Spectral anomalies of even-mass Hg isotopes

S. T. Hsieh and H. C. Chiang
Department of Physics, National Tsing Hua University, Hsinchu, Taiwan 30043, Republic of China

M. M. King Yen
Department of Nuclear Engineering, National Tsing Hua University, Hsinchu, Taiwan 30043, Republic of China
(Received 21 April 1992)

The $^{184-200}$Hg isotopes are studied in the interacting-boson-approximation-plus-fermion-pair model. In general, energy levels up to $I=22$ can be reproduced quite well. In the low excitation energy region both the shape evolution and the $(8^+,10^+,12^+)$ triplet structure can be reproduced. The $(8^+,10^+,12^+)$ triplet structure can be understood as the saturation of isospin 1 fermion-pair interaction at the high spin end. The termination of triplet structure at $I=12$ strongly supports the $(i_{13/2})^2$ band crossing. Striking agreements between the observed energy gaps of $(14_1^+,12_1^+)$ and $(2_1^+,0_1^+)$ states are considered as an indication of weak coupling between the boson core and the fermion pair. The $B(E2,I\rightarrow I-2)$ are calculated and the spin dependence of the $B(E2)$ values can be reproduced qualitatively. However, the detailed mass dependence is not correctly reproduced.

PACS number(s): 21.60.Ev, 27.80.+w, 27.70.+q

Recently, a lot of attention has been focused on the mercury isotopes. Identification of band structures and decay patterns through heavy-ion-induced reactions and related theoretical works uncovered many interesting facets of this chain of transitional nuclides, which is situated between the spherical and prolate regions. In this paper we consider only the positive-parity states of even-mass Hg isotopes. The systematics of the known low-lying energy spectra is nicely shown in Fig. 1, which is taken from Ref. [1]. For $182\leq A\leq 190$, the ground-state vibrational bands are crossed by the excited rotational bands built on the second 0^+ states. Separation of the two bandheads has a minimum at $A=182$ [2]. The two bands coexist above and below the crossing point. The ground-state bands are weakly oblate ($\beta\simeq 0.12$), and the excited bands are prolate and well deformed ($\beta\simeq 0.25$) [1,3,4]. They thus establish the nuclear shape coexistence. For the heavier isotopes, $192\leq A\leq 198$, there is no low-lying prolate band. Rather, a closely spaced triplet 8^+, 10^+, and 12^+ intervenes between the ground oblate band and the $\nu i_{13/2}^2$ aligned band. The mass dependence of energy gaps and transition probabilities between them indicate a reduction of the collective motion of the 12^+ state in the lighter ones [5].

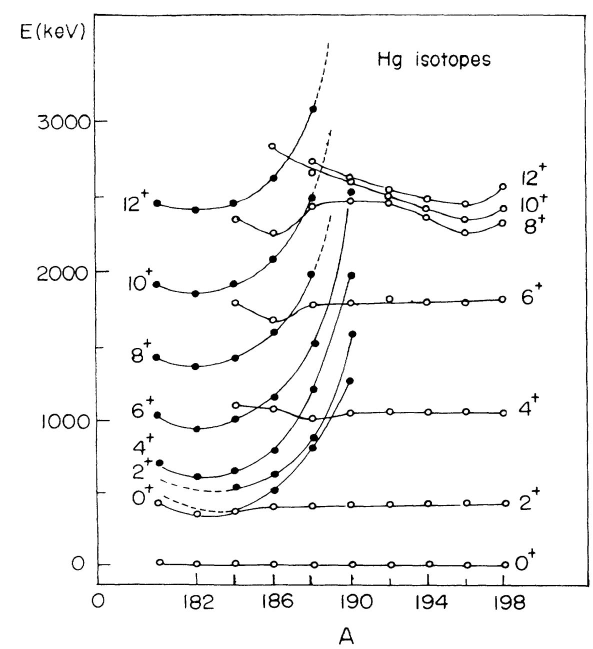

FIG. 1. Systematics of strongly deformed bands (solid circles) and weakly deformed bands (open circles) in the even-mass Hg isotopes. This figure is taken from Ref. [1].

