$E0$ decays of β vibrations in the O(6) limit of the neutron-proton interacting boson model

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Expressions are derived for the $E0$ transition strengths for the symmetric and antisymmetric β-vibrational bandhead states to the ground state for O(6)-like nuclei. These $E0$ transitions are the only ones allowed to the ground state, and we predict them to be of similar magnitude. The isomer shifts for these β-vibrational states are also discussed, as well as the isotope shifts for the ground states. We propose the Ba isotopes around $A \approx 132$ as possible candidates for observing these $E0$ decays.

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I. INTRODUCTION

In an earlier paper [1], we discussed the structure and decay of antisymmetric β vibrations in the O(6) limit of the neutron-proton interacting boson model [2] (also referred to as the IBM-2). In the O(6) limit, these symmetric and antisymmetric β vibrations do not belong to the lowest O(6) representations $\langle N,0\rangle$ and $\langle N-1,1\rangle$, respectively ($N=N_\pi+N_\nu$, N_ρ=number of ρ bosons, where $\rho=\pi$ or ν), but to the next-higher-lying O(6) representation, being $\langle N-2,0\rangle$ in both cases. Because of O(6) selection rules, these β vibrations cannot decay to the 0^+ ground state (g.s.) or to other ground-band configurations with O(6) symmetry $\langle N,0\rangle$ by $M1$ or $E2$ electromagnetic transitions, and in particular, the lowest-lying antisymmetric β vibration, i.e., $0^+(M')$, can only decay to the 1^+ and 2^+ mixed-symmetry states of the same F spin [2]. Because of all of these restrictions, no strong signature was found for the $M1$ and $E2$ decay modes of the $0^+(M')$ state. The distinguishing feature of the $M1$ and $E2$ transition strengths for the $0^+(M')$ state was that they are proportional to $(N_\pi-N_\nu)^2$. Hence they vanish—i.e., the $0^+(M')$ state becomes an isomeric state—for $N_\pi=N_\nu$.

The purpose of this paper is to report on the strengths of the $E0$ decays of these symmetric and antisymmetric β vibrations. Because of its one-body nature, as has been verified by the intrinsic state argument by Leviatan and Ginocchio [3], $E0$ decays will connect only the lowest 0^+ states in the symmetric and antisymmetric β-vibrational bands to the 0^+ g.s. Hence, the observation of strong $E0$ transitions would provide a signature for these β-vibrational configurations, since the $E0$ transition probability from a two-quasiparticle state to the g.s. is hindered as compared to the decay of β-bandhead states.

II. FORMALISM AND RESULTS

We consider the O(6) limit of the IBM-2 corresponding to the group chain

$$\begin{array}{ccccccccc} \mathrm{U}_\pi(6)\times\mathrm{U}_\nu(6) & \supset & \mathrm{U}_{\pi+\nu}(6) & \supset & \mathrm{O}_{\pi+\nu}(6) & \supset & \mathrm{O}_{\pi+\nu}(5) & \supset & \mathrm{O}_{\pi+\nu}(3)\,, \\ [N-f,f] & & (\sigma_1,\sigma_2) & & \langle\tau_1,\tau_2\rangle & & L & & \end{array} \tag{1}$$

where the quantum numbers for each step in the subgroup chain are listed below each group classification [4]. Detailed studies [5] have been made of the low-lying properties of O(6) nuclei within the IBM-2 for the totally symmetric states $[N]$ (i.e., the IBM-1 configurations or, equivalently, the states of maximal F spin [2], $F_{\max}=N/2$) and the first set of mixed-symmetry states $[N-1,1]$ (i.e., those with an F-spin value of $F_{\max}-1$). A possible spectrum of a "typical" nucleus with the dynamical symmetry chain (1) is illustrated in Fig. 1.

In this study we concern ourselves only with the symmetric β-vibrational bandhead state, which we denote as (using the notation of Van Isacker *et al.* [6])

$$|0^+(\beta)\rangle \equiv |[N_\pi][N_\nu];[N]\langle N-2\rangle(0)0^+\rangle\,, \tag{2}$$

and in the antisymmetric β-vibrational bandhead state, which we earlier denoted [1] as

$$|0^+(M')\rangle \equiv |[N_\pi][N_\nu];[N-1,1]\langle N-2\rangle(0)0^+\rangle\,. \tag{3}$$

For convenience and conciseness, we hereafter omit second quantum numbers (e.g., f, σ_2, or τ_2) which are zero. We use M' to distinguish the $\langle N-2\rangle$ mixed-symmetry states from the $\langle N-1,1\rangle$ mixed-symmetry states, which are usually denoted by a subscript M.

