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Signature inversion driven by wobbling motion in negative-gamma nuclei

Masayuki Matsuzaki
Department of Physics, Fukuoka University of Education, Munakata, Fukuoka 811-41, Japan
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Effects of the wobbling motion on the signature splitting in triaxial odd-A nuclei are studied analytically and numerically by means of the quasiparticle-vibration coupling model that is an extension of the cranking model. The signature inversion in negative-gamma three-quasiparticle bands is shown to be driven by the rotational K mixing in the wobbling mode.

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g-s band crossings in rotating nuclei cause shape change depending on the Fermi surface (λ) of aligning two quasiparticles. The first crossing in the $A \simeq 160$ region is usually due to the $i_{13/2}$ quasineutrons and the triaxial deformation after the crossing is $\gamma > 0$ in the nuclei with $\lambda_\nu < \epsilon_{3/2}$ and $\gamma \lesssim 0$ in those with $\lambda_\nu > \epsilon_{3/2}$. This holds for both even-even and odd-Z nuclei but, in the latter, their shapes depend also on the Fermi surface of the odd quasiproton.

Signature inversion, a phenomenon that a $\Delta I = 1$ rotational band decouples into two signature [$r = \exp(-i\pi\alpha)$, $I = \alpha \bmod 2$] sequences invertedly, has been observed systematically in the $(\pi h_{11/2})^1(\nu i_{13/2})^2$ bands in the nuclei around Tm and Lu isotopes. The rotational-frequency range where the inversion survives is wider and its magnitude is bigger in the positive-gamma (low-λ_ν) nuclei than in the negative-gamma (mid-λ_ν) nuclei [1]. In the case of the positive-gamma nuclei, signature inversion has been explained in terms of the static triaxial deformation of the rotating mean field as done in odd-odd nuclei [2–6]. But these theories are insufficient to explain the inversion in negative-gamma nuclei, ^{165,167}Lu, for example. Then, a natural extension of the theory is to incorporate the fluctuations, related to the gamma degree of freedom, around the mean field: the shape fluctuation (gamma vibration) and the fluctuation of the rotational axis (wobbling motion) in triaxial nuclei. The effects of the former on signature inversion were examined in Ref. [7]. The authors obtained signature inversion using a particle-rotor model with gamma vibration but they needed to alter the γ dependence of moments of inertia so as to simulate the positive-gamma rotation even in negative-gamma nuclei.

The rotational K mixing is an essential property of the wobbling motion in our quasiparticle-vibration coupling (QVC) model based on the random-phase approximation (RPA) in the rotating frame [8], although this mode reduces to a gamma-vibrational mode with $K = 2$ and $r = -1$ in the limit of vanishing $\omega_{\rm rot}$ and static triaxial deformation [9]. Then, is it possible to find experimentally the effects of K mixing? The signature dependence of the intraband $B(E2{:}\ \Delta I = 1)$ in odd-A nuclei has been shown to be determined by the coupling with this mode, but only the $K = 2$ component is concerned in this case [10]. The relative sign between the $K = 2$ and the rotationally induced $K = 1$ components in the wave function was discussed for the first time in Ref. [11], and it was pointed out that this relative sign determines the $\omega_{\rm rot}$ dependence of $B(E2{:}\ \Delta I = 1)$ between the wobbling and the yrast bands in even-even nuclei. Unfortunately, it is difficult to measure these interband $B(E2{:}\ \Delta I = 1)$ values. As discussed in the following, the signature inversion in negative-gamma nuclei is related intimately to the signature dependence of the QVC wave function that stems from the K mixing in the wobbling mode. Therefore signature inversion offers an opportunity for observing the rotational K mixing.

