QCD sum rules for nucleons in nuclear matter

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The self-energies of quasinucleon states in nuclear matter are studied using QCD sum-rule methods. A correlator of nucleon interpolating fields, evaluated in the finite-density ground state, is calculated using both an operator product expansion and a dispersion relation with a spectral ansatz. This approach relates the nucleon spectral properties (such as the quasinucleon self-energies) to matrix elements of QCD composite operators (condensates). With increasing nucleon density, large changes in Lorentz scalar and vector self-energies arise naturally; the self-energies are found to be comparable to those suggested by relativistic nuclear physics phenomenology. The most important phenomenological inputs are the baryon density and the value of the nucleon σ term divided by the average current mass of the light quarks. However, the successful comparison to relativistic phenomenology is sensitive to assumptions made about the density dependence of certain four-quark condensates.

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I. INTRODUCTION

The Dirac phenomenology of proton-nucleus scattering provides a simple yet quantitatively accurate model of spin observables over a wide range of energies and target nuclei [1,2]. In this approach, nucleon propagation in the nuclear medium is described by a Dirac equation with an optical potential that has large (several hundred MeV) and cancelling isoscalar Lorentz scalar and vector components [3]. Relativistic models of bound-state properties (e.g., nuclear matter saturation), which feature scalar and vector self-energies similar to the optical potentials, have also had wide success [4]. However, despite the appeal of the phenomenology, it has not been clear how this physics might be motivated from quantum chromodynamics (QCD).

In a recent Letter, we showed how large scalar and vector self-energies could arise naturally in finite-density QCD due to changes in the scalar quark condensate and the quark density [5]. We used QCD sum-rule techniques to calculate the self-energies of nucleons propagating in nuclear matter and found them to be comparable to the Dirac optical potentials and to self-energies in relativistic bound-state models. In the present work, we give further details of the approach along with a more sophisticated analysis, including a variety of corrections outlined in Ref. [5].

The sum-rule approach at zero density [6–8] focuses on a Fourier-transformed correlation function of the interpolating field $\eta(x)$, which is constructed from quark fields such that it carries the quantum numbers of a nucleon [see Eqs. (3.3) and (3.4)]:

$$\Pi(q) \equiv i\int d^4x\, e^{iq\cdot x}\langle 0|T[\eta(x)\bar{\eta}(0)]|0\rangle \qquad (1.1)$$

$$\equiv \Pi_s(q^2)+\Pi_q(q^2)\not{q} \; . \qquad (1.2)$$

The state $|0\rangle$ is the physical nonperturbative vacuum, and T is the time-ordering operator. This function is analogous to a nucleon propagator in a hadronic model. Lorentz covariance and the discrete space-time symmetries imply that this vacuum correlator can be decomposed into two invariant functions of q^2, Π_s and Π_q, whose analytic properties (as functions of complex q^2) can be studied. Dispersion relations in q^2 for each function follow after inserting a complete set of physical intermediate states into Eq. (1.1). These spectral representations show that the correlator describes the propagation of nucleons (i.e., there is a nucleon pole) as well as higher-mass states with nucleon quantum numbers. By assuming a simple phenomenological ansatz for the spectral densities, the dispersion relations provide a model of the correlator in the spacelike (Euclidean) region ($q^2<0$).

On the other hand, the correlator at large spacelike momenta can be evaluated independently using an operator product expansion (OPE) [6,8]. This expansion expresses the correlator as a sum of coefficient functions, calculated in QCD perturbation theory, that multiply matrix elements of local composite operators. The operators are ordered according to their mass dimension. Examples of such operators are $\bar{q}q$ and $G^a_{\mu\nu}G^{a\mu\nu}$, where q is an up or down quark field and $G^a_{\mu\nu}$ ($a=1-8$) is the gluon field-strength tensor. In vacuum, these matrix elements are the nonperturbative quark and gluon condensates.

By equating these two different representations of the correlator, we obtain QCD sum rules, which directly relate the spectral parameters (i.e., the masses and other parameters of the phenomenological ansatz) of low-lying resonances to QCD Lagrangian parameters and the condensates [6,7].

To optimally extract the spectral parameters, one typically applies a Borel transform to the sum rules. On the QCD side it improves the convergence of the OPE by suppressing the contributions of higher-dimensional operators, while on the phenomenological side it emphasizes the contribution from the nucleon pole [6]. (It also eliminates undetermined polynomials in q^2.) This double improvement of the sum rules often seems miraculous; we will present an equivalent but more transparent formulation in Sec. III in terms of integrals of the correlator times a weighting function on a contour in the complex q^2 plane [9,10]. In this approach, the conventional Borel-transformed sum rules simply correspond to a particularly advantageous choice of weighting function.

To generalize the sum-rule analysis to finite density, we consider a correlator of the same interpolating field $\eta(x)$ evaluated in the ground state of finite nuclear matter $|\Psi_0\rangle$, rather than in the vacuum:

$$\Pi(q)\equiv i\int d^4x\, e^{iq\cdot x}\langle\Psi_0|T[\eta(x)\bar{\eta}(0)]|\Psi_0\rangle \tag{1.3}$$

$$\equiv\Pi_s(q^2,q\cdot u)+\Pi_q(q^2,q\cdot u)\not{q}+\Pi_u(q^2,q\cdot u)\not{u}\ . \tag{1.4}$$

We can still use Lorentz covariance and the discrete space-time symmetries to identify invariant functions and the kinematical invariants; however, there are now three functions of two independent invariants, q^2 and $q\cdot u$. Here u^μ is the four-velocity of the nuclear medium. If, for convenience, we specialize to the rest frame of nuclear matter, where $u^\mu=(1,\mathbf{0})$, we can consider the analytic properties of the correlator as a function of the complex "energy" variable q_0, with the three-momentum $\mathbf{q}$ held fixed. A Lehmann representation in q_0 follows just as in nonrelativistic many-body physics [11].

We again exploit the analytic properties of the correlator to derive sum rules, although we must be aware of the more complex spectrum at finite density. The nucleon is no longer an eigenstate of the many-body Hamiltonian, and nucleon and antinucleon spectral properties are not simply related by discrete symmetries, since the ground state is not invariant under charge conjugation. Nevertheless, we still argue for a simple ansatz for each spectral function, which will introduce Lorentz scalar and vector "self-energies" characterizing a quasinucleon excitation. As before, we relate these functions to an operator product expansion of the correlator in the Euclidean region, which now corresponds to taking q_0 large and imaginary. The expansion involves matrix elements of operators at finite density. These include new matrix elements that vanish in the vacuum as well as modifications to the vacuum condensates. The simplest new matrix element is the quark density $\langle q^\dagger q\rangle$.

Our assumptions about the finite-density spectral functions rely on basic features of observed nuclear phenomena. In particular, at finite density the positive-energy nucleon pole becomes a broadened peak in the medium, which reflects the spreading of strength into other states such as two-particle–one-hole excitations. Nevertheless, the peak apparently remains narrow on hadronic scales. Since the sum rule averages over the spectral function on such scales, a pole approximation is justified. Experiment also indicates that the pole position in energy remains essentially constant with density for low-energy nucleons—a result we should *predict* from the sum rules. In contrast, we expect the negative-energy nucleon pole, which corresponds to an antinucleon propagating in nuclear matter, to be shifted and broadened significantly in a manner not well constrained by experiment.

