Algebraic treatment of giant dipole resonances built on excited states

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An algebraic method for the description of giant dipole resonances built on discrete excited states of both spherical and deformed nuclei is presented within the framework of the interacting boson model. Applications to lanthanide and actinide isotopes are discussed and the effect of γ deformation is also investigated.

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I. INTRODUCTION

The excitation of the giant dipole resonance (GDR) in photoreactions or hadronic and heavy-ion collisions has represented for 40 years one of the major experimental tools in the investigation of nuclear structure properties. The GDR is one of the best known collective motions in nuclei both at zero and finite temperature. Because of its coupling to the low-energy surface degrees of freedom, it provides valuable information on the shape of ground and excited states, derived, for instance, by elastic and inelastic (Raman) photon scattering measurements. Moreover, in recent years [1], γ -ray and particle coincidence experiments following heavy-ion reactions have made it possible to study the GDR in hot nuclei at temperatures $T \approx 1-2$ MeV.

Therefore, mean resonance energies, widths, and strengths of the GDR have been determined with sufficient accuracy for a wide range of nuclei and spin values ($I \le 40\%$). In particular, the persistence of ground-state-like deformation up to temperatures of 1-1.5 MeV [2] has been observed in many cases. Many theoretical approaches have been developed so far in order to deal with GDR excitations at finite temperatures (see [3], and references quoted therein).

Mean-field theories predict a large variety of behaviors in nuclei, due to the interplay between shell effects (vanishing with increasing temperatures) and time-dependent shape fluctuations [3].

Proportionally, minor attention has been paid to the GDR's built on low-energy states of given spin, which are excited, for instance, in nucleon radiative capture reactions [4,5], that correspond to the time-reversed process of photoabsorption. In his often quoted thesis, Brink estimated the partial widths of neutron resonances due to electric dipole emission, assuming that the energy dependence of the photoeffect was independent of the detailed structure of the initial state (the so-called Brink-Axel hypothesis [6]) and applying the principle of detailed balance.

Recent experimental studies [7] seem to indicate a broadening of the GDR as the energy of the relevant excited state increases. Moreover, Brink himself has proposed [8] a simplified theoretical treatment, based on a coupled oscillator model, showing that his initial assumption is only approximately correct.

This matter is both of fundamental and applied interest, for instance, in the evaluation of radiative capture cross sections for shielding and heating in fission and fusion devices.

In the last years, we have developed an algebraic approach to GDR excitations within the framework of the interacting boson model (IBM) approximation [9] to the shell model. This treatment [10] is particularly suitable to the description of shape-transitional nuclei far from closed-shell configurations. With only minor changes, the IBM and related code can be easily extended to calculations of GDR excitation on low-lying excited levels. This topic will be specifically discussed in the present paper.

Section II will be devoted to the description of the relevant formalism, while numerical results will be presented in Sec. III. Finally, concluding remarks will be made in Sec. IV.

It is worth pointing out that evaluation of GDR shapes at finite temperature can rely upon the same IBM approach, once the calculated GDR profiles for a number of discrete levels are suitably averaged over a definite range of excitation energies and spin values. However, this application lies outside the purposes of this article and requires further developments.

II. FORMALISM

The IBM assumes correlated pairs of protons and neutrons, in the valence shells, coupled to angular momenta J=0 and 2 (s and d bosons, respectively), as building blocks [9]. This assumption corresponds to a suitable truncation of the complete shell-model space. If one is interested in giant resonances, in addition to low-energy

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collective motions, further degrees of freedom have to be included in the algebraic model. In particular, the isovector giant dipole resonance (GDR) arises from a coherent superposition of one-particle-one-hole (1p-1h) excitations across one major shell, in the shell-model language. Therefore, the GDR states in even-even nuclei have spin and parity $J^{\pi} = 1^{-}$.

The IBM limiting symmetry corresponding to deformed nuclei, studied in this work, is SU(3). The electric dipole operator behaves like a first-rank tensor under SU(3) transformations; in the IBM approach, it can be simulated by a *p* boson, carrying one unit of spin and negative parity [10,11] and belonging to the (1,0) irreducible representation of SU(3).

The model space is thus given by

$$\psi \to |(s,d)^N > \oplus |(s,d)^N p\rangle \oplus |(s,d)^N p^2\rangle \oplus \cdots, \qquad (1)$$

where N is the usual effective number of valence bosons.