The QVC vertices in the $r = -1$ sector are given by

$$\begin{aligned}\Lambda(\text{ wob} \otimes f, u) &= -(\kappa_1^{(-)} \tilde{T}_1^{(-)} \langle f | \tilde{Q}_1^{(-)} | u \rangle \\ &\quad + \kappa_2^{(-)} \tilde{T}_2^{(-)} \langle f | \tilde{Q}_2^{(-)} | u \rangle) , \\ \Lambda(\text{ wob} \otimes u, f) &= -(\kappa_1^{(-)} \tilde{T}_1^{(-)} \langle f | \tilde{Q}_1^{(-)} | u \rangle \\ &\quad - \kappa_2^{(-)} \tilde{T}_2^{(-)} \langle f | \tilde{Q}_2^{(-)} | u \rangle) ,\end{aligned} \tag{1}$$

where $\langle f | \tilde{Q}_K^{(-)} | u \rangle$ and $\tilde{T}_K^{(-)} \equiv \langle \phi | [\tilde{Q}_K^{(-)}, \ X_{\rm wob}^\dagger] | \phi \rangle$ ($K = 1, 2$) are the quasiparticle and phonon matrix elements, respectively, of the doubly stretched quadrupole operators, and $\kappa_K^{(-)}$ are the strengths of the doubly stretched $Q \cdot Q$ interaction (see Ref. [12] for details). The former vertex determines the coupling between $a_u^\dagger | \phi \rangle$ and $X_{\rm wob}^\dagger a_f^\dagger | \phi \rangle$ while the latter determines that between $a_f^\dagger | \phi \rangle$ and $X_{\rm wob}^\dagger a_u^\dagger | \phi \rangle$, where f and u denote the lowest-energy favored and unfavored states, respectively, and $X_{\rm wob}^\dagger$ denotes the creation operator of the wobbling mode excited on the even-even reference state $|\phi\rangle$. The coupling strengths are determined by the relative sign between the $K = 1$ and $K = 2$ terms in Eq. (1): The wobbling phonon mixes more into the unfavored (favored) state than into its signature partner when $\tilde{T}_1^{(-)} \langle f | \tilde{Q}_1^{(-)} | u \rangle$ and $\tilde{T}_2^{(-)} \langle f | \tilde{Q}_2^{(-)} | u \rangle$ have the same (opposite) sign, whereas the QVC becomes almost signature independent if the phonon has a good K quantum number. The partial phase rules are given by

$$\text{quasiparticle}: \ \frac{\langle f | \tilde{Q}_2^{(-)} | u \rangle}{\langle f | \tilde{Q}_1^{(-)} | u \rangle} > 0 , \tag{2}$$

except for the low-spin region of positive-gamma nuclei discussed later [10], and

$$\text{phonon}: \quad \frac{\tilde{T}_2^{(-)}}{\tilde{T}_1^{(-)}} = f(\omega_{\rm wob}) \frac{\sin\gamma_{\rm s.c.}}{\sin(\gamma_{\rm s.c.}+60^\circ)} , \qquad (3)$$

for triaxial nuclei [11]. Here $\gamma_{\rm s.c.}$ denotes the self-consistent deformation of $|\phi\rangle$ and $f(\omega_{\rm wob})$ is a negative-definite function of the excitation energy of the wobbling motion. An example of QVC wave functions is presented in Fig. 1 of Ref. [12]. Note that the presented wave functions were calculated adopting $\gamma^{(\rm pot)}=0$; however, $\tilde{T}_2^{(-)}/\tilde{T}_1^{(-)}$ is positive because of the shape driving effect to $\gamma<0$ of the collectively rotating zero-quasiparticle (0qp) reference state. This figure shows clearly that the wobblinglike phonon with $r=-1$ mixes more strongly into the unfavored state than into the favored state when $\gamma_{\rm s.c.}<0$ whereas the gamma-vibrational phonon with $r=+1$ mixes more into the favored state because of $|\langle f|\tilde{Q}_2^{(+)}|f\rangle| > |\langle u|\tilde{Q}_2^{(+)}|u\rangle|$. This result indicates that the energy of the lowest unfavored state is pushed down mainly by the coupling with the $r=-1$ phonon in negative-gamma nuclei. The effects of the mean field and the fluctuations on signature splitting in the odd-Z nuclei around Tm and Lu isotopes are summarized in Table I. The most characteristic feature is that the static deformation and the wobbling motion contribute oppositely to each other in most cases. In addition, although this table says nothing about the $\omega_{\rm rot}$ dependence, the effect of the wobbling on signature splitting is expected to increase at high spins as the rotational K mixing increases.