Our strategy is to approximate the spectral functions for the positive-energy nucleon by assuming a quasiparticle pole, allowing scalar and vector self-energies to change with density, while suppressing contributions from the region of the negative-energy nucleon excitation (antinucleon) as well as strength from higher-energy states. We achieve this suppression through the weighting function in the sum-rule integrals or, equivalently, by manipulating the real and imaginary parts of the Borel-transformed correlator. One of the fundamental assumptions made is that there is a separation of "hadronic" and "nuclear" energy scales and the physics corresponding to each. We expect that the real part of the quasinucleon self-energies can be associated with hadronic physics, while determining the quasinucleon width (or making a quantitative study of nuclear matter saturation) requires a detailed treatment of the nuclear many-body problem, which is well beyond the scope of QCD sum rules.

If we tried to map out the discontinuities of the correlator (spectral functions) in detail by using a timelike probe, we would naturally be sensitive to many-body physics. However, by concentrating on the spacelike correlator, we deal with an energy-averaged spectrum. If the averaging is comparable to or coarser than the spread in the nucleonlike excitation, a quasiparticle model is appropriate and useful. The idea is to focus on (Euclidean) time scales that are short enough so that an operator product expansion is reliable but long enough so that the implied energy averaging is not so coarse as to wash out the quasinucleon contribution. In practice this is accomplished by an appropriate weighting function.

Why should the QCD sum-rule approach work? Since a nucleon in free space is an excitation of the vacuum, its properties are ultimately related to vacuum properties. But the degree to which one must understand the nature of the vacuum is not clear; a full solution of QCD may be needed for a complete understanding of the strong-interaction properties of hadrons. However, to determine the *spectral* properties of many hadrons (e.g., the masses), it may be sufficient to characterize the vacuum in terms of a small number of parameters: the quark and gluon condensates. The sum-rule approach is based on this possibility [6]. While sum rules have not been *proved* to correctly represent QCD, they successfully account for many properties of hadrons in free space [7].

It is then natural to extend QCD sum-rule methods to calculate the scalar and vector self-energies of a nucleon quasiparticle in the nuclear medium. If a small number of condensates are sufficient to determine gross features of the low-energy structure in the zero-density spectrum, then changes in these condensates due to finite baryon density should be reflected in changes in the nucleon spectrum. Thus the condensates should determine the basic features of the nucleon self-energies in medium. In particular, we should be able to assess the predictions of relativistic phenomenology: Are scalar and vector nucleon self-energies large and cancelling in nuclear matter?

We find that a simplified version of the sum rules for the nucleon correlators, discussed previously in Ref. [5], predicts nucleon isoscalar self-energies qualitatively similar to those of relativistic phenomenology. In particular, large and cancelling scalar and vector self-energies arise naturally due to changes in the simplest scalar and vector quark condensates, $\langle \bar{q}q \rangle$ and $\langle q^{\dagger}q \rangle$. The leading density dependence of the scalar condensate is determined by the nucleon σ term in a model-independent way [12,13], and the density dependence of the vector "condensate" is trivially proportional to the nucleon density. We find that the more complete sum rules discussed in the present work imply, for the most part, relatively minor modifications of these basic results. If we consider the ratios of self-energies at finite density to the zero-density nucleon mass, the predictions are insensitive to details of the calculation, with one important exception. This exception is the assumed density dependence of a four-quark condensate, which occurs in a higher-order term in the OPE and also plays an important role in the zero-density sum rule [8]. Considering two possible scenarios for the density dependence of this condensate, we find either good qualitative agreement with relativistic phenomenology (Fig. 5) or else a basic disagreement with the empirical situation (Fig. 7).

There are several other recent applications of QCD sum-rule methods to finite-density problems [12,14–17]. In Refs. [14] and [15], sum rules were applied to account for the Nolen-Schiffer anomaly in terms of changes in the neutron-proton mass difference with density. The implication that nucleon sum rules predict in-medium self-energies similar to those of relativistic phenomenology was also noted in Ref. [14]. Recent work by Hatsuda and Lee studies the properties of vector mesons in the nuclear medium [16].

In Refs. [12] and [17], Drukarev and Levin use finite-density sum rules to describe nuclear matter saturation properties. Thus, rather than focusing on individual scalar and vector self-energies as we do, they study the shift of the quasinucleon pole. Cancellations between scalar and vector contributions are still present in the sum rules and were pointed out in Ref. [17]. Since the empirical pole shift is quite small on hadronic scales, its determination from sum rules is likely to be very uncertain. We can establish that QCD predicts large and cancelling scalar and vector self-energies more reliably than we can quantitatively predict the net single-particle energy. Therefore, we concentrate on the self-energies in this paper. We also note that the Drukarev-Levin approach is based on a dispersion relation in q^2 rather than one in q_0, as presented here.

The outline of this paper is as follows. In Sec. II, the elements of relativistic nuclear phenomenology that we might expect to test using QCD sum-rule techniques are reviewed, and some notation is introduced. The conventional QCD sum-rule treatment of the nucleon in free space is outlined in Sec. III, and the added complications of working at finite density are noted. The general approach at finite density, including the spectral decomposition of the finite-density correlator and its operator product expansion, are presented in Sec. IV. We reserve many details of the OPE and the models of higher-dimensional condensates for discussion elsewhere [18,19]. Basic results are presented in Sec. V (a more exhaustive analysis will be given in Ref. [19]) and discussed further in Sec. VI. Section VII is a summary.

II. RELATIVISTIC NUCLEAR PHYSICS PHENOMENOLOGY

In this section, we review some elements of relativistic nuclear physics phenomenology and introduce some basic concepts and notation that we will use to generalize the QCD sum rules to finite density. The discussion is intended to build intuition about what we might except in the QCD analysis and to motivate our spectral ansatz. To this end, we consider the Dirac phenomenology of proton-nucleus scattering and some aspects of quantum hadrodynamics (QHD) [4]. QHD is a class of relativistic quantum field theories using baryons and mesons as Lagrangian degrees of freedom. We emphasize that our sum-rule approach does not assume that QHD is valid but, rather, tests some basic features of successful relativistic phenomenology against QCD predictions.

The most acclaimed success of relativistic nuclear physics in the past decade has been the economical description of proton-nucleus spin observables over a wide range of energies and target nuclei. These analyses have been performed using a purely phenomenological global parametrization of the scattering [3] and using a meson-theoretical framework [2]. The essential ingredient in both approaches is that nucleon propagation in the nuclear medium is described by a Dirac equation featuring large Lorentz scalar and vector optical potentials. In the simplest case, the nucleon wave function ψ satisfies

$$(E\gamma^0 - \boldsymbol{\gamma}\cdot\mathbf{q} - M_N - U)\psi = 0 , \tag{2.1}$$

where $U \approx S + V\gamma^0$ are the complex optical potentials, M_N is the nucleon mass, and E is the nucleon energy.

Although there are differences in detail, the qualitative characteristics of phenomenological optical potentials for intermediate-energy protons and those obtained from meson-exchange models are similar [1,2,20]: (i) Attractive scalar ($\mathrm{Re}\, S < 0$) and repulsive vector ($\mathrm{Re}\, V > 0$) potentials are the dominant features, with magnitudes increasing with density, reaching several hundred MeV at nuclear matter saturation density. (ii) There is significant cancellation between the potentials, so that the effective nonrelativistic central potential is only tens of MeV in magnitude. (iii) The imaginary parts also exhibit significant cancellation, and are each smaller than the real parts. (iv) The real parts of the potentials have relatively weak energy dependence.

