

and, consequently, the NN differential cross section given by Eq. (B3) reads

$$\frac{d\sigma}{d\Omega}=\left[\frac{m}{4\pi}\right]^2|T_{\rm OB}(\mathbf{p}',\mathbf{p})|^2\ , \tag{B18}$$

which is the nonrelativistic expression for the NN cross section.

NN cross section from a nonrelativistic T matrix

The nonrelativistic T matrix, $T_{\rm nr}$, obeys the LS equation which in momentum space reads (in the NN c.m. frame)

$$T_{\rm nr}(\mathbf{p}',\mathbf{p})=V_{\rm nr}(\mathbf{p}',\mathbf{p})+\int d^3p''V_{\rm nr}(\mathbf{p}',\mathbf{p}'')\times\frac{1}{\mathbf{p}^2/m-\mathbf{p}''^2/m+i\eta}T_{\rm nr}(\mathbf{p}'',\mathbf{p})\ . \tag{B19}$$

It is clear that in the above equation, if the NN potential $V_{\rm nr}$ is Galilean invariant, then $T_{\rm nr}$ will also be Galilean invariant. In other words, the LS equation preserves the Galilean transformation property of the NN potential $V_{\rm nr}$. With such a T matrix, the NN cross section can be obtained trivially and consistently from Eq. (B1) *in the nonrelativistic limit* with

$$T(\mathbf{p}',\mathbf{p})=T_{\rm nr}(\mathbf{p}',\mathbf{p})\ . \tag{B20}$$

In particular, for the NN differential cross section we have

$$\frac{d\sigma}{d\Omega}=\left[\frac{m}{4\pi}\right]^2|T_{\rm nr}(\mathbf{p}',\mathbf{p})|^2\ , \tag{B21}$$

which is the nonrelativistic expression used to fit the NN cross-section data. The T matrix based on the Paris potential [40] was, for example, constructed using Eqs. (B19) and (B21).

We now ask whether it is possible to use consistently such a nonrelativistic T matrix within a relativistic approach. In other words, we seek a relationship between the transition amplitude T in Eq. (B1) and the T matrix $T_{\rm nr}$ in Eq. (B19) such that $\tilde{T}$ given by Eq. (B2) is a Lorentz invariant. This is not a trivial question. First of all, most nonrelativistic T matrices are based on phenomenological NN potentials whose Lorentz structures are undefined. Second, even if we knew them, the resulting T matrices might (and probably would) have quite different Lorentz structures from the associated potentials, for the LS equation does not preserve the Lorentz transformation property of the NN potential. Despite these facts, we shall show in the following that a simple relationship between T in Eq. (B1) and $T_{\rm nr}$ in Eq. (B20) can be obtained which satisfies the requirement that $\tilde{T}$ in Eq. (B2) be a Lorentz invariant and yet reproduce the NN cross-section data according to Eq. (B21).

The LS equation can be cast into a covariant BbS equation type if we define the quantities

$$\bar{V}_{nr}(\mathbf{p}',\mathbf{p})=\left[\frac{\varepsilon_{p'}}{m}\right]^{1/2}V_{nr}(\mathbf{p}',\mathbf{p})\left[\frac{\varepsilon_p}{m}\right]^{1/2},$$
$$\bar{T}_{nr}(\mathbf{p}',\mathbf{p})=\left[\frac{\varepsilon_{p'}}{m}\right]^{1/2}T_{nr}(\mathbf{p}',\mathbf{p})\left[\frac{\varepsilon_p}{m}\right]^{1/2}. \tag{B22}$$

In terms of the above quantities Eq. (B21) becomes

$$\bar{T}_{nr}(\mathbf{p}',\mathbf{p})=\bar{V}_{nr}(\mathbf{p}',\mathbf{p})+\int\frac{d^3p''}{\varepsilon_{p''}}\bar{V}_{nr}(\mathbf{p}',\mathbf{p}'')\times\frac{m}{\mathbf{p}^2/m-\mathbf{p}''^2/m+i\eta}\bar{T}_{nr}(\mathbf{p}'',\mathbf{p}), \tag{B23}$$

which is formally identical to the relativistic BbS equation. We, therefore, see that, with the introduction of a minimal degree of relativity [the factor $\sqrt{\varepsilon_{p'}/m}\sqrt{\varepsilon_p/m}$ in Eq. (B22)] in the nonrelativistic *NN* potential, it is possible to convert the nonrelativistic LS equation into a relativistic BbS equation. In fact, this way of converting the LS equation has been a common procedure [41] in order to satisfy the relativistic elastic unitarity relation.

Unlike in the previous section, where the BbS equation has been cast into the LS equation, the conversion of the LS equation into a relativistic integral equation, as given in Eq. (B23), is by no means unique. However, the above procedure allows us, with the introduction of a minimal degree of relativity, to assure us that the resulting interaction $\bar{T}_{nr}$ obeys the relativistic elastic unitarity relation and retains the Lorentz transformation property of $\bar{V}_{nr}$. Therefore, if $\bar{V}_{nr}=\tilde{V}_{nr}$ is Lorentz invariant, $\bar{T}_{nr}$ will also be Lorentz invariant $(=\tilde{T}_{nr})$ and the relation between the transition amplitude T in Eq. (B1) and $\bar{T}_{nr}=\tilde{T}_{nr}$ will be the same as that between T and $\tilde{T}_{OB}$ in the previous section [Eq. (B13)], i.e.,

$$T(\mathbf{p}',\mathbf{p})=\left[\frac{m}{\varepsilon_{p'}}\right]^{1/2}\left[\frac{m}{\varepsilon_{p'}}\right]^{1/2}\tilde{T}_{nr}(\mathbf{p}',\mathbf{p})\left[\frac{m}{\varepsilon_p}\right]^{1/2}\left[\frac{m}{\varepsilon_p}\right]^{1/2}. \tag{B24}$$

