# COMMENTS

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## Proton 2p-2h intruder excitations and the modified vibrational intensity and selection rules

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It is pointed out that strong mixing between proton two-particle two-hole (2p-2h) $0^+$, $2^+$, intruder configurations, and the quadrupole phonon vibrational states modifies the original quadrupole vibrational intensity and selection rules. This approach explains the possibility for certain states to exhibit apparently both intruder *and* vibrational characteristics and results mainly from strong mixing of the $0^+$ states. We present interacting boson model calculations and compare in detail, for $^{114}$Cd, the quintuplet of levels near $E_x \cong 1.2$ MeV.

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In a recent study of even-even Cd nuclei, concentrating in particular on the two-quadrupole phonon region with its quintuplet of states [1,2], it was pointed out that even though extra $0^+$ and $2^+$ states are present, a picture including large mixing between the intruder $0^+,2^+$ proton 2p-2h excitations and the $0^+,2^+,4^+$ two-phonon triplet does not result in a consistent picture.

In the present Comment we point out that a description in which proton 2p-2h excitations across the $Z=50$ closed shell are incorporated [3,4] together with strong mixing, in particular for the $0^+$ states, leads to a rather good description of both energies, $B(E2)$ values, and quadrupole moments. Moreover the observation of intruder $0^+,2^+,\ldots$ proton 2p-2h states is supported rather strongly by the observation of similar intruder excitations in the Sn region (In, Ag, Sb, I nuclei) and in other mass regions and by two-proton transfer reactions as a general phenomenon near (or at) closed shells [5,6].

Proton 2p-2h intruder excitations can be introduced in the interacting boson model (IBM-2) by including both $\bar{N}_\pi=1$ and $(\bar{N}_\pi=2)+(N_\pi=1)$ states, where $\bar{N}_\pi$ ($N_\pi$) describes the number of hole (particle) bosons, respectively, in the Cd ($Z=48$) nuclei, which are then mixed through the Hamiltonian [7]

$$H_{\text{mix}}=\alpha(s^\dagger s^\dagger+\text{H.c.})+\beta[(d^\dagger d^\dagger)^{(0)}+\text{H.c.}]+\Delta \ . \quad (1)$$

A consistent approach, fixing the quantity $\Delta$ which determines the unperturbed energy of the proton 2p-2h excitations relative to the regular states, is given by the expression

$$\Delta=2(\varepsilon_p-\varepsilon_h)-\Delta E_{\text{pair}}+\Delta E_{\text{monopole}}\cong 4 \text{ MeV} \ , \quad (2)$$

which is presented in Ref. [4]. In Eq. (2), the single-particle energy is modified by pairing energy and monopole energy corrections as discussed in detail in Ref. [3].

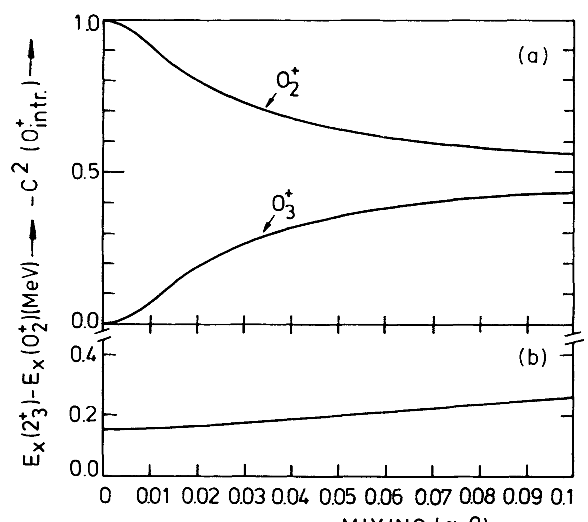


FIG. 1. (a) The intruder $c^2(|0_1^+;\bar{N}_\pi=3\rangle)$ amplitude squared in the $0_2^+$ and $0_3^+$ states when mixing the intruder ($\bar{N}_\pi=3$) and regular ($\bar{N}_\pi=1$) vibrational configurations in $^{114}$Cd, as a function of mixing strength $\alpha,\beta$ [see Eq. (1)] (in units MeV). (b) The corresponding $2_3^+$-$0_2^+$ energy separation.