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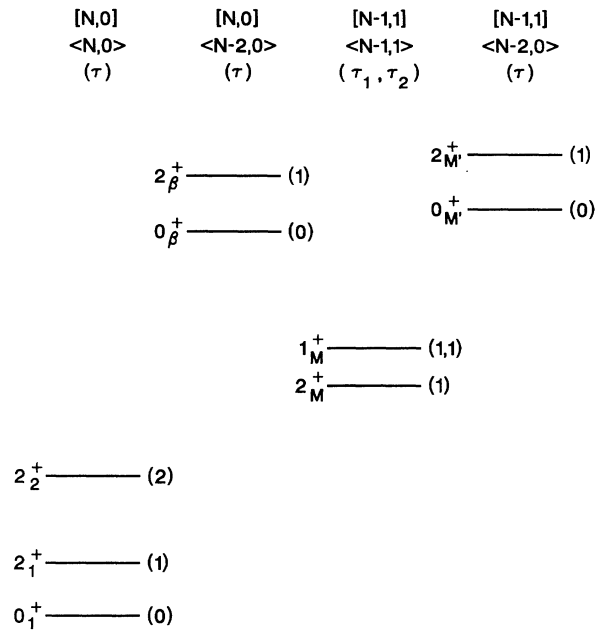

FIG. 1. "Typical" spectrum of a nucleus with the dynamical symmetry chain (1), including the $\langle N-2,0\rangle$ O(6) configurations.

States of the possible structure (2) have been reported in the investigations of Casten and von Brentano [7] regarding the spectra of the Xe and Ba isotopes.

The structure of the intrinsic states, from which the states $|0^+(M')\rangle$ and $|0^+(\beta)\rangle$ and the other states in their corresponding bands can be projected, was reported in Ref. [1]. Also in Ref. [1] we constructed the $|0^+(\beta)\rangle$ and $|0^+(M')\rangle$ states corresponding to $N_\nu=N-1$ and $N_\pi=1$, using the formalism of Van Isacker *et al.* [6] and the isoscalar factors of Van Isacker, Frank, and Sun [8]. In a similar manner, one can construct the corresponding $|0^+(1)\rangle$ g.s.

Within the IBM-2 the "effective" $E0$ operator is of the form [5]

$$T(E0)=f_\pi\hat{n}_{d_\pi}+\gamma_{0_\pi}N_\pi+f_\nu\hat{n}_{d_\nu}+\gamma_{0_\nu}N_\nu\ , \tag{4}$$

where f_ρ is the monopole effective charge of a ρ boson and $\hat{n}_{d_\rho}$ is the number operator for a d boson of type ρ, where $\rho=\pi$ or ν. The terms $\gamma_{0_\rho}N_\rho$ are constant and do not give rise to transitions. We choose the parameters f_ρ and γ_{0_ρ} to be in units of fm^2, since, in configuration space, that part of the $E0$ operator that gives rise to transitions is proportional to r^2. Although the terms in the $E0$ operator depending upon N_ρ do not contribute to transitions, they definitely do contribute to the calculated values of the nuclear radii.

As Van Isacker *et al.* [8], have shown, it is straightforward to calculate the reduced matrix elements of transition operators, such as (4), once one has constructed the states $|0^+(1)\rangle$, $|0^+(\beta)\rangle$, and $|0^+(M')\rangle$ for $N_\nu=N-1$ and $N_\pi=1$. The present situation, however, is a little tricky [9,10], because $\hat{n}_{d_\rho}$ is not a [2,1,1,1,1] tensor under U(6), which is the condition for using the procedure of Van Isacker *et al.* [6]. Instead, $\hat{n}_{d_\rho}$ is a combination of a [2,1,1,1,1] tensor, namely,

$$\tilde{N}_\rho=\hat{n}_{s_\rho}-\hat{n}_{d_\rho}=\hat{T}_\rho\ , \tag{5a}$$

and a [1,1,1,1,1,1]=[0] tensor (i.e., a scalar), namely, the number operator N_ρ, given by