The relative magnitudes of these individual contributions vary depending on the quasiparticle number and λ_ν. In the 1qp bands, the contributions from the $\gamma(+)$ vibration and the $\gamma(-)$ vibration (wobbling motion) almost cancel each other because they have similar collectivity [13]. In the 3qp bands, on the other hand, the $\gamma(+)$ vibration becomes less collective in this mass region [14]. Then the sign of the signature splitting is expected to be determined by competition between the contributions from the static deformation and the wobbling.

This competition is examined numerically in the following. The calculations were performed for two Yb (s band) plus $(\pi h_{11/2})^1$ systems that represent the average property of adjacent Tm and Lu nuclei for which experimental data were available. The "cr" values were calculated by diagonalizing the cranked triaxial Nilsson plus the BCS potential. The phonons were constructed from the pairing plus the doubly stretched $Q\cdot Q$ interaction by means of the RPA in the rotating frame, using the interaction strengths which reproduce at $\omega_{\rm rot}=0$ the experimental $E_{2_2^+}$ and $E_{0_2^+}$. The QVC Hamiltonian was calculated within the second-order perturbation or diagonalized in a model space consisting of zero-, one-, and two-phonon states; the results were denoted by "pert" or "full", respectively. The details of the formulation were given in Ref. [8] but the deformations were treated as input parameters in the present study. Their values are presented in Table II. Figure 1 shows the results for the yrast $(\pi h_{11/2})^1(\nu i_{13/2})^2$ band in the ^{166}Yb plus $(\pi h_{11/2})^1$ system, a representative of mid-λ_ν, negative-gamma nuclei. In the cranking calculation, signature splitting $\Delta e'$ is always positive because of negative-gamma deformation. But the QVC effect is strong enough to reverse the sign of $\Delta e'$ both in the perturbation and the diagonalization. The result of the diagonalization almost reproduces the data for ^{167}Lu [1]. (Those for ^{165}Tm have not been reported.) The partial contributions from the $r=-1$ wobbling and the $r=+1$ gamma-vibrational phonons, calculated by the second-order perturbation theory, are shown in Fig. 2. Note that their sum gives the difference between the "cr" and the "pert" curves in Fig. 1. This proves the qualitative expectation presented above that the wobbling motion is the cause of the signature inversion in the 3qp bands of negative-gamma nuclei. The crucial difference from the 1qp bands is that the effect of the $r=+1$ phonon is too small to cancel that of the $r=-1$ phonon.

The remaining problem is how the spectra of the 3qp bands with positive-gamma deformation, in which the static deformation has driven signature inversion already, will be modified by the QVC. Two partial phase rules, Eqs. (2) and (3), give the result that the wobbling motion contributes to normal signature splitting, $\Delta e'>0$, in positive-gamma cases, and the contribution increases with $\omega_{\rm rot}$. This is true for small-gamma cases and for high spins even if gamma is large. On the other hand, there is another mechanism that favors signature inversion at low spins of large-gamma cases. In order to see this mechanism, we review the origin of Eq. (2). It can be decomposed into two steps [10]:

$$\frac{\langle f|iJ_y|u\rangle}{\langle f|J_z|u\rangle} \approx -\frac{\Delta e'_{\rm cr}}{\hbar\omega_{\rm rot}} , \qquad (4)$$

derived by using a small-gamma approximation, and

$$\frac{\langle f|\tilde{Q}_2^{(-)}|u\rangle}{\langle f|\tilde{Q}_1^{(-)}|u\rangle} \approx -\frac{\langle f|iJ_y|u\rangle}{\langle f|J_z|u\rangle} , \qquad (5)$$

derived by using a single-j approximation. These rela-

TABLE I. Partial contributions to signature splitting in the odd-Z nuclei with $A\simeq160$ from the static triaxial deformation, the wobbling motion ($r=-1$), and the gamma vibration with $r=+1$; n and i denote contributions to the normal ($\Delta e'=e'_u-e'_f>0$) and inverted ($\Delta e'<0$) splittings, respectively.