 

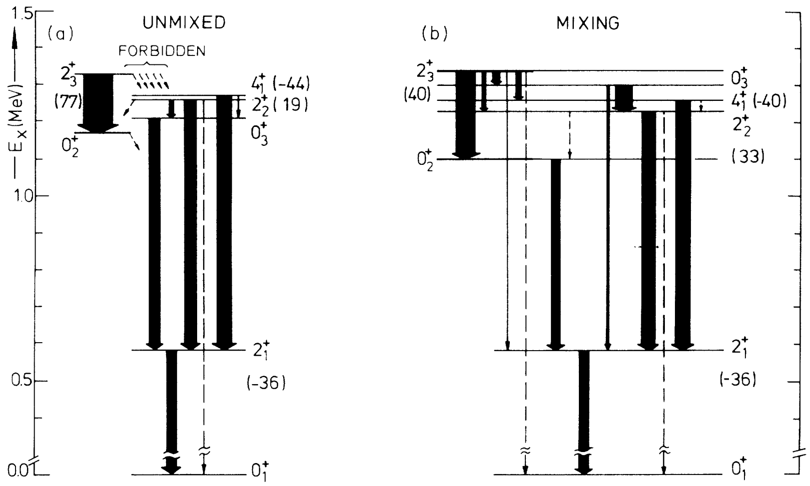


FIG. 2. (a) The $^{114}$Cd intruder and quadrupole phonon $B(E2)$ values. The widths are normalized to the $B(E2;2_1^+\rightarrow 0_1^+)$ value ($\simeq 1000\ e^2\ \mathrm{fm}^4$). Values between brackets give the $Q(2_i^+)$ and $Q(4_1^+)$ quadrupole moments in units $e$ fm$^2$. Forbidden transitions also are indicated [4]. (b) Analogous $^{114}$Cd $B(E2)$ values and quadrupole moments for the case $\alpha=\beta=0.08$ and charges $e_\pi, e_\nu$ as discussed in the text [4].

The IBM-2 Hamiltonians in the $\bar{N}_\pi=1$ and $\bar{N}_\pi=3$ subspaces contain parameters that are determined as closely as possible by observed properties in adjacent even-even nuclei [4]. We present some of the salient features relevant for the present discussion in $^{114}$Cd in pointing out the modifications brought about by strong mixing, in particular for the $0^+$ states.

The important mixing between the $0^+$ two-phonon and $0^+$ proton 2p-2h intruder states does *not* modify the $0_2^+$-$2_3^+$ intruder energy difference in an important way. This observation is most dramatically illustrated in Fig. 1. In a two-level mixing model, the residual interaction strength $\langle H_{\mathrm{mix}}\rangle$ does not appear in the final energies in a linear way since the eigenvalues are determined by the expression

$$E_{0_i^+}=\frac{\varepsilon_{0_2^+}+\varepsilon_{0_3^+}}{2}\pm\frac{1}{2}\sqrt{(\Delta E)_{\mathrm{unp}}^2+4\langle H_{\mathrm{mix}}\rangle^2}\quad (i=2,3)\ , \tag{3}$$

where $(\Delta E)_{\mathrm{unp}}=|\varepsilon_{0_2^+}-\varepsilon_{0_3^+}|$. Moreover, the $2_2^+$ and $2_3^+$ are affected by mixing such that over a large interval of $\alpha,\beta$ strengths $0\ \mathrm{MeV}<\alpha,\beta<0.1$ MeV, the $0_2^+$-$2_3^+$ energy difference changes only very smoothly.