Theoretical works have been performed using the cranking model, calculating the potential-energy surfaces, or in the framework of the interacting boson model. Cranked shell-model calculations suggested that the two upbendings in the prolate deformed band of ^{184}Hg can be associated with $\nu i_{13/2}^2$ and $\pi h_{9/2}^2$ pairs. Similarly, for ^{186}Hg, the gentle upbending in the prolate band was also due to the $\nu i_{13/2}^2$ pair. In addition, there is a superband in ^{184}Hg. Its lower part was interpreted as members of a two-quasiparticle $\pi h_{9/2}^2$ band, and its higher-spin members were speculated as $\pi i_{13/2}^2$ excitation coupled to the prolate deformed band [3]. Cranking calculations have also been performed on $^{189-198}$Hg [6]. For the positive-parity yrast sequences, Routhians, band-crossing frequencies, and aligned angular momenta were well reproduced. Later on, this kind of analysis was extended to new band intersections at higher spins in the five isotopes $^{190-194}$Hg [7]. Potential-energy surface calculations [4,8] have obtained theoretical deformations for mercury isotopes which agree with experimentally deduced β values, and the shape coexisting excited 0^+ states can be identified as a secondary minimum.

On the other hand, the interacting boson model IBM-2 has been applied to even isotopes of mercury with $182 \le A \le 202$ [9]. Two kinds of configurations, with proton boson number equal to 1 and 3, respectively, were mixed in order to treat shape coexistence in the light isotopes. The calculated energy spectra and reduced quadrupole transition rates reasonably reflected the observed mass dependence. Comparisons of calculated $B(E2)$ branching ratios and quadrupole moments of the first excited state with the available data were also included. Another systematic study was based on the interacting-boson-fermion-approximation (IBFA) model. A unified description of $^{189-198}$Hg isotopes was presented where up to two- and three-quasiparticle states were involved for even- and odd-mass isotopes, respectively [10]. With one set of interaction parameters, it was possible to reproduce the main features of the yrast spectra of all isotopes as well as the reduced quadrupole transition rates and g factors. However, the 20^+ states in ^{190,192,194}Hg which exhibit a second anomaly were too high in the calculation. The most recent interacting-boson-model approach included not only one broken pair, but also two broken pairs (or four quasiparticles). A first calculation was done on the high-spin states in ^{194}Hg [11]. Both positive- and negative-parity states can be calculated simultaneously with good results. In contrast with cranking-model calculations, the interacting boson calculations being performed in the laboratory frame yield results that can be compared with experiments directly. This has the advantage that all states, and not just the bandhead energies, can be compared with the experimental data directly.

In this work we study the positive-parity states of even mercury isotopes with mass numbers $184 \le A \le 200$ using the interacting boson approach with one broken pair. This one broken pair is treated as two fermions as we have done in a series of papers. We also do not distinguish protons and neutrons. The purpose is to see if it is possible to use such a simpleminded combination of the interacting boson and shell models with minimum parameter fitting to describe the spectra variation from light isotopes to heavy ones, including shape coexistence. Particular interest will be focused on the explanations of the $(8^+,10^+,12^+)$ triplet structure in the energy spectra of $^{188-198}$Hg.

The model space is spanned by basis states with bosons only and those including two fermions:

$$|n_s n_d \nu I\rangle + |\tilde{n}_s \tilde{n}_d \tilde{\nu} L;(j_1,j_2)J;I\rangle , \tag{1}$$

where the s- and d-boson numbers satisfy the relation $n_s+n_d=N=\tilde{n}_s+\tilde{n}_d+1$. The active boson number N is equal to 12 and 4 for ^{184}Hg and ^{200}Hg, respectively, taking ^{208}Pb as the inert core. The I here is the total angular momentum, and the L and J are the intermediate angular momenta for the boson core and fermion pair, respectively. The ν and $\tilde{\nu}$ stand for the additional quantum numbers needed to specify the particular d-boson states. The fermion single-particle orbits are denoted by (j_1,j_2). Since the $(\nu i_{13/2})^2$, $(\pi h_{9/2})^2$, and $(\pi i_{13/2})^2$ pair excitations are most relevant to the high-spin states, we include $j_1=j_2=\frac{13}{2}$ or $\frac{9}{2}$. A third orbital $f_{5/2}$ is also included. Therefore the cases of $j_1=j_2=\frac{5}{2}$ and $j_1=\frac{9}{2},j_2=\frac{5}{2}$ are also included.