In general, one adopts the one-boson approximation to the giant resonances and, therefore, considers only states of the form $|(sd)^N\rangle$ for the low-energy levels and $|(s,d)^Np\rangle$ for the GDR components. More than one p boson are needed only when descriptions of multiple giant resonances are attempted.

The energies of the GDR components can be easily computed by diagonalizing the following Hamiltonian [10,11]

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{s,d} + \hat{\mathcal{H}}_p + \hat{\mathcal{H}}_{s,d,p} , \qquad (2)$$

where $\hat{\mathcal{H}}_{s,d}$ is the usual IBM-1 [9] Hamiltonian and $\hat{\mathcal{H}}_p$ corresponds to the unperturbed *p*-boson energy, $\varepsilon_p \hat{n}_p$, with \hat{n}_p the number of *p* bosons, equal to 0 or 1 for lowor high-energy (GDR) levels, respectively. $\hat{\mathcal{H}}_{s,d,p}$ describes the coupling between low- and high-lying degrees of freedom and is responsible for the fragmentation of the GDR:

$$\hat{\mathcal{H}}_{s,d,p} = b_0 [d^{\dagger} \times \tilde{d}]^{(0)} \cdot [p^{\dagger} \times \tilde{p}]^{(0)} + b_1 [d^{\dagger} \times \tilde{d}]^{(1)} \cdot [p^{\dagger} \times \tilde{p}]^{(1)} + b_2 \hat{Q} \cdot [p^{\dagger} \times \tilde{p}]^{(2)}, \qquad (3)$$

where the sd-boson quadrupole operator,

$$\hat{Q} = [d^{\dagger} \times \tilde{s} + s^{\dagger} \times \tilde{d}]^{(2)} + \chi [d^{\dagger} \times \tilde{d}]^{(2)}, \qquad (4)$$

is defined as usual [9]. It is worth recalling that $\tilde{s} = s, \tilde{d}_m = (-1)^m d_{-m}$ and $\tilde{p}_m = (-1)^{m+1} p_{-m}$. The leading term in Eq. (3) is the quadrupole-

The leading term in Eq. (3) is the quadrupolequadrupole interaction; the $\{b_i\}$ coefficients are treated as adjustable parameters and the free *p*-boson energy, ε_p , is assumed to follow the semiempirical law $\varepsilon_p \simeq 70 A^{-1/3}$ MeV.

In general, the Hamiltonian (2) has to be diagonalized numerically in the basis (1). Moreover, one must evaluate the reduced matrix elements of the electric dipole operator, $\hat{D}^{(1)}$, between the low-lying states and the GDR components. It has the following form [10]:

$$\hat{D}^{(1)} = D_0 [p^{\dagger} + \tilde{p}]^{(1)} , \qquad (5)$$

with D_0 adjustable parameter.

In order to compare the IBM predictions about the GDR fragmentation with the experimental data, an intrinsic width must be associated with each calculated GDR component, due to the coupling to more complicated nuclear configurations, such as 2p-2h doorway states, or the continuum. Since these configurations lie outside the IBM, their effect has to be accounted for by means of a phenomenological recipe such as

$$\Gamma(E) = k E^{\alpha} , \qquad (6)$$

where the excitation energy, E, is expressed in MeV, Γ is the intrinsic width, and k and α are adjustable parameters [10].

Once GDR excitation energies and dipole transition strengths have been evaluated within the framework of the IBM approach, photon absorption and scattering cross sections can be obtained by means of standard techniques. The photon absorption cross section can be derived from the optical theorem and reads

$$\sigma_{\gamma abs}(E, I_i) \approx \frac{4\pi e^2}{\hbar c} \frac{E^2}{3(2I_i + 1)} \sum_{j,k} \frac{2E_{j,k} \Gamma_{j,k}}{(E_{j,k}^2 - E^2)^2 + \Gamma_{j,k}^2 E^2} |\langle I_{j,k} || \hat{D}^{(1)} || I_i \rangle|^2 , \qquad (7)$$

where E is the incident photon energy, I_i is the spin of the initial nuclear state on which the GDR is built, the indices j and k refer, respectively, to the allowed angular momentum values of the GDR ($|I_i - 1| \le I_j \le I_i + 1$), and the different GDR states at fixed I_j . Therefore, $I_{j,k}$, $E_{j,k}$, and $\Gamma_{j,k}$ are the spin, energy, and width, respectively, of each component into which the GDR is split up.