The vibrational $E2$ intensity and selection rules of a quadrupole two-phonon triplet are modified in a *decisive* way through mixing with the intruder $0_2^+,2_3^+$ states (see also [8]). In Fig. 2 we present the two-phonon and intruder unperturbed and mixed structures using effective charges $e_\pi=e_\nu$ $(\bar{N}_\pi=1)=0.086$ $e$b and $e_\pi(\bar{N}_\pi=3)/e_\pi$ $(\bar{N}_\pi=1)=1.2$ and all IBM-2 parameters identical with Ref. [4]. In the unmixed case, the two sets of subspaces are disconnected while anharmonicities in the $\bar{N}_\pi=1$ system are clearly indicated [$E2$ transitions inside the two-phonon $0_3^+,2_2^+,4_1^+$ multiplet, nonvanishing $Q(2_1^+)$ mo-

TABLE I. Comparison of the theoretical $B(E2)$ values without coupling Hamiltonian ($\alpha=\beta=0$ MeV) and with the coupling as indicated ($\alpha=\beta=0$ MeV). The experimental data are taken from Ref. [9]. $B(E2;J_i^\pi\rightarrow J_f^\pi)$ ($e^2\mathrm{Im}^4$).

| $J_i^\pi\rightarrow J_f^\pi$ | Expt. | $\alpha=\beta=0$ MeV | $\alpha=\beta=0.08$ MeV |
|---|---|---|---|
| $2_1^+\rightarrow 0_1^+$ | 1020 | 955 | 971 |
| $0_2^+\rightarrow 2_1^+$ | 900 | | 895 |
| $2_2^+\rightarrow 0_1^+$ | 17 | 2.2 | 2.2 |
| $\rightarrow 2_1^+$ | 930 | 1362 | 1373 |
| $\rightarrow 0_2^+$ | 60 | | 14 |
| $4_1^+\rightarrow 2_1^+$ | 2020 | 1571 | 1623 |
| $\rightarrow 2_2^+$ | 136 | 41 | 3.7 |
| $0_3^+\rightarrow 2_1^+$ | 0.1 | 1091 | 314 |
| $\rightarrow 2_2^+$ | 3590 | 762 | 1765 |
| $2_3^+\rightarrow 0_1^+$ | 11 | | 0.0 |
| $\rightarrow 2_1^+$ | 1.3 | | 50 |
| $\rightarrow 0_2^+$ | 520 | 3216 | 2134 |
| $\rightarrow 2_2^+$ | 1010 | | 439 |
| $\rightarrow 0_3^+$ | 230 | | 811 |
| $\rightarrow 4_1^+$ | 420 | | 604 |
| $4_2^+\rightarrow 2_1^+$ | 13 | | 6.7 |
| $\rightarrow 2_2^+$ | 1053 | | 564 |
| $\rightarrow 2_3^+$ | 3785 | 4537 | 3420 |
| $\rightarrow 4_1^+$ | 430 | | 179 |
| $6_1^+\rightarrow 4_1^+$ | 3915 | | 1825 |
| $6_2^+\rightarrow 4_2^+$ | 4245 | | 2701 |

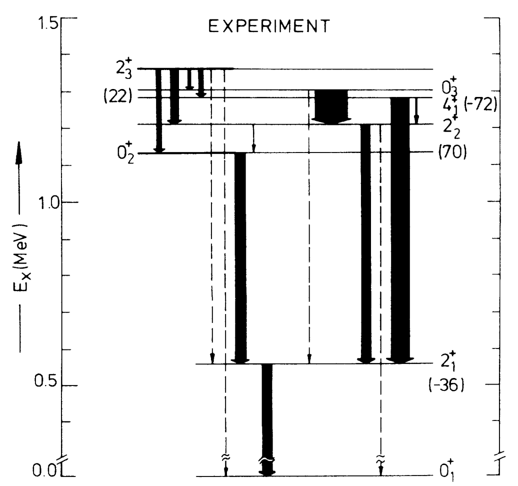


FIG. 3. The corresponding experimental $B(E2)$ values and quadrupole moments following the conventions discussed in the caption to Fig. 2.

ment]. The mixing, implied by the Hamiltonian (1) modifies *both* the vibrational intensity and selection rules *and* gives rise to important $E2$ transitions connecting the original $0_2^+$ and $2_3^+$ states to the original two-phonon $0_3^+,2_2^+,4_1^+$ and one-phonon $2_1^+$ states. Some dramatic (almost paradoxical) changes appear.