$$\hat{N}_\rho=\hat{n}_{s_\rho}+\hat{n}_{d_\rho}\ . \tag{5b}$$

The operator $\tilde{N}_\rho$ has $\sigma=2$, while $\hat{N}_\rho$ has $\sigma=0$, and so it is the $\hat{T}_\rho$ part of $T(E0)$, which produces the $E0$ decays of the β-vibrational bandhead states. The operator $\hat{N}_\rho$ does not contribute to transitions. Taking these points into consideration, we obtain, for the reduced matrix elements of $T(E0)$,

$$\langle 0^+(1)||T(E0)||0^+(\beta)\rangle=[f_\pi N_\pi+f_\nu N_\nu]\times\sqrt{(N+2)(N+3)(N-1)}/2N(N+1)\ , \tag{6a}$$

$$\langle 0^+(1)||T(E0)||0^+(M')\rangle=[f_\pi-f_\nu]\times\sqrt{(N-2)(N+3)N_\pi N_\nu/(N+1)}/2N\ , \tag{6b}$$

$$\langle 0^+(\beta)||T(E0)||0^+(M')\rangle=2[f_\pi-f_\nu]\times\sqrt{(N-2)(N+2)N_\pi N_\nu/[(N-1)(N+1)]}/N(N-1)\ . \tag{6c}$$

For large N we note that

$$\langle 0^+(1)||T(E0)||0^+(\beta)\rangle\xrightarrow[N\to\infty]{}[f_\pi N_\pi+f_\nu N_\nu]/2\sqrt{N}\to f\sqrt{N}/2\quad\text{for } f\equiv f_\pi=f_\nu\ , \tag{7a}$$

$$\langle 0^+(1)||T(E0)||0^+(M')\rangle\xrightarrow[N\to\infty]{}[f_\pi-f_\nu]\sqrt{N_\pi N_\nu/N}/2\ , \tag{7b}$$

$$\langle 0^+(\beta)||T(E0)||0^+(M')\rangle\xrightarrow[N\to\infty]{}[f_\pi-f_\nu]2\sqrt{N_\pi N_\nu}/(N^2)\ . \tag{7c}$$

So, depending upon the values of f_π and f_ν, (7a) and (7b) are of the same order in N, while (7c) is smaller by a factor of $N^{-3/2}$. We have checked formulas (6a)–(6c) by explicit IBM-2 numerical calculations using the computer code NPBOS [11] and obtain exact agreement in all cases. The expressions for $\rho(E0)$ are obtained by multiplying the reduced matrix elements (6a)–(6c) by Z/R^2, where R is the nuclear radius, given by $r_0A^{1/3}$ and $r_0=1.2$ fm.

If $f_\pi=f_\nu$, then the antisymmetric β-vibrational bandhead state $0^+(M')$ has no $E0$ strength in the O(6) limit.

As a "maximal" approximation, we will take $f_\nu=0$ and $f_\pi\neq0$ and determine the $E0$ strength, given by

$[\rho(E0)]^2$, for the nucleus ^{132}Ba ($Z=56$), considered in our previous study [1]. Because it has $N_\pi=N_\nu=3$, the $0^+(M')$ state in ^{132}Ba cannot decay by either $M1$ or $E2$ transitions. Then, in units of $[f_\pi]^2$,

$$[\rho(E0,0^+(1)\rightarrow 0^+(\beta))]^2=1.0333 , \tag{8a}$$

$$[\rho(E0,0^+(1)\rightarrow 0^+(M'))]^2=0.7233 , \tag{8b}$$

$$[\rho(E0,0^+(\beta)\rightarrow 0^+(M'))]^2=0.0823 . \tag{8c}$$

So the $E0$ transitions to the g.s. of ^{132}Ba from the symmetric and antisymmetric β-vibrational bandhead states have similar strengths, while the $E0$ transition between these two β-vibrational states is an order of magnitude smaller. Three $E0$ decays, similar to ours, were discussed by Scholten *et al.* [12], but in the SU(3) limit of the IBM-2 for ^{156}Gd, using two different choices for the form of the Majorana interaction.

In Table I we list the values of $\rho(E0,0_i^+\rightarrow 0_f^+)$ for *all* the even Ba isotopes from $N=54$ to 78 using the proton and neutron boson effective monopole charges determined by Otsuka [13], namely, $f_\pi=0.200$ fm^2 and $f_\nu=0$.