With these ingredients, it is a simple matter to evaluate the GDR shapes built on excited discrete levels for eveneven nuclei with open-shell configurations. Numerical results for deformed and shape-transitional isotopes in the lanthanide and actinide regions will be presented and discussed in the following section. Here, we intend to work out in some detail a particular case, namely, the GDR built on the β bandhead of an axially symmetric rotor, corresponding to the SU(3) IBM dynamical symmetry [9], which can be handled by analytical methods. Our procedure is analogous to the one developed by Rowe and Iachello [11] for the GDR built on the ground state of deformed nuclei.

In fact, if $\mathcal{H}_{s,d}$ in Eq. (2) has an SU(3) symmetry [9], it is possible to cast the Hamiltonian (2) in terms of quadratic Casimir operators of SU(3) and SO(3) [11,12] by retaining in Eq. (3) only the quadrupole-quadrupole interaction, and to solve analytically the eigenvalue problem. Moreover, the reduced matrix elements of the dipole operator (5) are proportional to the Wigner coefficients of the $SU(3) \supset SO(3)$ reduction. Thus, Rowe and Iachello [11] recovered by this group-theoretical technique the well-known result that the GDR splits into two components because of its coupling with low-lying quadrupole degrees of freedom. For a prolate nucleus, the higher energy component is doubly degenerate and, therefore, its dipole transition strength from the ground state is nearly twice that of the lower energy component. The opposite happens in the case of oblate nuclei.

Following the notation of Ref. [12], the Hamiltonian (2) can be reduced to the form $(b_0=b_1=0)$

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{s,d} + \varepsilon_p \hat{n}_p + b_2 \hat{Q} \cdot [p^{\dagger} \times \tilde{p}]^{(2)} , \qquad (8)$$

where \hat{Q} is defined as in Eq. (4) with $\chi = \pm \sqrt{7}/2$ and $\hat{Q}_p = \pm \sqrt{3}/4 [p^{\dagger} \times \tilde{p}]^{(2)}$. In the previous expressions, one has to take simultaneously the upper, or the lower signs. We assume, moreover, $b_2 > 0$.

Choosing the minus sign in both operators, which corresponds to the usual assumption for ground-state prolate shapes [9], one has

$$\hat{\mathcal{H}}_{s,d,p} \simeq -\frac{b_2}{2\sqrt{3}} [\hat{C}_2(s,d,p) - \hat{C}_2(s,d) - \hat{C}_2(p)], \qquad (9)$$

where the Casimir operator, $\hat{C}_2 \equiv 2\hat{Q}\cdot\hat{Q} + \frac{3}{4}\hat{L}\cdot\hat{L}$ (\hat{L} is defined as in Refs. [9,12]), is diagonal in the (λ,μ) irreducible representation (irrep) of SU(3). Its eigenvalue is

$$C_2(\lambda,\mu) = \lambda^2 + \mu^2 + \lambda(\mu+3) + 3\mu .$$

Since the β bandhead belongs to the (2N-4,2) irrep [9], with N effective number of valence bosons, the coupling of the p boson, belonging to the (1,0) irrep, with the β band gives rise to the following set of states:

$$(2N-4,2)\otimes(1,0) = (2N-4,1)\oplus(2N-5,3)$$

 $\oplus(2N-3,2)$. (10)

All the irreps on the right-hand side of Eq. (10) contain 1^- states, whose excitation energies are therefore given by

$$E_{1}-E_{0_{\beta}^{+}} = \varepsilon_{p} - \frac{b_{2}}{2\sqrt{3}} [C_{2}(2N-4,1)-C_{2}(2N-4,2) - C_{2}(1,0)]$$

$$= \varepsilon_{p} + \frac{b_{2}}{2\sqrt{3}} (2N+6) ,$$

$$E_{2}-E_{0_{\beta}^{+}} = \varepsilon_{p} - \frac{b_{2}}{2\sqrt{3}} [C_{2}(2N-5,3)-C_{2}(2N-4,2) - C_{2}(1,0)]$$