(i) The $0_2^+$ final state collects the characteristics of *both* an intruder *and* vibrational excitation since the $0_2^+\to 2_1^+$ $E2$ rates becomes almost as large as the original, unperturbed, two-phonon $0_3^+\to$ one-phonon $2_1^+$ $E2$ reduced transition probability. Moreover, many $E2$ transitions deexciting the $2_3^+$ state emerge in the mixed case. These particular features compare quite well with the experimental situation (Fig. 3).

(ii) The very strong $0_3^+\to 2_2^+$ and very weak $0_3^+\to 2_1^+$ $E2$ transitions, as observed experimentally, are a feature fully at variance with *coexisting* intruder and two-phonon vibrational structures, but are rather well obtained in the mixed case. The difference is even more accentuated in the data for $^{114}$Cd. The interfering $E2$ components, giving rise to these $B(E2)$ values, depend in a rather sensitive way on some of the model parameters ($\Delta,\ldots$), and it is consistent that a further decrease in the $B(E2;2_3^+\to 0_2^+)$ value will result in increasing $B(E2;0_3^+\to 2_2^+)$ and decreasing $B(E2;0_3^+\to 2_1^+)$ values. These arguments are made more quantitative by making some approximations to the realistic wave functions obtained in the case of $^{114}$Cd [4]. The $0_2^+$ and $0_3^+$ wave functions are almost equally mixed, i.e.,

$$
\begin{aligned}
|0_2^+\rangle &\cong \frac{1}{\sqrt{2}}|0_2^+;\bar{N}_\pi=1\rangle+\frac{1}{\sqrt{2}}|0_1^+;\bar{N}_\pi=3\rangle\ ,\\
|0_3^+\rangle &\cong -\frac{1}{\sqrt{2}}|0_2^+;\bar{N}_\pi=1\rangle+\frac{1}{\sqrt{2}}|0_1^+;\bar{N}_\pi=3\rangle\ ,
\end{aligned}
\tag{4}
$$

with the $2_1^+$ wave function given as

$$
|2_1^+\rangle \cong |2_1^+;\bar{N}_\pi=1\rangle+\varepsilon|2_1^+;\bar{N}_\pi=3\rangle\ . \tag{5}
$$

This results in the following $E2$ matrix elements

$$
\langle 2_1^+\|\mathbf{M}(E2)\|0_2^+\rangle \cong \frac{1}{\sqrt{2}}\langle 2_1^+;\bar{N}_\pi=1\|\mathbf{M}(E2)\|0_2^+;\bar{N}_\pi=1\rangle+\frac{\varepsilon}{\sqrt{2}}\langle 2_1^+;\bar{N}_\pi=3\|\mathbf{M}(E2)\|0_1^+;\bar{N}_\pi=3\rangle\ , \tag{6}
$$

$$
\langle 2_1^+\|\mathbf{M}(E2)\|0_3^+\rangle \cong -\frac{1}{\sqrt{2}}\langle 2_1^+;\bar{N}_\pi=1\|\mathbf{M}(E2)\|0_2^+;\bar{N}_\pi=1\rangle+\frac{\varepsilon}{\sqrt{2}}\langle 2_1^+;\bar{N}_\pi=3\|\mathbf{M}(E2)\|0_1^+;\bar{N}_\pi=3\rangle\ , \tag{7}
$$

with the "unperturbed" $E2$ matrix element $\langle 2_1^+;\bar{N}_\pi=1\|\mathbf{M}(E2)\|0_2^+;\bar{N}_\pi=1\rangle=-0.33$ $eb$ and $\langle 2_1^+;\bar{N}_\pi=3\|\mathbf{M}(E2)\|0_1^+;\bar{N}_\pi=3\rangle=1.26$ $eb$ having opposite sign and, with the numerical value of $\varepsilon\cong -0.12$ [4], the matrix elements (6) and (7) result in constructive and destructive interfering parts, respectively. In the same spirit, the $2_2^+$ ($2_3^+$) wave functions can be approximated as

$$
\begin{aligned}
|2_2^+\rangle &\cong \gamma|2_2^+;\bar{N}_\pi=1\rangle+\delta|2_1^+;\bar{N}_\pi=3\rangle\ ,\\
|2_3^+\rangle &\cong -\delta|2_2^+;\bar{N}_\pi=1\rangle+\gamma|2_1^+;\bar{N}_\pi=3\rangle\ ,
\end{aligned}
\tag{8}
$$