The Hamiltonian is

$$H=H_B+H_F+V_{BF} . \tag{2}$$

H_B contains the single-boson energies and boson-boson interactions:

$$H_B=\epsilon_d n_d+a_1 P^\dagger\cdot P+a_2 L\cdot L+a_3 Q\cdot Q , \tag{3}$$

where the pairing, angular momentum, and quadrupole interactions are the most important terms in the multipole expansion. The four parameters ϵ_d, a_1, a_2, and a_3 are determined mainly from the most low-lying part of the spectra. H_F contains the single-fermion energies and fermion-fermion interactions:

$$H_F=\epsilon_j n_j+\frac{1}{2}\sum_J V_J\sqrt{2J+1}[(a_j^\dagger\times a_j^\dagger)^{(J)}\times(\tilde{a}_j\times\tilde{a}_j)^{(J)}]^{(0)} , \tag{4}$$

where ϵ_j is the fermion single-particle energy in the j orbit and the V_J's are the fermion-fermion interactions. To calculate V_J, we have taken the Yukawa potential form with the Rosenfeld mixture. The oscillator constant is $0.96(190)^{-1/3}\times10^{22}$ sec^{-1}. The overall strength of the V_J's is determined by requiring that the energy of the two identical particles in the $i_{13/2}$ orbit coupled to $J=0$ be 2 MeV lower than that of the $J=2$ state. V_{BF} is a quadrupole boson-fermion interaction:

$$V_{BF}=Q\cdot\{\alpha(a_j^\dagger\times\tilde{a}_j)^{(2)}+\beta[(a_j^\dagger\times a_j^\dagger)^{(4)}\times\tilde{d} -d^\dagger\times(\tilde{a}_j\times\tilde{a}_j)^{(4)}]^{(2)}\} , \tag{5}$$

where

$$Q=(d^\dagger\times\tilde{s}+s^\dagger\times\tilde{d})^{(2)}-\frac{\sqrt{7}}{2}(d^\dagger\times\tilde{d})^{(2)} . \tag{6}$$

For each Hg isotope, the Hamiltonian is then diagonal-

TABLE I. Adopted interaction parameters in MeV.

	^{184}Hg	^{186}Hg	^{188}Hg	^{190}Hg	^{192}Hg	^{194}Hg	^{196}Hg	^{198}Hg	^{200}Hg
ϵ_d	0.688	0.693	0.772	0.772	0.739	0.733	0.728	0.695	0.675
a_1	0.035	0.049	0.049	0.049	0.057	0.083	0.099	0.119	0.145
β	0.035	0.035	0.035	-0.005	-0.005	-0.005	-0.005	-0.005	-0.005
$\epsilon_{13/2}$	1.626	1.442	1.334	1.295	1.268	1.242	1.220	1.220	1.416

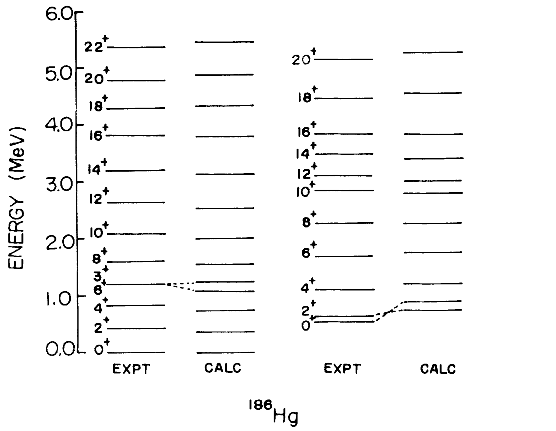

FIG. 2. Calculated and observed energy spectra of ^{186}Hg. Data were taken from Ref. [13].

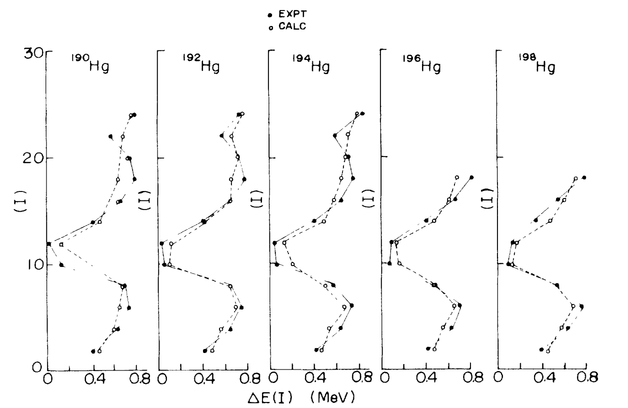

FIG. 4. Plots of the angular momentum I vs $\Delta E(I)=E(I)-E(I-2)$ for the yrast states of ^{190,192,194,196,198}Hg. Data were taken from Refs. [16–20].