One of our previous predictions [1] for the antisymmetric β-vibrational 0^+ state was that it should be an isomeric state for $N_\pi=N_\nu$. Consequently, another property of interest regarding the $0^+(\beta)$ and $0^+(M')$ states is their isomer shifts relative to the g.s., given by

$$\delta(r^2)_i=\langle 0^+(i)|r^2|0^+(i)\rangle-\langle 0^+(1)|r^2|0^+(1)\rangle , \tag{9}$$

where $i=\beta$ or M' and

$$\langle r^2\rangle=\langle r^2\rangle_{\rm core}+\langle T(E0)\rangle , \tag{10}$$

and $T(E0)$ is the full $E0$ operator given by Eq. (4). But, again, the $\gamma_{0_\rho}N_\rho$ ($\rho=\pi$ or ν) terms do not contribute because the difference in Eq. (9) involves matrix elements for the same values of N_π and N_ν. Using the intrinsic state formalism (see Ref. [1]), we find that the isomer shifts for the symmetric and antisymmetric β-vibrational bandhead states are of order $1/N$, so that they vanish in the limit of large N.

The isomer shift calculation within the algebraic formalism is more complicated [8,10] than for the $E0$ transitions, because the product $\langle N-2\rangle\langle 2\rangle$ contains $\langle N-2\rangle$ *twice* in the evaluation of $\langle 0^+(M')||\hat{n}_{d_\rho}||0^+(M')\rangle$. This is not true of the other two cases for $0^+(1)$ and $0^+(\beta)$. The results are

$$\langle 0^+(1)||\hat{n}_{d_\rho}||0^+(1)\rangle=\frac{N-1}{2(N+1)}N_\rho , \tag{11a}$$

$$\langle 0^+(\beta)||\hat{n}_{d_\rho}||0^+(\beta)\rangle=\frac{N^3-2N^2+N+8}{2N(N+1)(N-1)}N_\rho , \tag{11b}$$

$$\langle 0^+(M')||\hat{n}_{d_\rho}||0^+(M')\rangle$$

$$=\frac{2(N-4)}{(N-1)(N-2)}+\frac{N^3-5N^2+6N+16}{2N(N-1)(N-2)}N_\rho . \tag{11c}$$

From the form of Eqs. (11a)–(11c), it is clear that the isomer shifts, as given by Eq. (9), vanish for large N, as predicted by the intrinsic state formalism.

Using Eqs. (9) and (11a)–(11c), we obtain, for the isomer shifts in the O(6) limit,

$$\delta(r^2)_\beta=4[f_\pi N_\pi+f_\nu N_\nu]/N(N-1)(N+1) , \tag{12a}$$

$$\delta(r^2)_{M'}=\sum_\rho f_\rho\left[\frac{2(N-4)}{(N-1)(N-2)}-\frac{2(N^2-6N-4)}{(N+1)N(N-1)(N-2)}N_\rho\right] . \tag{12b}$$

Equations (12a) and (12b) give the isomer shifts for particles. At midshell for N_π (N_ν), one switches from particle proton (neutron) bosons to hole proton (neutron) bosons. The nuclear radius given by Eq. (10) depends on two effects: (1) the mean field which increases with A [and, hence, with N_π and N_ν as *particle* numbers in Eq. (4) for the γ terms] and (2) the deformation which is simulated in the IBM-2 by the $\hat{n}_{d_\rho}$ terms in Eq. (4). Because the mean-field effect is linear in the nucleon number in lowest order, while the deformation effect is quadratic, the γ_{0_ρ}

TABLE I. Values of $\rho(E0,0_i^+\rightarrow 0_f^+)$ for all even Ba isotopes for even neutron numbers from 54 to 78 using the reduced matrix elements (6a)–(6c) and $f_\pi=0.200$ fm^2 and $f_\nu=0$ from Ref. [13]. The overbars in column 3 denote neutron-hole bosons.