with the $0_3^+\to 2_2^+$ $E2$ matrix element

$$
\langle 2_2^+\|\mathbf{M}(E2)\|0_3^+\rangle \cong -\frac{\gamma}{\sqrt{2}}\langle 2_2^+;\bar{N}_\pi=1\|\mathbf{M}(E2)\|0_2^+;\bar{N}_\pi=1\rangle+\frac{\delta}{\sqrt{2}}\langle 2_1^+;\bar{N}_\pi=3\|\mathbf{M}(E2)\|0_1^+;\bar{N}_\pi=3\rangle\ . \tag{9}
$$

Using the numerical values of $\gamma\cong -0.9$, $\delta\cong -0.3$, and the two-phonon vibrational ($\bar{N}_\pi=1$) $0_2^+\to 2_2^+$ $E2$ matrix element $\langle 2_2^+;\bar{N}_\pi=1\|\mathbf{M}(E2)\|0_2^+;\bar{N}_\pi=1\rangle=-0.28$ $eb$ [4], the matrix element (9) also gives rise to constructive interference, even more strongly than in the case of the $0_2^+\to 2_1^+$ $E2$ matrix element.

In the same spirit, the analysis for the $2_3^+,2_2^+\to 2_1^+$ $E2$ transitions can be made using the $2^+$ wave functions as depicted in Eqs. (5) and (8). The reduced $E2$ matrix elements become

$$
\begin{aligned}
\langle 2_1^+\|\mathbf{M}(E2)\|2_2^+\rangle &\cong \gamma\langle 2_1^+;\bar{N}_\pi=1\|\mathbf{M}(E2)\|2_2^+;\bar{N}_\pi=1\rangle+\delta\varepsilon\langle 2_1^+;\bar{N}_\pi=3\|\mathbf{M}(E2)\|2_1^+;\bar{N}_\pi=3\rangle\ ,\\
\langle 2_1^+\|\mathbf{M}(E2)\|2_3^+\rangle &\cong -\delta\langle 2_1^+;\bar{N}_\pi=1\|\mathbf{M}(E2)\|2_2^+;\bar{N}_\pi=1\rangle+\gamma\varepsilon\langle 2_1^+;\bar{N}_\pi=3\|\mathbf{M}(E2)\|2_1^+;\bar{N}_\pi=3\rangle\ .
\end{aligned}
\tag{10}
$$

Using the above numerical values of $\gamma$, $\delta$, and $\varepsilon$ and the separate reduced matrix element

$$
\begin{aligned}
&\langle 2_1^+;\bar{N}_\pi=1\|\mathbf{M}(E2)\|2_2^+;\bar{N}_\pi=1\rangle=-0.83\ eb\ ,\\
&\langle 2_1^+;\bar{N}_\pi=3\|\mathbf{M}(E2)\|2_1^+;\bar{N}_\pi=3\rangle=0.90\ eb\ ,
\end{aligned}
$$

constructive and destructive interference for the two transitions in Eq. (10), respectively, occur. With small changes to the numerical values obtained in [4], the destructive interference can even give rise to an almost vanishing $2_3^+\to 2_1^+$ $E2$ matrix element.

So, a number of specific effects at variance with the simple quadrupole two-phonon picture can be understood in a consistent way. This can be judged best by comparing the mixing situation of Fig. 2 with the data, presented in Fig. 3. Details of this comparison are presented in Table I also.

In conclusion, we stress again that strong mixing between the low-lying intruder $0^+$, $2^+$ proton 2p-2h excitations with the quadrupole two-phonon states modifies the two unperturbed structures of Fig. 2(a) resulting in a totally new picture. Thereby we are able to describe most aspects of a complicated system like the quintuplet of levels in $^{114}$Cd rather well. Moreover, this picture is not only adjusted to this well-documented case but applies to the other even-even Cd nuclei too, encompassing the changes implied by the specific energy dependence of the proton 2p-2h excitations near $Z=50$ as a function of neutron number $N$. A more extensive study, using the methods of Ref. [4] and applied to the larger set of even-even Cd nuclei, is in progress.

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