$$= \varepsilon_{p} + \frac{b_{2}}{2\sqrt{3}} (2N-3) ,$$

$$E_{3}-E_{0_{\beta}^{+}} = \varepsilon_{p} - \frac{b_{2}}{2\sqrt{3}} [C_{2}(2N-3,2)-C_{2}(2N-4,2) - C_{2}(1,0)]$$

$$= \varepsilon_{p} - \frac{b_{2}}{\sqrt{3}} (2N-3) . \qquad (11)$$

According to our choice for the sign of $b_2, E_1 > E_2 > E_3$. It is worth recalling the expressions for the energies of the two GDR components built on the 0_1^+ ground state, namely [11],

$$\overline{E}_1 = \varepsilon_p - \frac{2b_2}{\sqrt{3}}N ,$$

$$\overline{E}_2 = \varepsilon_p + \frac{b_2}{2\sqrt{3}}(2N+3) ,$$
(12)

which look like Eqs. (11), since $\overline{E}_1 \simeq E_3$ in the large N limit and \overline{E}_2 corresponds to the average of E_1 and E_2 , approximately.

Finally, the dipole matrix elements for transitions between the basis of the β band and the three GDR components are proportional to the following Wigner coefficients [13]:

$$D_{1} = \langle (2N-4,2), K = 0, L = 0; (1,0), K = 0, L = 1 | | (2N-4,1), K = 1, L = 1 \rangle$$

= $-\left[\frac{2(2N-1)}{18N}\right]^{1/2}$,
$$D_{2} = \langle (2N-4,2), K = 0, L = 0; (1,0), K = 0, L = 1 | | (2N-5,3), K = 1, L = 1 \rangle$$

$$= -\left[\frac{4(2N-4)}{9(2N-3)}\right]^{1/2},$$
(13)

 $D_3 = \langle (2N-4,2), K=0, L=0; (1,0), K=0, L=1 | | (2N-3,2), K=0, L=1 \rangle$

$$= \left[\frac{(2N+1)(N-1)}{3N(2N-3)}\right]^{1/2}$$



FIG. 1. GDR shapes built on yrast positive-parity states of 238 U, belonging to the rotational ground-state band.



FIG. 3. GDR shapes built on positive-parity states of 238 U, belonging to the rotational β band.



FIG. 2. GDR shapes built on $0^+ - 12^+$ levels of 238 U in steps of 2 spin units, belonging to the rotational (A) ground state and (B) β bands. The curve peaks decrease with increasing spin.



FIG. 4. GDR shapes built on positive-parity states of 238 U, belonging to the rotational γ band.

The resulting ratio between lower and upper components, namely, $R \equiv |D_3|^2/(|D_1|^2 + |D_2|^2)$, approaches $\frac{1}{2}$ in the limit of large N, as expected for prolate shapes, and corresponds to a first approximation to the shape of the GDR built on the ground state, thus confirming in this particular case the Brink-Axel hypothesis. In the case of the deformed nucleus ¹⁵⁴Sm, N = 11 and R = 0.58for the GDR components built on the 0⁺ state of the β band, while the corresponding ratio for the two GDR components excited from the 0⁺ ground-state is



FIG. 5. GDR profiles built on 0_i^+ states of 238 U (i = 1, ..., 5).

 $R = \frac{1}{2}(2N+3)/2N \simeq 0.57$, in agreement with the geometrical factor for a prolate shape in both cases.

Analogous results can be obtained for GDR's built on higher spin states of the (2N-4,2) irrep. It is worth recalling that the β and γ bands belong to this irrep and their states with the same spin values are degenerate. Therefore, the energies and transition strengths for the GDR's built on these levels are the same.

These findings are confirmed by refined numerical calculations, as discussed in the next section.

III. RESULTS AND COMMENTS

A. Deformed nuclei

The ²³⁸U nucleus is a suitable example of rigid rotator, close to the IBM SU(3) symmetry [12,14]. The *p*-boson model was able [14] to describe both the GDR twohumped photoabsorption cross section and the elastic scattering distribution by the 0_1^+ ground state and the Raman inelastic scattering cross section to the 2_1^+ excited level, thus giving a first insight into the reliability of the algebraic approach when coupling of the collective dipole mode to states other than the ground state is considered.