These characteristics naturally suggest that a nucleon above the Fermi sea can be regarded as a quasiparticle with large scalar and vector self-energies (corresponding to the optical potentials). The imaginary parts of the optical potential indicate that the width in energy of the quasinucleon excitation is relatively small on hadronic scales, e.g., small compared to the spacing between the free-space nucleon and the Roper resonance. As discussed below, a similar picture emerges from the phenomenology for nucleons below the Fermi sea [21].

Why should we expect such a picture of nucleon propagation in the medium? The empirical low-energy *NN* scattering amplitudes are conventionally parametrized using Galilean invariants. However, if the amplitudes are decomposed using Lorentz invariants, one finds Lorentz scalar and vector components that are much larger than the amplitudes deduced from the nonrelativistic decomposition. In spin-saturated nuclear matter, other Lorentz components of the *NN* interaction average essentially to zero; these include terms arising from one-pion exchange. So the dynamics of neutral scalar and vector components are the most important for describing nucleons in bulk nuclear matter [4].

Relativistic hadronic field theories of nuclear phenomena (QHD) provide a qualitative explanation of this physics. Large scalar and vector self-energies have their origin at the mean-field level as the interaction of a nucleon with all other nucleons in the Fermi sea via the exchange of isoscalar scalar and vector mesons. This simple picture has many phenomenological successes; relativistic mean-field models provide a quantitatively accurate description of many bulk properties of nuclei [4].

It is important to note that we will not try to use QCD sum rules to test this picture of meson exchange as the origin of the quasinucleon self-energies. Concerns about such issues as the use of the Dirac equation to describe composite nucleons (e.g., suppression of *Z*-graph physics) and the validity of a meson-exchange picture at short distances have dominated the past discussion of relativistic nuclear physics and its connection to QCD. *Such questions are not addressed in our sum-rule analysis.* Instead, we focus on the *spectral* properties themselves, i.e., the self-energies, which we can study outside the context of a hadronic model, but still compare to the predictions of Dirac phenomenology or QHD.

To put this more concretely, we consider the nucleon propagator in a QHD theory

$$G(q)=-i\int d^4x\, e^{iq\cdot x}\langle\Psi_0|T[\psi(x)\bar{\psi}(0)]|\Psi_0\rangle \ , \tag{2.2}$$

where $|\Psi_0\rangle$ is the nuclear matter ground state and $\psi(x)$ is a nucleon field [4]. The analytic structure of G in the mean-field and more sophisticated approximations can suggest what we should find for an analogous QCD correlator. The nucleon self-energy Σ can be defined from the formal solution of Dyson's equation for the inverse propagator:

$$[G(q)]^{-1}=\gamma_\mu q^\mu - M_N - \Sigma(q) \ . \tag{2.3}$$

This self-energy can be identified directly from the analytic properties of the propagator $G(q)$. In particular, the discontinuities of G across the real q_0 axis will be used to extract the on-shell self-energy. We illustrate this point explicitly, since this is how it will be identified from the correlator Π in the QCD sum-rule analysis.

We start with a general decomposition of $G(q)$ as

$$G(q)=G_s(q^2,q\cdot u)+G_q(q^2,q\cdot u)\not{q}+G_u(q^2,q\cdot u)\not{u} \ , \tag{2.4}$$

where u^μ is the four-velocity of the nuclear matter ground state. This form is determined by Lorentz covariance and the assumed invariance of the ground state under parity and time reversal (see Sec. IV). The self-energy can be decomposed similarly; this will define the notation used in subsequent sections. The nucleon self-energy is written as

$$\Sigma(q)=\tilde{\Sigma}_s(q^2,q\cdot u)+\tilde{\Sigma}_v^\mu(q)\gamma_\mu \ , \tag{2.5}$$

where

$$\tilde{\Sigma}_v^\mu(q)=\Sigma_u(q^2,q\cdot u)u^\mu+\Sigma_q(q^2,q\cdot u)q^\mu \ . \tag{2.6}$$

We also define

$$\Sigma_v \equiv \frac{\Sigma_u}{1-\Sigma_q}, \quad M_N^* \equiv \frac{M_N+\tilde{\Sigma}_s}{1-\Sigma_q} \ , \tag{2.7}$$

and an in-medium scalar self-energy,

$$\Sigma_s \equiv M_N^* - M_N \ . \tag{2.8}$$

The combinations in Eq. (2.7) appear naturally when one solves for the nucleon pole.[1] Strictly speaking, the "scalar self-energy" of the nucleon in the medium is M_N^*, which is therefore the scalar quantity we will calculate using the sum rules. However, we will follow nuclear physics convention and refer to Σ_s as the scalar self-energy in the medium.

In the mean-field approximation, Σ_s and Σ_v are real and independent of momentum, while Σ_q is identically zero. Thus nucleons of any three-momentum appear as stable quasiparticles with self-energies that depend on the

[1] The self-energy is often parametrized in QHD models as

$$\Sigma(q)=\Sigma^s(q)-\gamma_\mu\Sigma^\mu(q)$$
$$=\Sigma^s(q^0,|\mathbf{q}|)-\gamma^0\Sigma^0(q^0,|\mathbf{q}|)+\boldsymbol{\gamma}\cdot\mathbf{q}\Sigma^v(q^0,|\mathbf{q}|)$$

in the rest frame of infinite nuclear matter. This decomposition follows generally from the translational and rotational invariance of the ground state, and its assumed invariance under parity and time reversal [4]. These self-energies are related to our definitions in the rest frame by $\Sigma^s \to \tilde{\Sigma}_s$, $\Sigma^0 \to -\Sigma_u - q^0\Sigma_q$, and $\Sigma^v \to \Sigma_q$. The definitions of Σ_v and M_N^* with the factor $(1-\Sigma_q)^{-1}$ are analogous to the definitions of the $\tilde{S}$ and $\tilde{V}$ potentials, for example, in the IA2 analysis of Tjon and Wallace [20], which absorb the "space-vector potential" C into the usual scalar and vector potentials S and V.

density. For mean-field models that provide quantitative fits to bulk properties of finite nuclei, the self-energies are typically several hundred MeV in magnitude at nuclear matter saturation density: $\Sigma_s \sim -350$ MeV and $\Sigma_v \sim +300$ MeV. These are both essentially linear in the density up to nuclear matter density. In this approximation, the self-energies Σ_s and Σ_v also correspond to energy-independent optical potentials S and V. Note that the effective nucleon mass M_N^* defined in Eq. (2.7) is *not* equivalent to the usual nonrelativistic effective nucleon mass that is connected with energy-level spacing. It is, however, equivalent to the Walecka-model effective mass at the mean-field level. We refer the reader to Refs. [22,23] for further discussion of the relations between different effective masses.

Although mean-field models successfully describe a wide range of phenomena, their simplicity leads us to question whether the basic physics survives in a more sophisticated analysis. The most detailed relativistic calculations of nuclear matter have been performed in what is usually known as the relativistic Brueckner-Hartree-Fock (RBHF) approximation [4,21]. This approximation incorporates effects from short-range correlations, which are critical in the nonrelativistic description of nuclear matter saturation. While these calculations involve some untested assumptions, they provide a unified and quantitative description of NN scattering observables and nuclear matter saturation properties.