				$\rho(E0,0_i^+\rightarrow 0_f^+)$		
A	N_π	N_ν	N	$i=1$, $f=\beta$	$i=1$, $f=M'$	$i=\beta$, $f=M'$
110	3	2	5	0.2535	0.1660	0.0776
112	3	3	6	0.2268	0.1898	0.0640
114	3	4	7	0.2059	0.2047	0.0528
116	3	5	8	0.1890	0.2144	0.0441
118	3	6	9	0.1751	0.2209	0.0374
120	3	7	10	0.1633	0.2252	0.0321
122	3	8	11	0.1533	0.2281	0.0308
124	3	$\bar{7}$	10	0.1598	0.2204	0.0314
126	3	$\bar{6}$	9	0.1676	0.2114	0.0358
128	3	$\bar{5}$	8	0.1771	0.2007	0.0413
130	3	$\bar{4}$	7	0.1886	0.1875	0.0484
132	3	$\bar{3}$	6	0.2033	0.1701	0.0574
134	3	$\bar{2}$	5	0.2223	0.1455	0.0681

values in Eq. (4) change sign at midshell, while the f_ρ values do *not* change sign at midshell.

Since the isomer shifts are independent of the γ_{0_ρ} terms and since $f_\nu=0$ for the Ba isotopes [13], only the $\hat{n}_{d_\pi}$ term contributes to the isomer shifts for these isotopes. Table II lists the isomer shifts for the Ba isotopes, using the monopole effective charges in Ref. [13]. As is clear from Eqs. (12a) and (12b), $\delta(r^2)_\beta$ decreases as $1/N^2$ for large N, while $\delta(r^2)_{M'}$ decreases only as $1/N$, so that the isomer shifts for the $0^+(M')$ state in the Ba isotopes are significantly larger than those for the $0^+(\beta)$ state. It is worth noting that the isomer shifts are symmetric about midshell for N_ν, because N_π is constant, f_π is kept constant, and the same O(6) symmetry is assumed for all nuclei. Consequently, the largest difference between $\delta(r^2)_\beta$ and $\delta(r^2)_{M'}$ occurs at midshell ($N_\nu=8$). In a more realistic treatment, the O(6) symmetry is most suitable for Ba isotopes with $66 \lesssim N \lesssim 80$.

Another physical quantity of interest is the isotope shift given by

$$\Delta(r^2)=\langle 0^+(1)|r^2|0^+(1)\rangle_{(N_\pi,N_\nu)} - \langle 0^+(1)|r^2|0^+(1)\rangle_{(N_\pi,N_\nu-1)} \tag{13a}$$

for the beginning to the middle of the neutron shell and by

$$\Delta(r^2)=\langle 0^+(1)|r^2|0^+(1)\rangle_{(N_\pi,\bar{N}_\nu)} - \langle 0^+(1)|r^2|0^+(1)\rangle_{(N_\pi,\bar{N}_\nu+1)} \tag{13b}$$

for the middle to the end of the neutron shell, where $\bar{N}_\nu$ now is the number of neutron-hole bosons and is decreasing as one approaches the end of the shell. The theory of isotope shifts in the IBM-2 is discussed in detail in Ref. [13], and the interested reader is referred to this article.

TABLE II. Values of the isomer shifts $\delta(r^2)_\beta$ and $\delta(r^2)_{M'}$ in fm^2 for the even Ba isotopes for even neutron numbers from 54 to 78 using Eqs. (12a) and (12b) for the neutron-particle bosons and $f_\pi=0.200$ fm^2 and $f_\nu=0$ from Ref. [13]. For neutron-hole bosons we use the particle-hole conjugation described following Eq. (12b). The overbars in column 3 denote neutron-hole bosons.

A	N_π	N_ν	N	$\delta(r^2)_\beta$	$\delta(r^2)_{M'}$
110	3	2	5	0.0200	0.0633
112	3	3	6	0.0115	0.0457
114	3	4	7	0.0071	0.0379
116	3	5	8	0.0047	0.0334
118	3	6	9	0.0033	0.0303
120	3	7	10	0.0025	0.0279
122	3	8	11	0.0018	0.0259
124	3	$\bar{7}$	10	0.0025	0.0279
126	3	$\bar{6}$	9	0.0033	0.0303
128	3	$\bar{5}$	8	0.0047	0.0334
130	3	$\bar{4}$	7	0.0071	0.0379
132	3	$\bar{3}$	6	0.0115	0.0457
134	3	$\bar{2}$	5	0.0200	0.0633