According to the formalism presented in the previous section, we have calculated the GDR profiles built on excited levels of ²³⁸U, by numerical diagonalization of the full Hamiltonian (2), where the relevant parameters slightly violate the SU(3) limit symmetry. These IBM parameters, namely $\{b_i\}$ and χ in Eqs. (3) and (4), D_0 , k, and α in Eqs. (5) and (6), have been taken from Ref. [14]. They have been assumed independent of the nuclear excitation energy, since it is well known from microscopic calculations [15] that the effective nucleon-nucleon interaction is only weakly dependent on nuclear temperature in the range 0–15 MeV.

Figure 1 shows the calculated GDR shapes in correspondence with the yrast levels of ²³⁸U belonging to the rotational ground-state band, up to spin $J = 12\hbar$. The resulting two-humped structure is typical of a prolate rotor with axial symmetry; however, by increasing spin and excitation energy of the basis level, this shape is smoothed out because of the increased intrinsic width associated with each GDR component [see Eq. (6)]. No sizeable broadening of the total GDR curve appears, as clearly shown in the two-dimensional plot of Fig. 2(a). An analogous behavior, even if the two peaks are less pronounced than in the previous case, is exhibited by the profiles relevant to the states of the rotational β band, shown in Fig. 2(b). In this case, the deformed structure is smoothed in a nearly spherical GDR shape at spin values $J \simeq 10$ h. This transitional behavior, as a function of the excitation energy of the level on which the GDR is built, is also evident in Figs. 3 and 4, where the calculated GDR shapes on β and γ band states, respectively, are shown.

Therefore, it is possible to state that in realistic calculations for ²³⁸U the effect relevant to dynamical γ deformation in the side rotational bands does not alter the GDR two-humped shape corresponding to an underlying prolate structure with axial symmetry, thus confirming the



FIG. 6. GDR profiles built on 2_i^+ states of 238 U $(i=1,\ldots,5)$. Note that GDR shapes relevant to the 2_2^+ , 2_3^+ , and 2_4^+ , 2_5^+ states, respectively, are nearly coincident, because these doublets are degenerate in the SU(3) scheme.

analytical results of Sec. II, obtained in the exact SU(3) limit.

The main observed effect is the development of a single-peaked shape, approximating a Lorentzian distribution, because of the increasing of $\Gamma(E)$ for each GDR component, due to the higher excitation energy, E, and consequently to the increased number of exit or doorway states coupled to the GDR states, which determine their intrinsic widths.

In fact, if one considers GDR built on states with the



FIG. 7. GDR shapes built on yrast positive-parity states of ¹⁵⁰Sm, belonging to successive phonon multiplets.

same spin, as in Figs. 5 and 6 for $J=0^+$ and 2^+ , respectively, a similar trend from deformed to spherical profiles arises. The dependence on the spin values has a different, opposite effect, as discussed in the next subsection. In the case of 238 U, it is overcome by the excitation energy effect.

Finally, it is worth noticing that, in Fig. 6, the GDR shapes for 2_2^+ , 2_3^+ , and 2_4^+ , 2_5^+ states are nearly coincident, as expected on the basis of the degeneracy of the above levels in the SU(3) scheme [9], even if they belong to different rotational bands.



FIG. 8. GDR profiles built on 0_i^+ states of ¹⁵⁰Sm $(i=1,\ldots,5)$.



FIG. 9. GDR profiles built on 2_i^+ states of ¹⁵⁰Sm (i = 1, ..., 5).

B. Transitional nuclei

The photoabsorption cross sections in the ground states of even-even Sm nuclei has been studied in Ref. [16] within the framework of the p-boson IBM.

The Sm isotope chain provides a classical example of transitional nuclei ranging from a spherical $(^{146,148}$ Sm) to an axially symmetric deformed shape $(^{154}$ Sm) [9], thus allowing us to investigate how the GDR profiles built on excited states vary with the nuclear deformation. It is worth recalling that IBM predictions of photon inelastic scattering cross sections to 2_1^+ levels of $^{148-154}$ Sm have

been recently compared with the corresponding measured data [17]; however, no definite conclusion can be drawn from that work because the experiments have been performed only at one incident photon energy, $E_{\gamma} = 11.4$ MeV, lower than the GDR peaks and cannot discriminate between the available theoretical models [17].

We focus our present analysis on ¹⁵⁰Sm, which is intermediate between a spherical vibrator and a deformed rotor, and on ¹⁵⁴Sm, where rotational bands are already fully developed. The same IBM parameters as in Ref. [16] have been used, according to the above-mentioned remarks.