ized in the complete model space with the interaction parameters varied to get a good agreement with the low-lying energy spectra. From one isotope to another, we want to unify the parameters as much as possible. It was found that $a_2=\alpha=0$, $a_3=-0.005$ MeV can be fixed for all isotopes. Other parameter values are listed in Table I. Generally speaking, values of ϵ_d, a_1, β, and $\epsilon_{13/2}$ change slightly as the neutron number changes.

The calculated spectra of a typical light isotope ^{186}Hg was compared with experimental data in Fig. 2. Both the weakly oblate ground-state band and the prolate band were reproduced quite well. Only the closely spaced 0_2 and 2_2 states were inverted. For the heavier isotopes, the yrast states with even spins were reproduced very well, as can be seen in Figs. 3 and 4, where the spin I vs $\Delta E(I)=E(I)-E(I-2)$ were plotted. The difficulty encountered in Ref. [10] about the position of the 20^+ states in ^{190,192,194}Hg did not appear in this work. For the heaviest isotope ^{200}Hg, the spectra cannot be reproduced so well as the others. In particular, all the 1^+ states were way off. Note that the number of bosons for ^{200}Hg is only 4, and so the IBA type of model is not expected to work very well.

In the energy spectra of the $^{186-198}$Hg isotopes, the most striking feature is the closely spaced 8_1^+, 10_1^+, and 12_1^+ triplet states. In Fig. 5 we plot the experimental and calculated energies for the triplet states for comparison. In general, the triplet structure and its mass dependence can be reproduced quite well. In our model this triplet structure can be understood by $(i_{13/2})^2$ band crossing and the saturation of $T=1$ fermion-pair interaction at the high-J end. It was shown in Ref. [11] that the wave function of ^{194}Hg can be interpreted as two quasiparticle states weakly coupled with the boson core. In this picture the dominant configurations for these triplet states should be $J=8,10,12$ $(i_{13/2})^2$ fermion-pair states coupled with $L_B=0$ boson states. Since the two-body matrix elements for the $(i_{13/2})^2$ fermion pair with $J=8$, 10, and 12 are very close, this produces the triplet structure of the $I=8$, 10, and 12 states. This is consistent with the general saturation behavior for $T=1$ fermion-pair interaction at the high-spin end [12]. On the other hand, the

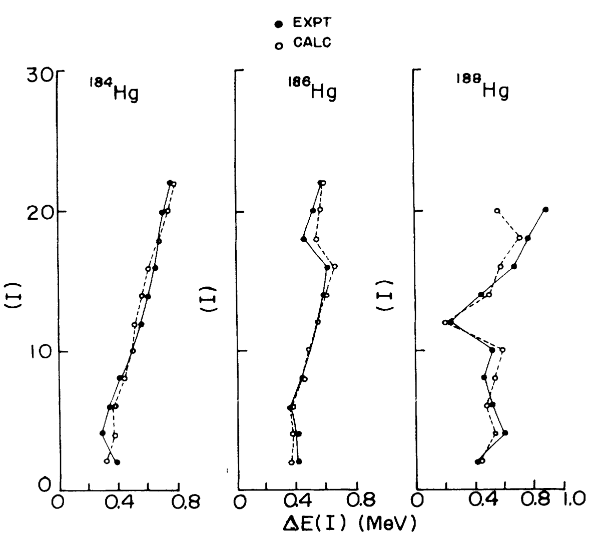

FIG. 3. Plots of the angular momentum I vs $\Delta E(I)=E(I)-E(I-2)$ for the yrast states of ^{184,186,188}Hg. Data were taken from Refs. [13–15].

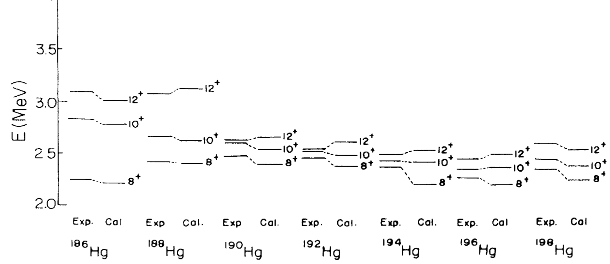

FIG. 5. Experimental and calculated 8^+, 10^+, and 12^+ triplet states for $^{186-198}$Hg.