Relativistic Brueckner calculations generally find that the (on-shell) self-energies are only weakly dependent on the three-momentum $\mathbf{q}$. (This corresponds to a weak energy dependence for the real parts of the relativistic scalar and vector optical potentials seen by a scattered nucleon.) Here, "on-shell" means that the self-energies are evaluated at the q_0 corresponding to the pole position, which is found by solving a transcendental equation for the self-consistent single-particle energy [4]. The self-energies Σ_s and Σ_v are found to be similar in magnitude, sign, and density dependence to those from mean-field calculations. Furthermore, the magnitude of the dimensionless Σ_q, which is zero in the mean-field approximation, is typically much less than one in RBHF calculations (see, however, Ref. [24]). Thus the mean-field quasiparticle picture is qualitatively unchanged in the RBHF approximation. We use this picture to guide us in formulating our QCD sum-rule spectral ansatz.

In the mean-field approximation, the propagator with real self-energies in the rest frame of nuclear matter is

$$G(q)=\frac{1}{\not q-M_N-\Sigma(q)}\to\lambda^2\frac{\not q+M_N^*-\not u\Sigma_v}{(q_0-E_q)(q_0-\bar{E}_q)}\ , \qquad (2.9)$$

where E_q and $\bar{E}_q$ are defined in Eqs. (2.10) and (2.11) below. We have introduced a common residue factor λ^2, which is unity here and in more general approximations includes the factor $(1-\Sigma_q)^{-1}$. We can identify the functions G_q, G_s, and G_u directly from Eq. (2.9) using Eq. (2.4). A Lehmann representation obtained by inserting a complete set of intermediate states between the ψ and $\bar{\psi}$ in Eq. (2.2) shows that G_q, G_s, and G_u have the same singularity structure. In general, the entire real q_0 axis is cut, but in the mean-field approximation we have only two simple poles. There are no singularities elsewhere in the complex q_0 plane. The discontinuities of the G_i's across the real q_0 axis are proportional to the spectral functions.

The positive- and negative-energy poles in q_0 are at

$$E_q=\Sigma_v+\sqrt{\mathbf{q}^2+M_N^{*2}}\equiv\Sigma_v+E_q^*\ , \qquad (2.10)$$

$$\bar{E}_q=\Sigma_v-\sqrt{\mathbf{q}^2+M_N^{*2}}\equiv\Sigma_v-E_q^* \qquad (2.11)$$

in the mean-field approximation. The discontinuities of the propagator functions across the real q_0 axis, for real, fixed $|\mathbf{q}|$, are delta functions in this limit:

$$\Delta G_s(q_0)=-2\pi i\frac{M_N^*\lambda^2}{2E_q^*}[\delta(q_0-E_q)-\delta(q_0-\bar{E}_q)]\ , \qquad (2.12)$$

$$\Delta G_q(q_0)=-2\pi i\frac{\lambda^2}{2E_q^*}[\delta(q_0-E_q)-\delta(q_0-\bar{E}_q)]\ , \qquad (2.13)$$

$$\Delta G_u(q_0)=+2\pi i\frac{\Sigma_v\lambda^2}{2E_q^*}[\delta(q_0-E_q)-\delta(q_0-\bar{E}_q)]\ . \qquad (2.14)$$

These define the mean-field spectral densities up to a constant factor. As evident from Eq. (2.9), the *relative* residues of the $\not q$, scalar, and $\not u$ poles gives us the self-energies M_N^* and Σ_v. Thus, ignoring the overall residue λ^2, these are the two independent quantities to extract; these are the quantities we want to extract via our spectral ansatz for the QCD nucleon correlator. There is an important distinction: In the mean-field approximation, the on-shell self-energies are independent of $\mathbf{q}$ while the self-energies we extract will depend explicitly on $\mathbf{q}$ (although the dependence is apparently weak).

Evidently, to match the empirical fact that quasinucleon single-particle energies are mostly unchanged at nuclear matter density, we must find significant cancellation between Σ_s and Σ_v to keep E_q roughly constant. In contrast, a mean-field ansatz for the quasinucleon also predicts a significant shift of the negative-energy "pole" position $\bar{E}_q$ with increasing density. In reality, we expect a broad distribution of strength rather than a narrow excitation, so the simple ansatz is more realistic for the positive-energy quasinucleon.

One might imagine that the large self-energies predicted in relativistic models would have definite experimental signatures. It is found, however, that the *individual* self-energies are not manifest in nuclear observables. So signatures of a modified relativistic effective mass M_N^*, for example, are difficult to identify experimentally. As a result, we choose to adopt a more theoretical approach. We extract QCD predictions for the individual scalar and vector self-energies by considering the analytic structure of something like the nucleon propagator in QHD models. This leads us to the natural analog in QCD of the QHD propagator: a correlator of interpolating fields with nucleon quantum numbers.

In Sec. IV, we consider the QCD correlator at finite density, which can be decomposed as in Eq. (2.4). A Lehmann representation once again tells us that the discontinuities in q_0 across the real axis define the spectral densities, which determine the correlator everywhere in the complex q_0 plane. We use a quasiparticle model to represent the region of the cut corresponding to the energy of a nucleon in nuclear matter. That is, we take Eq. (2.9) as our ansatz for the quasinucleon contribution to the correlator (which also implies a negative-energy quasinucleon), allowing the self-energies to depend on the three-momentum **q**. We assume that the pole approximation provides a reasonable representation of the relative strength in the three spectral densities.

Lorentz covariance and the assumed invariance of the nuclear matter ground state under time reversal and parity constrain the form of the spectral functions for the nucleon propagator in QHD models. In particular, they imply that only Lorentz scalar and vector self-energies are associated with a quasiparticle pole. These same constraints also apply to the QCD correlator of nucleon interpolating fields. They *do not* involve further assumptions about hadronic degrees of freedom or other aspects of relativistic phenomenology. Therefore, the principal issue we address here is not whether there are scalar and vector self-energies that characterize the nucleonlike excitation in medium; this is given once we believe that a quasiparticle approximation is reasonable. The real question is: What are the magnitudes, signs, and density dependencies of the self-energies?

Direct experimental evidence and successful relativistic phenomenology imply that a nucleon propagating in the nuclear medium is well described as a relativistic quasiparticle with scalar and vector self-energies changing with increasing baryon density. This motivates the simple quasinucleon pole ansatz we adopt for the spectral densities of the QCD correlator. This ansatz defines self-energies that characterize the physical spectrum independent of QHD dynamics. Again, the existence of both scalar and vector pieces is not an assumption but follows from the fundamental symmetries and invariances of the nuclear system. Phenomenology suggests that these self-energies should be large at saturation density; if true, this should be *predicted* by the sum-rule analysis. Furthermore, we know from the weak binding of nuclear matter, the empirical nonrelativistic optical potentials (which are small), and information on single-particle energies from $(e,e'p)$ and $(p,2p)$ experiments that the quasiparticle energy of a nucleon in nuclear matter is only fractionally shifted from the free nucleon energy and that the excitation is only tens of MeV wide at most. This small shift will not be a built-in constraint, but should be predicted as well.

Although the true QCD correlator at finite density will not just have simple poles on the real axis, we expect the width of the positive-energy quasinucleon excitation to be small on hadronic scales (and compared to the energy over which we average), so we are justified in making a pole ansatz. However, since we will not explicitly include in our ansatz any background at these energies, the effective self-energies we extract will account for *all* of the strength in the nuclear domain. We have much less information about the spectral density on the negative-energy side, where the zero-density pole corresponding to an antinucleon becomes a broad distribution as the density of the nuclear system is increased. Therefore, to minimize our sensitivity to this part of the spectral density, we construct our sum rule to suppress this contribution relative to the positive-energy side

III. QCD SUM RULES FOR THE NUCLEON IN VACUUM

In this section, we briefly review some basic features of QCD sum-rule calculations at zero density, with particular emphasis on those elements that will be modified when we generalize to finite density. Sum-rule analyses of the nucleon mass in vacuum have been made by Ioffe [8] and many others [25,7]. We assume that these treatments are valid and build on their assumptions and conclusions. However, we will make note of potential problems with the sum rules and how they might affect the interpretation of our finite-density results.