An interesting feature is provided by Fig. 7, where the



FIG. 10. GDR shapes built on yrast positive-parity states of ¹⁵⁴Sm, belonging to the rotational ground-state band.

GDR's built on the yrast levels of ¹⁵⁰Sm are shown. The GDR curve relevant to 0_1^+ ground state has a singlehumped shape characteristic of a spherical vibrator [16], but—proceeding to higher spins—the relevant levels assume a dynamic deformation as reflected by the increased momenta of inertia [18], thus giving rise to a splitting of the GDR profile into two peaks whose distance increases with spin as a result of the increased nuclear deformation. In this particular case, the concurrent effect due to the larger intrinsic widths provided by the higher excitation energies to be taken into account, which tends to smooth out any GDR structure from deformation splitting (see Figs. 1 and 2 for ²³⁸U), is overcome by the spininduced larger deformation.

For the sake of comparison, Figs. 8 and 9 show the



FIG. 11. GDR profiles built on (a) 0_i^+ and (b) 6_i^+ states of 154 Sm (i = 1, ..., 5).

GDR shapes built on excited 0_i^+ and 2_i^+ states $(i=1,\ldots,5)$, respectively. Here, at fixed spin value, when the curves depend only on the different excitation energies and nuclear-level structures, a single-humped GDR shape persists, whose total width is nearly constant in the 0-4 MeV excitation energy range.

As far as statically deformed nuclei are concerned, like ¹⁵⁴Sm, close to the SU(3) limit, this kind of effect is even more pronounced, as clearly shown in Figs. 10 and 11. In the latter case, exemplifying the general behavior, for fixed spin values the GDR profile evolve from two- to one-humped shape as the excitation energy of the basis level increases.

In all the considered cases, the GDR total width



FIG. 12. GDR shapes built on levels of 154 Sm, belonging to the rotational (a) ground-state and (b) β bands.



FIG. 13. GDR shapes built on positive-parity states of 154 Sm, belonging to the rotational γ band.

remains almost constant, only the detailed structure is smoothed out because of the increased intrinsic width associated with each GDR component [see Eq. (6)]. The effect of spin is to enlarge the deformation splitting, thus resulting in a broader width than for the ground state.

This behavior is elucidated in Figs. 10, 12(a), and 12(b) for rotational ground state and β bands, respectively. In the latter case, the two peaks, not completely resolved for the low spins, arise from the three GDR components of Eq. (11), where E_2 and E_3 components are not separate because of the overlapping intrinsic widths, and are clearly visible for spins $J \simeq 10\hbar$. This trend is further confirmed by the results for the γ band, shown in Fig. 13. It is worth remembering that the even-spin levels of the γ band are nearly degenerate with the corresponding states of the β band, belonging to the same SU(3) irreps [9].

IV. CONCLUDING REMARKS

The previous IBM calculations of GDR curves built on excited states of deformed and transitional nuclei show that the GDR profile has quite a regular behavior as a function of level excitation energy and spin. In particular, the first two moments of the GDR strength distribution, which determine its centroid energy and total width, do not vary too much as far as different basis levels are considered for a given nucleus. A minor spin effect is found, resulting in a larger deformation of the nucleus with increasing spin, which could eventually lead to a two-humped shape starting from a single-humped one. We have studied this behavior in transitional nuclei like Sm isotopes, belonging to the U(5) \rightarrow SU(3) IBM transitional chain; an analogous analysis for the other transitional classes of the algebraic model (from spherical oscillators to γ soft nuclei and from γ soft to rigid rotors) has already been performed [19], complementing and supporting the present results.

The degree of freedom given by the dynamical γ deformation does not have important consequences on the GDR shape and, in general, a GDR pattern similar to that for the ground state arises.

To sum up, the Brink-Axel hypothesis can be assumed as a reasonable first-order approximation, even if its validity is confined to low spin and excitation energy regions. Particular caution has to be taken in performing, for instance, calculations of nucleon radiative-capture cross sections to specific levels, mainly isometric states with high spin, for which the angular-momentum effects resulting in a rather large deformation of the nucleus could provide a broad splitting of the GDR and, consequently, affect the production cross sections, analogously to the findings for the high-spin levels of yrast, β , and γ bands of deformed nuclei and, more dramatically, of the multiphonon excitations in vibrational and transitional nuclei, previously described.

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