The only question now is whether $\bar{V}_{nr}$ given by Eq. (B22) can be a Lorentz invariant quantity for a given nonrelativistic *NN* potential V_{nr} whose explicit Lorentz structure is unspecified. We argue that it is possible to write $\bar{V}_{nr}$ as an invariant such that in the *NN* c.m. frame with the nucleons on the mass shell it will be consistent with its definition given by Eq. (B22). For example, suppose that we have chosen a simple Yukawa form for V_{nr}, i.e.,

$$V_{nr}(\mathbf{p}',\mathbf{p})=\frac{A}{(\mathbf{p}'-\mathbf{p})^2+\mu^2}. \tag{B25}$$

We, then, have

$$\bar{V}_{nr}(\mathbf{p}',\mathbf{p})=\left[\frac{\varepsilon_{p'}}{m}\right]^{1/2}\frac{A}{(\mathbf{p}'-\mathbf{p})^2+\mu^2}\left[\frac{\varepsilon_p}{m}\right]^{1/2}. \tag{B26}$$

Now, this can be written as an invariant

$$\bar{V}_{nr}(p_1',p_2';p_1,p_2)=\left\{\frac{\sqrt{(p_1'+p_2')^2}}{2m}\frac{\sqrt{(p_1+p_2)^2}}{2m}\right\}^{1/2}\left\{\frac{-A}{(p_1'-p_1)^2-\mu^2-[\sqrt{(p_1'+p_2')^2}-\sqrt{(p_1+p_2)^2}]^2/4}\right\}$$
$$=\tilde{V}_{nr}(p_1',p_2';p_1,p_2). \tag{B27}$$

In the *NN* c.m. system with the nucleons on the mass shell we have

$$\mathbf{p}_1+\mathbf{p}_2=\mathbf{p}_1'+\mathbf{p}_2'=0,$$
$$(p_0)_i=\varepsilon_i=\sqrt{\mathbf{p}_i^2+m^2}. \tag{B28}$$

It is, then, easy to verify that Eq. (B27) reduces to Eq. (B26).

We, therefore, have

$$\bar{T}_{nr}(\mathbf{p}',\mathbf{p})=\tilde{T}_{nr}(\mathbf{p}',\mathbf{p}). \tag{B29}$$

In terms of the original nonrelativistic T matrix, T_{nr}, which obeys the LS equation, Eq. (B24) becomes

$$T(\mathbf{p}',\mathbf{p})=\left[\frac{m}{\varepsilon_{p'}}\right]^{1/2}T_{nr}(\mathbf{p}',\mathbf{p})\left[\frac{m}{\varepsilon_p}\right]^{1/2}, \tag{B30}$$

which is the relation used in Ref. [14]. As has been shown [14], this relation yields the correct result *on the energy shell* for it reproduces the nonrelativistic *NN* cross-section formula [substituting Eq. (B30) into Eq. (B3)] as given by Eq. (B21).

We observe that, although $\bar{V}_{nr}$ can be put in an invariant form $(=\tilde{V}_{nr})$, knowledge of its explicit form is not required, since we still solve the LS equation for T_{nr} with the *NN* potential V_{nr}. All we do, then, is use Eq. (B30) to obtain T.

We should also mention that when we refer to the on- and off-shell behaviors of the T-matrix interaction, we are always referring to the on- and off-shell behaviors of the transition amplitude T which enters Eq. (B1). Of course, this transition amplitude has a different off-shell behavior than, for example, the T-matrix interaction T_{nr}. However, if it is desired, one can always invert Eq. (B30) to obtain T_{nr} and, consequently, the on- and off-shell behaviors of T_{nr}.

At this point, it should be noted that the TRIUMF group [2,3] has used the relation between the transition amplitude T and the Paris potential based T matrix ($T_{\rm nr}$) given by Eq. (B20) instead of Eq. (B30) within a relativistic approach [11]. Therefore, their transition amplitudes based on the Paris potential are larger than ours by a factor of $\sqrt{\varepsilon_{p'}/m}\sqrt{\varepsilon_p/m}$. This seems also to be the case of the transition amplitudes based on the Hamada-Johnston potential used in Refs. [12,42] as has been found empirically in Ref. [14]. Using Eq. (B20) in a relativistic approach is inconsistent with the way in which the nonrelativistic NN potentials were constructed. In addition, even if we argue that the nonrelativistic potential $V_{\rm nr}$ has a Lorentz transformation property consistent with Eqs. (B1), (B2), and (B20), the associated T matrix $T_{\rm nr}$ may (and probably will) be inconsistent with those equations, for the LS equation does not preserve the Lorentz transformation property of the NN potential $V_{\rm nr}$.

In summary, we have shown a simple procedure for constructing a Lorentz invariant amplitude from a nonrelativistic T matrix which obeys the LS equation. Although this procedure is not unique it allows us to convert the nonrelativistic LS equation into a relativistic integral equation by introducing a minimal degree of relativity in the nonrelativistic NN potential. The resulting interaction retains the Lorentz transformation property of the minimally modified NN potential and the NN cross section is reproduced in a consistent way.

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