TABLE II. Comparisons of the experimental $(2_1^+,0_1^+)$ and $(14_1^+,12_1^+)$ energy gaps (in MeV) of the Hg isotopes.

	^{184}Hg	^{186}Hg	^{188}Hg	^{190}Hg	^{192}Hg	^{194}Hg	^{196}Hg	^{198}Hg
$E_{2_1^+}-E_{0_1^+}$	0.367	0.405	0.413	0.416	0.423	0.428	0.426	0.412
$E_{14_1^+}-E_{12_1^+}$	0.604	0.582	0.437	0.420	0.417	0.413	0.403	0.348

most dominant configuration for the 14^+ states is the $J=12$ fermion-pair state coupled with the $L_B=2$ boson state since the highest J value for the $i_{13/2}$ pair is 12. Therefore we expect a larger energy gap between 14_1^+ and 12_1^+ states. In fact, if this picture is correct, the 14_1^+-12_1^+ energy gaps can be estimated from the corresponding 2_1^+-0_1^+ energy gaps. In Table II we list the experimental energy gaps between 2_1^+ and 0_1^+ states together with those between 14_1^+ and 12_1^+ states. For light mass isotopes such as ^{184}Hg and ^{186}Hg, the 14_1^+-12_1^+ gaps are considerably larger than those of 2_1^+-0_1^+ gaps. From ^{188}Hg to ^{198}Hg, the corresponding two energy gaps agree with each other strikingly. In our calculation the calculated energy gaps agree quite nicely with the experimental values. Furthermore, an analysis of the wave functions shows that the 14_1^+ and 12_1^+ states of the $^{188-198}$Hg isotopes are really dominated by a $(i_{13/2})^2$, $J=12$ fermion pair coupled with $L_B=2$ and 0 configurations, respectively. The positions of the spins which the $(i_{13/2})^2$-plus-boson configuration begin to dominate vary quite quickly from ^{184}Hg to ^{190}Hg. In Table III we list these spins for reference. Note that from ^{190}Hg the crossing spins are $I=8$ and the triplet structure of the 8^+, 10^+, and 12^+ states becomes quite prominent.

In the calculation, although the $h_{9/2}$ and $f_{5/2}$ orbitals are included, very few states, however, have significant contributions from the configurations related to these two orbitals. It is interesting to mention that the wave function of the 8_1^+ state of ^{200}Hg has about 35% intensity of the $(h_{9/2})^2$ configuration. This suggests that the 10_1^+ and 12_1^+ states of ^{200}Hg which have not been identified experimentally may have more complicated structures and the disappearance of the $(8^+,10^+,12^+)$ triplet structure can be understood.

The model wave functions can be further tested against the experimental reduced quadrupole transition rates $B(E2,I\rightarrow I-2)$. The electric quadrupole transition operator in this model is given by

$$T^{(2)}=e_BQ+\alpha e_F(a_j^\dagger\times\tilde{a}_j)^{(2)} +\beta e_B[(a_j^\dagger\times a_j^\dagger)^{(4)}\times\tilde{d}-d^\dagger\times(\tilde{a}_j\times\tilde{a}_j)^{(4)}]^{(2)} . \quad (7)$$

In our calculation, since $\alpha=0$, e_F is immaterial and e_B can be left as an overall normalization. Figure 6 shows the $B(E2,I\rightarrow I-2)$ plot for ^{184}Hg, where the calculated values were normalized to the data point at $I=12$ (that corresponds to $e_B\simeq0.094$). The overall trend was repro-

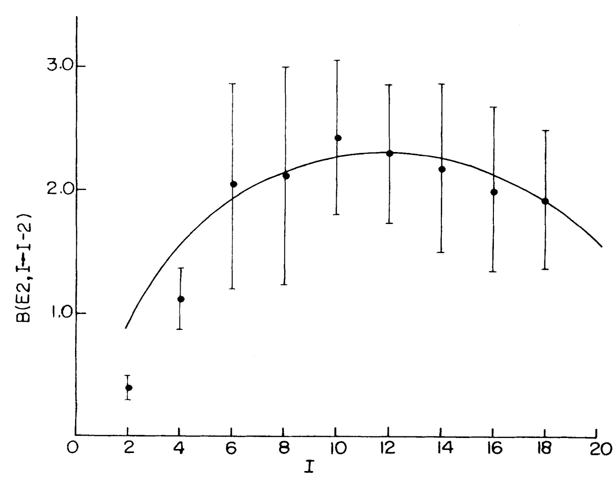

FIG. 6. Reduced quadrupole transition rates of ^{184}Hg as a function of angular momentum. Data with experimental errors were taken from Ref. [3]. The curve represents the calculated result with $e_B=0.094$.