We start once more with a correlator of the nucleon interpolating field $\eta(x)$ evaluated in the physical vacuum:

$$\Pi_{ij}(q) \equiv i\int d^4x\, e^{iq\cdot x}\langle 0|T[\eta_i(x)\bar{\eta}_j(0)]|0\rangle \qquad (3.1)$$

$$\equiv \Pi_s(q^2)\delta_{ij} + \Pi_q(q^2)\not{q}_{ij} \; . \qquad (3.2)$$

We have exhibited the Dirac indices i and j and introduced the two independent invariant functions of q^2. This decomposition is standard (see Ref. [26], for example) and relies on the transformation properties of $\eta(x)$ under Lorentz boosts and the discrete space-time symmetries, along with the Lorentz invariance of the vacuum. (We neglect violations of parity due to the weak interaction, which would introduce additional functions.) We generalize this result in the next section to the case of a finite-density ground state.

We follow the arguments of Ioffe and use an interpolating field $\eta(x)$ for a proton constructed from up and down quark fields as [8,27]

$$\eta(x) = \epsilon_{abc}[u^{aT}(x)C\gamma_\mu u^b(x)]\gamma_5\gamma^\mu d^c(x) \qquad (3.3)$$

$$= 4\epsilon_{abc}[(u_R^{aT}Cd_R^b)u_L^c - (u_L^{aT}Cd_L^b)u_R^c] \; , \qquad (3.4)$$

where a, b, and c are color indices (running from 1 to 3), T means transpose, and C is the usual charge-conjugation matrix [26]. In the second line, we have written $\eta(x)$ in terms of left- and right-handed quark fields, suppressing the x dependence. The analogous interpolating field for a neutron follows by interchanging the up and down quark fields.

This choice of interpolating field for the nucleon is motivated by the goal of maximizing the coupling to the nucleon intermediate state relative to other (continuum) states while minimizing the contributions of higher-order corrections from the operator product expansion [8,27]. Furthermore, we want the two invariant functions Π_s and Π_q to be more-or-less equally dominated by the nucleon contribution. For simplicity, we first restrict ourselves to interpolating fields that contain no derivatives and couple to spin-$\frac{1}{2}$ only. Such a composite field can be represented as a linear combination of fields corresponding to a scalar or pseudoscalar diquark coupled to an up quark:

$$\eta_t = 2\epsilon_{abc}[(u^{aT}Cd^b)\gamma^5 u^c + t(u^{aT}C\gamma^5 d^b)u^c] , \tag{3.5}$$

where t is an arbitrary parameter. To avoid direct instanton effects, we restrict the possibilities to $t=\pm 1$ [28]; Eq. (3.4) corresponds to $t=-1$ after some rearrangement. This choice is favored because the $t=+1$ field does not get a low-order chiral-symmetry breaking contribution in the operator product expansion (from $\langle \bar{q}q\rangle_{\rm vac}$ in particular). Therefore, the $t=+1$ field should either couple weakly to the nucleon resonance or receive compensating contributions from negative-parity states. In a future work [19], we will consider the general linear combination in Eq. (3.5).

The analytic properties of $\Pi(q)$ can be studied through a Lehmann representation, which reveals that all singularities in q^2 lie on the positive real axis. Writing a Cauchy relation for the contour of Fig. 1 with $s_0 \to \infty$, we obtain dispersion relations in q^2 for each of the Π_i's ($i=\{s,q\}$) of the form [26,29]

$$\Pi_i(q^2) = \frac{1}{2\pi i}\int_0^\infty ds \frac{\Delta\Pi_i(s)}{s-q^2} + \text{polynomial} , \tag{3.6}$$

where $\Delta\Pi_i(q^2)$ is the discontinuity across the positive real q^2 axis. The discontinuity is defined by

$$\Delta f(x) \equiv \lim_{\eta\to 0^+} [f(x+i\eta) - f(x-i\eta)] . \tag{3.7}$$

In this case, $\Delta\Pi_i(q^2) = 2i\,\mathrm{Im}\Pi_i(q^2+i0^+)$ contains the spectral information on the nucleon and higher-mass states.

The dispersion relations allow us to relate information contained in the spectral functions about physical intermediate states to the correlator evaluated elsewhere in the complex q^2 plane. Of particular interest is the deep spacelike region, for which $Q^2 \equiv -q^2$ is large and positive, where we can calculate the correlator using an operator product expansion (OPE). The OPE correlator takes the general form [6]

$$\Pi_i(Q^2) = \sum_n C_n^i(Q^2)\langle \hat{O}_n\rangle_{\rm vac} , \tag{3.8}$$

where the $C_n^i(Q^2)$ are c-number functions, calculated in QCD perturbation theory, and the $\langle \hat{O}_n\rangle_{\rm vac} \equiv \langle 0|\hat{O}_n|0\rangle$ are vacuum expectation values of QCD operators—the condensates. We have suppressed the dependence on the normalization scale μ. For sufficiently large Q^2, the correlator is reliably calculated with the OPE truncated at a small number of lower-dimensional operators *and* the Wilson coefficients evaluated in QCD perturbation theory.[2]

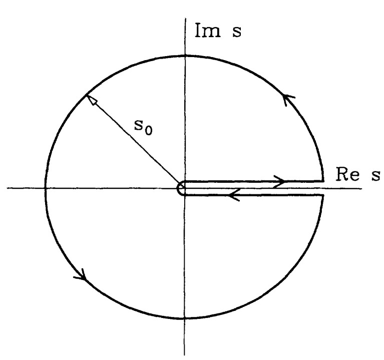

FIG. 1. Contour in the complex s plane used in the derivation of Eq. (3.6) and for the sum-rule integral in Eq. (3.28).

In principle, by equating Eqs. (3.6) and (3.8) for $i=\{s,q\}$ we have two sum rules. In practice, they are not useful. There are unknown polynomials, and the nucleon contribution does not dominate the dispersion integral for values of Q^2 large enough that the truncated OPE is both tractable and accurate. The standard procedure for improving the situation is to make a Borel transform of each representation of the correlator and then to equate the transformed equations. We review this standard approach first and then present an alternative derivation of the same Borel sum rules that is physically more transparent and more readily generalized.