In the case of the isotope shift, $\langle r^2\rangle$ actually refers to the charge radius which is related to the proton distribution. However, it is assumed that the proton and neutron distributions are more or less the same. For the isotope shift, the physical interpretation is that $\langle r^2\rangle$ has two parts: (1) a spherical part $\langle r^2\rangle_{\text{core}}+\gamma_{0_\pi}N_\pi+\gamma_{0_\nu}N_\nu$, which includes the spherical inert core and mean-field terms which increase linearly with the number of valence proton and neutron bosons, and (2) a deformed part $f_\pi\hat{n}_{d_\pi}+f_\nu\hat{n}_{d_\nu}$, as discussed earlier for the isomer shifts. Hence, using Eqs. (4), (10), and (11a), we find that

$$\Delta(r^2)=\gamma_{0_\nu}+\frac{f_\pi N_\pi+f_\nu N_\nu}{N(N+1)}+\frac{f_\nu(N-2)}{2N} \tag{14a}$$

for the first half of the shell and

$$\Delta(r^2)=\gamma_{0_\nu}-\frac{f_\pi N_\pi+f_\nu \bar{N}_\nu}{(N+2)(N+1)}-\frac{f_\nu N}{2(N+2)} \tag{14b}$$

for the second half of the shell, where $N=N_\pi+N_\nu$, as usual, and the sign change in γ_{0_ν} in going from particles to holes has already been included in Eq. (14b). Because $\langle r^2\rangle$ in the isotope shift is related to the charge radius, Otsuka and co-workers [13,14] assume that it can be represented by the proton deformation term $f_\pi\hat{n}_{d_\pi}$ and that the neutron deformation term $f_\nu\hat{n}_{d_\nu}$ can be neglected or renormalized somehow into the proton term. Consequently, they set $f_\nu=0$ in their investigations for the Xe, Ba, and Ce isotopes, and we will also make this assumption. In Table III we list our results for the g.s. isotope shifts for Ba using $f_\pi=0.200$ fm^2, $f_\nu=0$, and $\gamma_{0_\nu}=0.045$ fm^2 [13]. Figure 2 shows a comparison of our O(6) results for the isotope shifts for Ba with Otsuka's IBM-2 results [13] and with the known experimental values [15].

For $A>122$ we see that our O(6) results lie above the experimental and IBM-2 results, thereby indicating the degree to which the O(6) symmetry is broken by the Ba

TABLE III. Values of the ground-state isotope shifts $\Delta(r^2)$ in fm^2 for all even Ba isotopes for even neutron numbers from 56 to 80 using (14a) and (14b) and $f_\pi=0.200$ fm^2, $f_\nu=0$, and $\gamma_{0_\nu}=0.045$ fm^2 [13]. The overbars in column 3 denote neutron-hole bosons.

A	N_π	N_ν	N	$\Delta(r^2)$
112	3	3	6	0.059
114	3	4	7	0.056
116	3	5	8	0.053
118	3	6	9	0.052
120	3	7	10	0.051
122	3	8	11	0.050
124	3	$\bar{7}$	10	0.040
126	3	$\bar{6}$	9	0.039
128	3	$\bar{5}$	8	0.038
130	3	$\bar{4}$	7	0.037
132	3	$\bar{3}$	6	0.034
134	3	$\bar{2}$	5	0.031
136	3	$\bar{1}$	4	0.025

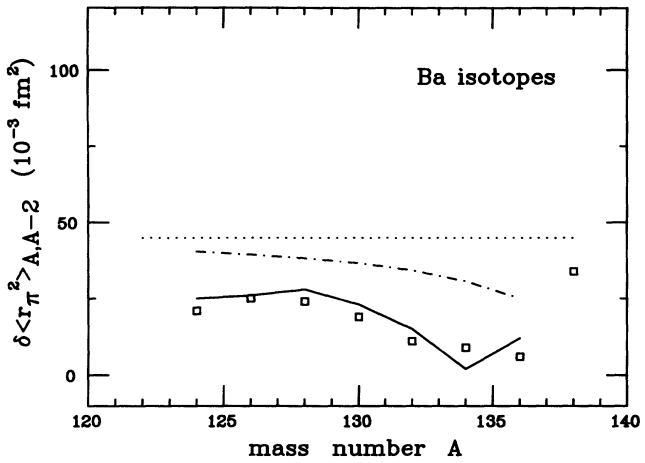

FIG. 2. Ground-state isotope shifts in the even-even Ba isotopes. The squares are the experimental values [15]. The solid line represents the calculated IBM-2 values, while the dot-dashed line represents the calculated values in the O(6) limit of the IBM-2. The dotted line shows the spherical shift (see Ref. [13]). Both calculations use $f_\pi=0.200$ fm^2, $f_\nu=0$, and $\gamma_{0_\nu}=0.045$ fm^2.