Band	$\gamma_{\rm s.c.}$	Static	Wobbling	$\gamma(+)$-vib.
$(\pi h_{11/2})^1$	$-$	n	i	n
$(\pi h_{11/2})^1(\nu i_{13/2})^2$, $\lambda_\nu \gtrsim \epsilon_{3/2}$	$-$	n	i	n
$(\pi h_{11/2})^1(\nu i_{13/2})^2$, $\lambda_\nu < \epsilon_{3/2}$	$+$	i	n[a]	n

[a]See text.

TABLE II. The parameters used in the calculations. The parametrizations were given in Ref. [15], where the sign of γ was defined oppositely. The presented $\beta^{(\rm den)}$ and $\gamma^{(\rm den)}$ were calculated at $\hbar\omega_{\rm rot}=0.2$ MeV.

Reference	$\beta^{(\rm pot)}$	$\gamma^{(\rm pot)}$	$\beta^{(\rm den)}$	$\gamma^{(\rm den)}$	Δ_p (MeV)	Δ_n (MeV)
^{166}Yb	0.27	-10°	0.32	-6°	1.10	0.73
^{160}Yb	0.20	$+20^\circ$	0.22	$+14^\circ$	1.24	0.75

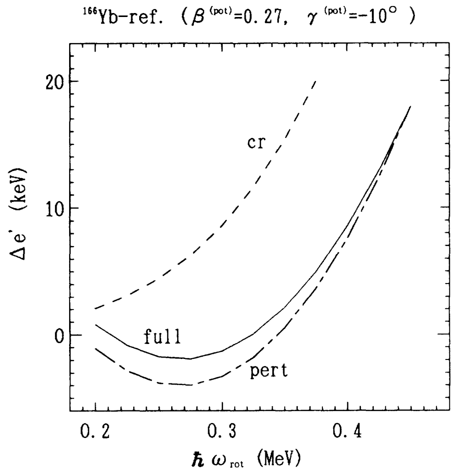

FIG. 1. Calculated signature splittings of the yrast $(\pi h_{11/2})^1(\nu i_{13/2})^2$ band in the ^{166}Yb plus $(\pi h_{11/2})^1$ system as a function of the rotational frequency. "Cr" denotes the result of the cranking calculation, while "pert" and "full" denote those of the quasiparticle-vibration coupling (QVC) calculations within the second-order perturbation and diagonalization, respectively. The QVC results were smoothed parabolically in order to remove accidental losses of collectivity in the RPA phonons as a result of mixings with other single-particle-like RPA solutions. The parameters used are compiled in Table II.

tions indicate that the inversion of $\Delta e'_{cr}$, brought about by large positive gamma, is accompanied by that of the sign of $\langle f|iJ_y|u\rangle/\langle f|J_z|u\rangle$, and consequently that of the sign of $\langle f|\tilde{Q}_2^{(-)}|u\rangle/\langle f|\tilde{Q}_1^{(-)}|u\rangle$. In numerical calculations, however, the breaking of rule (2) is confined in a relatively small region although the inversion of $\Delta e'_{cr}$ survives up to a high ω_{rot}. The calculated partial contributions to the signature splitting in the 3qp band of the ^{160}Yb plus $(\pi h_{11/2})^1$ system, which is a representative of low-λ_ν, positive-gamma nuclei, are shown in Fig. 3. The mechanism mentioned just above pushes $\delta(\Delta e')_{(-)}$ down