The Borel transform can be applied in practice using the operator $\mathcal{B}$ defined by [6]

$$\mathcal{B}[f(Q^2)] \equiv \lim_{\substack{Q^2, n\to\infty \\ Q^2/n = M^2}} \frac{(Q^2)^{n+1}}{n!}\left[-\frac{d}{dQ^2}\right]^n f(Q^2) \equiv \hat{f}(M^2), \tag{3.9}$$

which depends on the "Borel mass" M. One finds that

$$\mathcal{B}[(Q^2)^k] = 0 \quad \text{for } k \ge 0 , \tag{3.10}$$

$$\mathcal{B}\left[\frac{1}{(Q^2)^k}\right] = \frac{1}{(k-1)!}\left[\frac{1}{M^2}\right]^{k-1} \quad \text{for } k>0 , \tag{3.11}$$

$$\mathcal{B}[(Q^2)^k \ln(Q^2/\Lambda^2)] = k!(-M^2)^{k+1} \quad \text{for } k \ge 0 . \tag{3.12}$$

Finally, one finds

$$\mathcal{B}\left[\frac{1}{s+Q^2}\right] = e^{-s/M^2} . \tag{3.13}$$

From Eq. (3.10), any simple polynomial in Q^2 is eliminated by the Borel transform. This has two useful (and related) consequences: The subtraction terms accompanying the dispersion relation and any divergent (or renormalized) polynomials from the OPE are simultaneously eliminated. It is also evident that the higher-order terms in the OPE, which contain inverse powers of Q^2, are factorally suppressed by the Borel transform. The transform of the dispersion integral [Eq. (3.6)] is

$$\mathcal{B}[\Pi_i(q^2)] = \frac{1}{2\pi i}\int_0^\infty ds\, e^{-s/M^2}\Delta\Pi_i(s) . \tag{3.14}$$

For M near the mass of the nucleon, higher-mass contributions to the integral are exponentially suppressed.

[2]While in principle this is an overly simplistic approach to the operator product expansion, it has compelling phenomenological justification in QCD. In general, one has to be more precise about the definition and separation of perturbative and nonperturbative contributions to the Wilson coefficients and the condensates. See Refs. [30] and [31] for further discussion.

The correlator in spectral form can be evaluated by introducing a phenomenological model for the discontinuity. The lowest-mass contribution to the spectral function is from the nucleon pole. It can be found from the discontinuity in q^2:

$$\{\Delta\Pi(q)\}_{\text{nucleon}}=\Delta\left\{\frac{-\lambda_N^2}{\not q-M_N}\right\}$$

$$=\Delta\left\{\frac{-\lambda_N^2(\not q+M_N)}{q^2-M_N^2}\right\} \quad (3.15)$$

$$=2\pi i\lambda_N^2(\not q+M_N)\delta(q^2-M_N^2)\ , \quad (3.16)$$

where λ_N^2 is the residue. This is equivalent to constructing the spectral density from the matrix element

$$\langle 0|\eta(0)|q\rangle=\lambda_N u(q) \quad (3.17)$$

and from the related equation for the antinucleon [8]. Here $|q\rangle$ is a one-nucleon state with four-momentum q^μ ($q^2=M_N^2$) and $u(q)$ is a Dirac spinor for the nucleon. One can immediately read off the nucleon-pole contributions to the spectral functions for Π_q and Π_s. We note from Eq. (3.16) that their *ratio* is the nucleon mass M_N. Contributions to the spectral functions from higher-mass states are roughly approximated using the leading terms in the OPE, starting at a threshold s_0:

$$\Delta\Pi_i(s)=\{\Delta\Pi_i(s)\}_{\text{nucleon}}+\theta(s-s_0)\{\Delta\Pi_i(s)\}_{\text{OPE}}\ . \quad (3.18)$$

Equating the Borel transforms of the OPE and phenomenological descriptions yields two relations—one for each invariant function. For reference, we present the complete sum rule of Ioffe and Smilga (the reader is directed to Ref. [25] for more details):

$$\tilde{\lambda}_N^2 M_N e^{-M_N^2/M^2}=2aM^4-\frac{1}{12}ab+\frac{272}{81}\frac{\alpha_s}{\pi}\frac{a^3}{M^2}\ , \quad (3.19)$$

$$\tilde{\lambda}_N^2 e^{-M_N^2/M^2}=M^6L^{-4/9}+\frac{1}{4}bM^2L^{-4/9}+\frac{4}{3}a^2L^{4/9}-\frac{1}{3}a^2\frac{m_0^2}{M^2}\ . \quad (3.20)$$

An explicit expression for the nucleon mass is obtained by dividing Eq. (3.19) by Eq. (3.20). Here we have defined the vacuum matrix elements

$$a\equiv-(2\pi)^2\langle\bar{q}q\rangle_{\text{vac}}\ , \quad (3.21)$$

$$b\equiv(2\pi)^2\left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}\right\rangle_{\text{vac}} \quad (3.22)$$

at normalization scale μ, and defined

$$\tilde{\lambda}_N^2\equiv 32\pi^4\lambda_N^2\ . \quad (3.23)$$

α_s is the strong coupling constant evaluated at the Borel mass. The parameter m_0^2 is introduced to parametrize a mixed quark-gluon condensate [25]. Higher-dimensional condensates are approximated by inserting intermediate states and assuming that the vacuum state dominates. This factorization assumption expresses all condensates in terms of a, b, and m_0^2. For example, four-quark condensates such as $\langle\bar{q}\Gamma q\bar{q}\Gamma q\rangle_{\text{vac}}$ become proportional to a^2. This is an important assumption; we discuss it further in Sec. IV.

Perturbative corrections $\sim\alpha_s^n$ are taken into account in the leading logarithmic approximation through anomalous-dimension factors [6]. After the Borel transform, the effect of these corrections is through the factor L raised to an appropriate power, where

$$L\equiv\frac{\ln(M/\Lambda_{\text{QCD}})}{\ln(\mu/\Lambda_{\text{QCD}})}\ . \quad (3.24)$$

While these corrections can change the absolute predictions of the nucleon mass by 50–100 MeV, they have a relatively small effect on our results because we take ratios of finite-density self-energies to the zero-density mass. For simplicity, we do not consider these factors explicitly in the present treatment.

The dependence on the higher-mass states in Eq. (3.18) is transferred to the OPE side of each sum-rule equation [Eqs. (3.19) and (3.20)] using factors that depend on s_0, the continuum threshold [6]. These factors multiply the leading terms of the OPE for each sum rule. Terms with positive powers of M^2 are modified as follows:

$$M^2\to M^2(1-e^{-s_0/M^2})\ , \quad (3.25)$$

$$M^4\to M^4\left[1-e^{-s_0/M^2}\left[\frac{s_0}{M^2}+1\right]\right]\ , \quad (3.26)$$

$$M^6\to M^6\left[1-e^{-s_0/M^2}\left[\frac{s_0^2}{2M^4}+\frac{s_0}{M^2}+1\right]\right]\ . \quad (3.27)$$

All other terms are unaffected. In principle, the effective thresholds for the two rules could be different because negative-parity resonances enter with different signs [25]. We do not consider this possibility in the present work (see Ref. [19]).

An alternative path leading to these Borel sum rules starts with the contour integral [9,10]

$$\oint ds\ W(s)\Pi_i(s)=0\ , \quad (3.28)$$

where Π_i is one of the invariant functions, $W(s)$ is some analytic weighting function, and the contour is shown in Fig. 1. We also require $W(s^*)=[W(s)]^*$. Since Π_i is analytic except on the positive real axis, the total integral is zero. Then, by considering different pieces of the contour, we relate an integral along the positive real axis, whose integrand is the weighting function times the discontinuity, to an integral on a circle with $s=|s_0|$. The latter can be approximated by using the OPE for $\Pi_i(s=|s_0|)$ [32].

The equivalence of these integrals defines an infinite class of sum rules:

$$\int_{|s|=s_0}ds\ W(s)\Pi_i(s)=-\int_0^{s_0}ds\ W(s)\Delta\Pi_i(s)\ . \quad (3.29)$$

We stress that we can choose any analytic function for W. For example, if we take $W(s)=s^n$ ($n=0,1,2,\ldots$), we have the finite-energy sum rules (FESR) [32–34]. Nevertheless, some choices are more useful than others. Most useful for our case is the choice $W(s)=e^{-s/M^2}$, which is equivalent to the Borel-transformed sum rule, with all of its advantages. The same contribution from the usual continuum ansatz with threshold s_0 [Eq. (3.18)] follows by choosing the integration circle to have radius s_0. This shows that the Borel transform is not a miraculous trick but merely corresponds to an opportune choice of contour and weighting function.