isotopes. We note that our O(6) results have the same trends as the experimental and IBM-2 results, but do not produce so large an isotope shift as the latter two, indicating that the Ba isotopes are probably more deformed in their ground states than predicted by the O(6) limit of the IBM-2. Because we use Otsuka's values for the $T(E0)$ parameters [Eq. (4)], we would obtain similar agreement with his Xe and Ce results as we do not with his Ba isotope shifts. We could also compute isotope shifts in the Hg isotopes and compare with experimental results and with the IBM-2 calculations of Barfield *et al.* [16] (Fig. 7). However, in this case we would obtain even worse agreement, because the calculations of Barfield *et al.* indicate that configuration mixing plays an important role in the structure of the light even-even Hg isotopes, and we do not have configuration mixing in our pure O(6)-limit calculations. Clearly, the calculations of Barfield *et al.* are more realistic than ours, and so there is not point in our listing the pure O(6)-limit values for the isotope shifts using their parameters values.

For experimentalists interested in looking for these $E0$ properties of nuclei, such as the Xe, Ba, Pt, Os, and Hg isotopes, the main question is, at what energy should these symmetric and antisymmetric β-vibrational bandheads occur? In Ref. [1] we gave formulas for estimating the energy of the antisymmetry β-vibrational 0^+ state in terms of the energy of the symmetric β-vibrational 0^+ state. One of the formulas we gave was derived earlier by Balantekin, Barrett, and Halse [17], who used it to estimate that the antisymmetric β-vibrational 0^+ state in ^{134}Ba should be at roughly 3.1 MeV and in ^{196}Pt at approximately 2.1 MeV. We have performed IBM-2 calculations for ^{132}Ba and find that the second and third 0^+ states are symmetric and mixed almost equally. The lower of these two 0^+ states carries 66% of the $E0$ strength in the O(6) limit for the symmetric β vibration and, consequently, corresponds mainly to the state we have called $0^+(\beta)$. There are a few other 0^+ states around 3 MeV which have a summed $E0$ strength of only about 30% of the O(6) limit for the antisymmetric β vibration, and so the $E0$ strength for the mixed-symmetry states is strongly fragmented. But the $E0$ strength increases at the midshell, when the d-boson single-particle energy is smaller. Also, IBM-2 calculations for the symmetric β-vibrational 0^+ states have been done by Kirps *et al.* [18] for ^{126}Xe, ^{128}Xe, and ^{190}Os, indicating that the third 0^+ state is mainly $\sigma=N-2$. Similarly, the investigations of Casten and von Brentano [7] suggest that the third 0^+ state in ^{134}Ba is the symmetric $\sigma=N-2$ state. In particular, five 0^+ states have been found experimentally [19] in ^{134}Ba, and it is possible that one of these at approximately 4 times the energy of the first 2^+ state corresponds to the antisymmetric β-vibrational 0^+ state.

In this paper we have reported on the $E0$ decay properties of symmetric and antisymmetric β-vibrational states in the O(6) limit of the IBM-2. Assuming that the Ba isotopes are good O(6)-like nuclei, we predict that one should observe only two $E0$ transitions (of fairly large and similar strength) to the g.s. and that these excited 0^+ states should be connected by a much weaker $E0$ transition. Our IBM-2 calculations for the Ba isotopes indicated that the O(6) symmetry is broken for these nuclei, with the calculated $E0$ strength being below the O(6) prediction and strongly fragmented for the mixed-symmetry states. This O(6) symmetry breaking for the Ba isotopes is also apparent in our results for the isotope shifts (Fig. 2). On the other hand, the decay strength for mixed-symmetry states is always fragmented. Also, the $E0$ strength for the calculated $0^+(\beta)$ state is quite large and should be detectable. Finally, the O(6) symmetry should be better for the Ba isotopes near midshell (or other nuclei in this mass region), so that these nuclei would have larger $E0$ strength. A significant isomer shift is also predicted for most of the Ba isotopes for the antisymmetric β-vibrational bandhead state, but this shift would be difficult to measure experimentally. Consequently, measurement of $E0$ transition strengths in this mass region is still probably the best candidate for locating and identifying symmetric and antisymmetric 0^+ β-vibrational states, since the $M1$ and $E2$ transitions represent two-step processes.

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