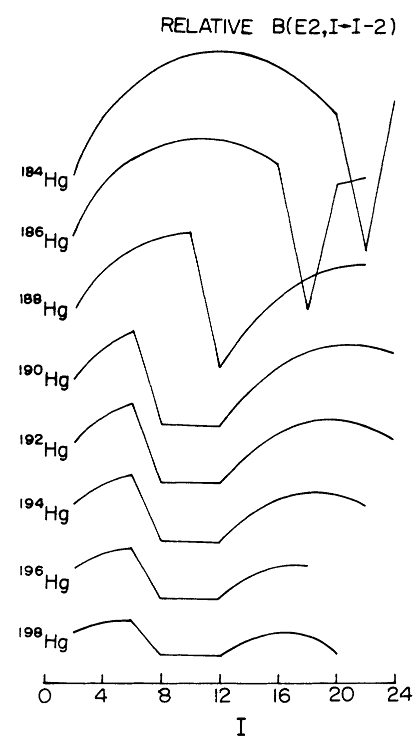

FIG. 7. Calculated $B(E2,I\rightarrow I-2)$ vs I for the even mercury isotopes. Each curve was plotted using an arbitrary convenient scale. The zero point is given by the minimum in each curve.

TABLE III. Crossing spins for the $(i_{13/2})^2$ band for the Hg isotopes.

	^{184}Hg	^{186}Hg	^{188}Hg	^{190}Hg	^{192}Hg	^{194}Hg	^{196}Hg	^{198}Hg	^{200}Hg
Crossing spin	22	18	12	8	8	8	8	8	8

duced, and all the calculated $B(E2)$ values fall well within the experimental errors except those at $I=2$ and 4.

Figure 7 shows the calculated behavior of $B(E2, I\to I-2)$ for $^{184-198}$Hg. For the lightest one, the sudden drop happens at $I=22$. As A increases, the drop moves toward smaller values of I. For $^{190-198}$Hg, $B(E2, I\to I-2)$ vanishes for all the triplet members 8, 10, and 12. Inspecting the relevant wave functions, we found that, for these isotopes, the $I=6$ state is almost a pure boson state (over 99%), whereas the $I=8$ state is almost a pure $i_{13/2}^2$ fermion-pair-plus-boson state (more than 90%). Likewise, $I=10$ and 12 states are almost pure $i_{13/2}^2$-plus-boson states. Since, in the V_{BF}, $\alpha=0$ and $\beta=0.005$, the boson-fermion coupling is very weak. Therefore the fermion angular momentum $(i_{13/2})^2$ J is almost a good quantum number for states near the yrast line. In calculating the $B(E2)$ values, this has the effect of eliminating the contributions from the boson part of the $T^{(2)}$ operator. Of course, if we adopted a nonzero α value, we may have some small contributions of $B(E2)$ values from the fermion part of the $T^{(2)}$ operator. However, we do not believe this can reproduce the mass variation of $B(E2)$ values mentioned in Ref. [5]. Nevertheless, the small values of these $B(E2)$ values are qualitatively in agreement with the result of our calculation. As a result, $B(E2, I\to I-2)$ for $I=8$, 10, and 12 in these five isotopes are all zero in our model and cannot account for the observed variations with mass.