There are various approaches one can take to analyze the sum rules. We apply a ratio method and an optimization method in Sec. IV. For example, the ratio of Eq. (3.19) to Eq. (3.20) provides an estimate of M_N, which should be weakly dependent on the Borel mass M. The most important constraint on the analysis is that we work in a region in M^2 for which both the nucleon contribution dominates and the operator product expansion is under control. These are opposing tendencies; the former condition argues for smaller M^2, while the latter requires higher M^2. In the vacuum case, Ioffe (and others) have concluded that the contributions of higher-dimensional condensates and the continuum are, in fact, sufficiently small for values of the Borel mass in the vicinity of M_N that meaningful predictions can be made. We take the results of Ref. [25] to be representative. The inputs are $a=0.55\ \text{GeV}^3$, $b=0.47\ \text{GeV}^4$, $m_0^2=0.8\ \text{GeV}^2$, $\Lambda_{\text{QCD}}=150$ MeV, and $\mu=0.5$ GeV. The fiducial interval is $0.8\ \text{GeV}^2 \le M^2 \le 1.4\ \text{GeV}^2$, and the predictions are $M_N=1.02\pm0.12$ GeV, $\tilde{\lambda}_N^2 \equiv 32\pi^4\lambda_N^2=3.6\pm1.2\ \text{GeV}^6$, and $s_0=2.3\ \text{GeV}^2$.

We have assumed that the current quark masses for the up and down quarks can be neglected. Then if chiral symmetry were not spontaneously broken by the vacuum, the quark condensate $\langle\bar{q}q\rangle_{\text{vac}}$ would be zero and the function Π_s would be identically zero. In the real world, where chiral symmetry is spontaneously broken, we expect a close correlation between $\langle\bar{q}q\rangle_{\text{vac}}$ and M_N, which characterizes the low-lying strength in Π_s. Such a relationship is also suggested in many QCD-inspired models, such as the Nanbu–Jona-Lasinio and Skyrme models [35].

In fact, the principal physical content of the full sum rule for the nucleon is that the scale of the nucleon mass is largely determined by the quark condensate. This is manifest in a simplified sum rule in which only the leading contributions from the OPE to each sum rule are kept and the continuum and anomalous dimensions are neglected [7]. In particular, one can divide Eq. (3.19) by Eq. (3.20) after these simplifications to obtain an expression for the nucleon mass [7],

$$M_N=-\frac{8\pi^2}{M^2}\langle\bar{q}q\rangle_{\text{vac}}\ , \tag{3.30}$$

where the formula is to be evaluated for $M^2\sim1\ \text{GeV}^2$. We generalize this formula to finite density in Sec. V. [Note that in this simple Ioffe formula, M_N is extremely sensitive to the Borel mass. One can see from the more sophisticated sum rule that effects of corrections to Eq. (3.30) tend to cancel for $M^2\sim1\ \text{GeV}^2$ [8,7].] The only quantitatively important physics missing from Eq. (3.30) is the contribution of the four-quark condensate [the a^2 term in Eq. (3.20)], which plays an important role in the full sum rule at finite density.

Before generalizing these results to finite density, we review some limitations and deficiencies of the vacuum sum rules for the nucleon. The sum rules for the nucleon are less than ideal for a number of reasons. First, we expect that the ratio sum-rule prediction for the nucleon mass, if well satisfied, should have a flat region as a function of the Borel mass, which is just an auxiliary parameter. In fact, the nucleon sum rule is not very flat (see Fig. 3), particularly if we truncate the OPE at dimension-six condensates (as we will do at finite density). The sum rule of Ioffe and Smilga [25], which includes various higher-dimensional condensates, becomes flatter, but the estimation of these condensates presumably is less reliable. The most favorable interpretation is that the change in the ratio within the fiducial region is a measure of the theoretical error bar for the mass. Second, perturbative corrections to many of the Wilson coefficients are large, although the net effect on the sum-rule predictions seems to be small[3] [36]. Third, the continuum contribution is unavoidably large because the gap between the nucleon and higher-mass states is only of order 500 MeV. This is generally undesirable, because we only use a crude model to account for the continuum states; thus we might expect a large uncertainty. Finally, there have been suggestions that the physics of the nucleon mass is really dominated by direct instanton contributions, which are not included in the conventional OPE [37].

Thus, despite the apparent phenomenological success of the vacuum sum rules for the nucleon (and other baryons), there are a variety of ways in which the sum rules could be less precise than often advertised or in which they could even fail. While this observation might make one hesitate before including the further complications of finite density, we think that the situation is actually quite favorable. Indeed, since we focus on *changes* in spectral properties with density, we are less sensitive to details that affect the *absolute* predictions of vacuum properties. Moreover, even predictions with large uncertainties (e.g., 50%) will be useful in assessing relativistic phenomenology. So we proceed to generalize the sum-rule formalism, assuming that the zero-density limit is valid.

IV. FINITE-DENSITY SUM RULES FOR THE NUCLEON

In this section, we generalize the vacuum QCD sum rules for the nucleon to finite density. The starting point is the same correlator considered in Sec. III, but with the

[3]We do not expect that a small coupling constant implies that the perturbative series for the coefficients *converges*. On the contrary, we expect it to be asymptotic. We assume that we can achieve reasonably accurate results by keeping a small number of terms—in this case, one or two.

finite-density ground state replacing the vacuum [see Eq. (1.3)]. The basic strategy is the same as in the vacuum: Use the analytic properties of the correlator to relate its value in a spacelike region, where it can be reliably calculated using an operator product expansion, to the spectral functions, which contain the information of interest about the physical states. Then use a Borel transform with a specified range of Borel masses to emphasize the quasinucleon contribution while improving the convergence of the truncated OPE.

A. Lehmann representation for the finite-density correlator

Consider the correlation function $\Pi(q)$ defined by

$$\Pi_{ij}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle \Psi_0 | T[\eta_i(x)\bar{\eta}_j(0)] | \Psi_0 \rangle \ , \quad (4.1)$$

where $|\Psi_0\rangle$ is the ground state of the infinite nuclear medium. The interpolating field $\eta(x)$ is given in Eq. (3.3), and we exhibit the Dirac indices i and j. The ground state is characterized by ρ_N, the nucleon density in the rest frame, and u^μ, the four-velocity of the nuclear medium. (Strictly speaking, we work at fixed volume and baryon number until the end, when we take the thermodynamic limit.) We also assume that the ground state has definite parity.

It is often remarked in the literature that one cannot work covariantly at finite density or temperature because of the existence of a preferred frame of reference, i.e., the rest frame of nuclear matter. This is a misconception. While the ground state is not *invariant* under all Lorentz transformations (unlike the vacuum state), matrix elements in this state do have well-defined Lorentz transformation properties. So two observers in different frames can still compare calculations or observations as prescribed by special relativity. The new feature is an additional four-vector u^μ that must be transformed when making the comparisons and must be included when building tensors or identifying invariant functions. (The situation here is analogous to considering diagonal matrix elements of a spin-averaged proton state, which is also characterized by a single four-vector, the four-momentum of the proton.)