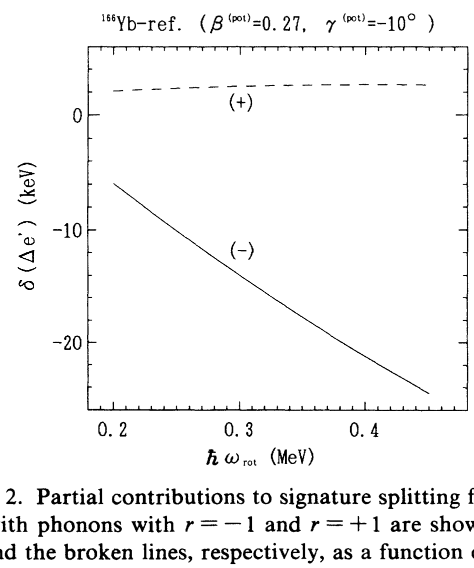

FIG. 2. Partial contributions to signature splitting from coupling with phonons with $r=-1$ and $r=+1$ are shown by the solid and the broken lines, respectively, as a function of the rotational frequency. Note that their sum gives the difference between the "cr" and "pert" curves in Fig. 1.

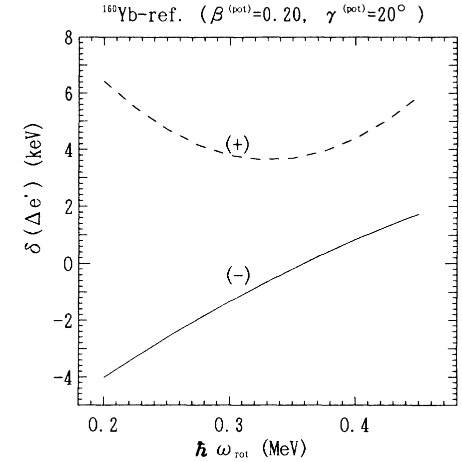

FIG. 3. The same as Fig. 2 but for the ^{160}Yb plus $(\pi h_{11/2})^1$ system.

overall to the direction of the inversion. The final results for this system are shown in Fig. 4 along with the cranking values. The inversion driven by the static deformation survives even after the inclusion of the QVC contribution. This result is qualitatively consistent with the data for ^{159}Tm and ^{161}Lu (compiled in Ref. [1]) but the calculated inversion is smaller than the measured one.

In summary, we have given an explanation of signature inversion in negative-gamma nuclei by means of the quasiparticle-vibration coupling model in the rotating frame analytically and numerically. The inversion in negative-gamma three-quasiparticle bands is driven by the wobbling motion, which reduces to gamma vibration with $r=-1$ in the limit of vanishing ω_{rot} and static γ. In particular, the rotational K mixing in it is the essential origin of the inversion. The present model has also been applied to a positive-gamma system and it has been proved that static deformation is the source of the inversion in this case as in usual explanations.

Fruitful discussions with Y. R. Shimizu are acknowledged.

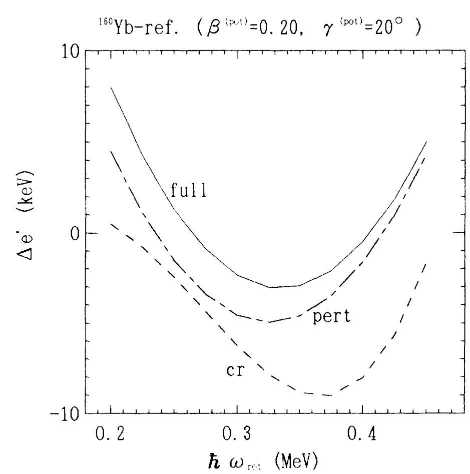

FIG. 4. The same as Fig. 1 but for the ^{160}Yb plus $(\pi h_{11/2})^1$ system.

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