In summary, the $^{184-200}$Hg isotopes are studied in the boson-plus-fermion-pair model. The energy spectra can be reproduced quite well in general. Both the prolate shape and oblate shape states can be reproduced. An analysis of the wave functions indicates that all states with $I\le 6$ are pure boson states. This again manifests the advantage of the interacting boson model in which different deformations can be treated simultaneously. The $(i_{13/2})^2$ alignment happens in all isotopes. From ^{184}Hg to ^{188}Hg, the crossing spin of the $(i_{13/2})^2$ band continues to reduce. From ^{190}Hg to ^{198}Hg, the $(i_{13/2})^2$ band crosses the ground-state band and becomes the yrast states. The observed $(8^+, 10^+, 12^+)$ triplet states are found to be the weak coupling between $(i_{13/2})^2$, $J=8,10,12$, states with the boson 0_1^+ states. The small energy gaps are understood as the saturation of $T=1$ fermion-pair interactions at the high-J end. The termination of the triplet structure at $I=12$ is a strong support for the $(i_{13/2})^2$ band structure. The striking agreements for the heavier isotopes between the energy gaps of $14_1^+, 12_1^+$ and $2_1^+, 0_1^+$ states are considered as a further support for the boson-fermion weak-coupling mechanism. The spin dependence of the $B(E2, I\to I-2)$ values can be reproduced qualitatively. However, the mass dependence of the $B(E2, I\to I-2)$ values within the 8^+, 10^+, and 12^+ triplets cannot be reproduced in this model. It is suggested that a more complete Hamiltonian which includes monopole and exchange terms may be needed for such delicate features.

This work was supported by the National Science Council, Republic of China, under Grant No. NSC 80-0208-M007-13.

[1] M. O. Kortelahti, E. F. Zganjar, J. L. Wood, C. R. Bingham, H. K. Carter, K. S. Toth, J. H. Hamilton, J. Kormicki, L. Chaturvedi, and W. B. Newbolt, Phys. Rev. C **43**, 484 (1991).

[2] G. D. Dracoulis, A. E. Stuchbery, A. O. Macchiavelli, C. W. Beausang, J. Burde, M. A. Deleplanque, R. M. Diamond, and F. S. Stephens, Phys. Lett. B **208**, 365 (1988).

[3] W. C. Ma, A. V. Ramayya, J. H. Hamilton, S. J. Robinson, J. D. Cole, E. F. Zganjar, E. H. Spejewski, R. Bengtsson, W. Nazarewicz, and J. Y. Zhang, Phys. Lett. **167B**, 277 (1986).

[4] R. Bengtsson, T. Bengtsson, J. Dudek, G. Leander, W. Nazarewicz, and J. Y. Zhang, Phys. Lett. B **183**, 1 (1987).

[5] M. Guttormsen, A. Von Grumbkow, Y. K. Agarwal, K. P. Blume, K. Hardt, H. Hubel, J. Recht, P. Schuler, H. Kluge, K. H. Maier, A. Maj, and N. Roy, Nucl. Phys. **A398**, 119 (1983).

[6] M. Guttormsen and H. Hubel, Nucl. Phys. **A380**, 502 (1982).

[7] H. Hubel, A. P. Byrne, S. Ogaza, A. E. Stuchberg, G. D. Dracoulis, and M. Guttormsen, Nucl. Phys. **A453**, 316 (1986).

[8] F. Hannachi, G. Bastin, M. G. Porquet, C. Schuck, J. P. Thibaud, C. Bourgeois, L. Hildingsson, D. Jerrestam, N. Perrin, H. Sergolle, F. A. Beck, T. Byrski, J. C. Merdinger, and J. Dudek, Nucl. Phys. **A481**, 135 (1988).

[9] A. F. Barfield, B. R. Barrett, K. A. Sage, and P. D. Duval, Z. Phys. A **311**, 205 (1983).

[10] S. Kuyucak, Amand Faessler, and M. Wakai, Nucl. Phys. **A420**, 83 (1984).

[11] F. Iachello and D. Vretenar, Phys. Rev. C **43**, R945 (1991).

[12] J. P. Schiffer and W. E. True, Rev. Mod. Phys. **48**, 191 (1976).

[13] R. B. Firestone, Nucl. Data Sheets **55**, 583 (1988).

[14] R. B. Firestone, Nucl. Data Sheets **58**, 243 (1989).

[15] B. Singh, Nucl. Data Sheets **59**, 133 (1990).

[16] B. Singh, Nucl. Data Sheets **61**, 243 (1990).

[17] V. S. Shirely, Nucl. Data Sheets **64**, 205 (1991).

[18] B. Singh, Nucl. Data Sheets **56**, 75 (1989).

[19] M. Sakai, *Table of Members of Quasi-Bands* (Institute for Nuclear Study, University of Tokyo, Tanashi, Tokyo, 1984).

[20] C. Zhou, Nucl. Data Sheets **60**, 527 (1990).