The correlation function is a 4×4 matrix in Dirac space, so we can expand it in the usual complete set of Dirac matrices. Using the transformation properties of $\eta(x)$ and keeping in mind the role of u^μ, we can constrain the form of $\Pi(q)$. The arguments are analogous to those in Chapter 16 of Ref. [26]. Lorentz covariance dictates that the general form of the correlator is

$$\Pi(q) = \Pi_s + \Pi_q \not{q} + \Pi_u \not{u} + \Pi_1 \gamma^5 + \Pi_2 \not{q}\gamma^5 + \Pi_3 \not{u}\gamma^5 + \Pi_4 (q_\mu u_\nu - q_\nu u_\mu)\sigma^{\mu\nu}, \quad (4.2)$$

where the Π_i's are scalar functions of the invariants q^2 and $q\cdot u$. We assume that the nuclear matter ground state is invariant under parity and time reversal in its rest frame. In a general frame, we must take $u^\mu \to u_\mu$ as well as $q^\mu \to q_\mu$ under these transformations; thus q^2 and $q\cdot u$ are unchanged. The parity constraint means that $\Pi_1 = \Pi_2 = \Pi_3 = 0$. In the vacuum, a term proportional to $\sigma^{\mu\nu}$ can be excluded, because it can only be contracted with the symmetric combination $q_\mu q_\nu$. In finite-density nuclear matter, this argument is no longer sufficient, but the assumed parity and time-reversal invariance implies that $\Pi_4 = 0$.

Thus Lorentz covariance, parity, and time-reversal imply that $\Pi(q)$ has the form announced in Eq. (1.4), namely,

$$\Pi(q) \equiv \Pi_s(q^2, q\cdot u) + \Pi_q(q^2, q\cdot u)\not{q} + \Pi_u(q^2, q\cdot u)\not{u} \ . \quad (4.3)$$

There are *three* distinct structures—scalar, $\not{q}$, and $\not{u}$—and thus three invariant functions of the two scalars q^2 and $q\cdot u$ or any convenient combination. Recall that in the vacuum there are only two structures: scalar and $\not{q}$. In the zero-density limit, $\Pi_u \to 0$ and Π_s and Π_q become functions of q^2 alone. For simplicity, we specialize the sum rules to the rest frame of the nuclear medium, where the variables q_0 and $\mathbf{q}^2$ are most useful. A covariant form can be recovered in general by repeating the analysis with $q_0 \to q\cdot u$ and $-\mathbf{q}^2 \to \tilde{q}^2 \equiv q^2 - (q\cdot u)^2$.

We can project out the individual invariant functions by taking traces:

$$\Pi_s = \frac{1}{4}\mathrm{Tr}(\Pi) \ , \quad (4.4)$$

$$\Pi_q = \frac{1}{q^2 - (q\cdot u)^2}\left[\frac{1}{4}\mathrm{Tr}(\not{q}\Pi) - \frac{q\cdot u}{4}\mathrm{Tr}(\not{u}\Pi)\right] \ , \quad (4.5)$$

$$\Pi_u = \frac{1}{q^2 - (q\cdot u)^2}\left[\frac{q^2}{4}\mathrm{Tr}(\not{u}\Pi) - \frac{q\cdot u}{4}\mathrm{Tr}(\not{q}\Pi)\right] \ . \quad (4.6)$$

These projections require that $q^2 - (q\cdot u)^2$ be nonzero; this means $\mathbf{q} \neq 0$ in the rest frame. If this is not the case, there are only two functions, and the second is projected by a trace with γ^0.

In the vacuum, where the invariant functions depend only on q^2, the separation of q_0 and $|\mathbf{q}|$ dependence is not necessary (or particularly useful), and we can derive a dispersion relation directly in q^2. (However, recall that in principle the dispersion relation derivation starts with an integral over q_0.) In contrast, the four-velocity of the medium u^μ makes the distinction important in our case. For instance, the magnitude of the three-momentum $|\mathbf{q}|$ labels *distinct* quasiparticle states with different self-energies.

In Ref. [17], the authors consider finite-density dispersion relations in q^2, with $s = (p+q)^2$ fixed. Here p^μ is the four-momentum of a nucleon in the medium. In this approach, the spacelike limit $q^2 \to -\infty$ is similar to the Bjorken limit of deep inelastic electron scattering. We take a different approach and consider dispersion relations in q_0, with the three-momentum $|\mathbf{q}|$ held fixed. This provides a clean identification of the intermediate quasinucleon states, which are naturally labeled by $|\mathbf{q}|$. The analytic structure of the correlator and the spectral content are revealed through a standard many-body Lehmann representation. The contribution from negative-energy quasinucleons (antinucleons) is clearly separated, which lets us isolate to a large degree the positive-energy quasinucleon contribution. Finally, the relevant spacelike (Euclidean) limit takes $q_0 \to i\infty$.

The correlator [Eq. (4.1)] is defined with Heisenberg-picture operators; in the zero-density limit, it becomes the usual vacuum correlator. The four-momentum dependence of $\eta(x)$ can be explicitly extracted by inserting a complete set of four-momentum eigenstates and using

$$\eta(x)=e^{i\hat{P}\cdot x}\eta(0)e^{-i\hat{P}\cdot x} . \tag{4.7}$$

$\hat{P}^{\mu}$ is the full four-momentum operator. (We work at large but finite volume and take the thermodynamic limit at the end.) The integral over the spatial coordinates in Eq. (4.1) ensures that the intermediate states have three-momentum $\mathbf{q}$. (The ground state has zero three-momentum in the rest frame.) Performing the time integration, we find that the correlator is analytic except for cuts on the entire real q_0 axis [11]. For q_0 off the real axis, one has

$$\Pi_i(q_0^*,|\mathbf{q}|)=[\Pi_i(q_0,|\mathbf{q}|)]^* , \tag{4.8}$$

which relates the function in the upper and lower half planes.

Using these results, a Cauchy relation for the contour of Fig. 2 with $\omega_0 \to \infty$ leads to dispersion relations of the form

$$\Pi_i(q_0,|\mathbf{q}|)=\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi_i(\omega,|\mathbf{q}|)}{\omega-q_0}+\text{polynomial} \tag{4.9}$$

for each invariant function Π_i, $i=\{s,q,u\}$. We will only be concerned with q_0 off the real axis, so keeping track of infinitesimals is not necessary. The discontinuity is defined by

$$\Delta\Pi_i(\omega,|\mathbf{q}|)\equiv \lim_{\eta\to 0^+}[\Pi_i(\omega+i\eta,|\mathbf{q}|)-\Pi_i(\omega-i\eta,|\mathbf{q}|)] , \tag{4.10}$$

with ω real; each of the $\Delta\Pi_i$'s is purely imaginary. In general, we have to worry about the convergence at large ω, which entails working with subtracted dispersion relations. However, this will not be an issue because we eventually introduce a weighting function that ensures convergence. This is equivalent to taking the Borel transform of the dispersion relation, which, as noted in the zero-density case, eliminates the polynomials and the need to consider subtractions.

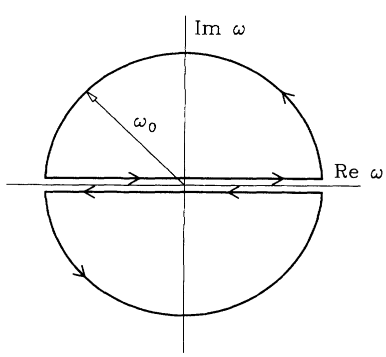

FIG. 2. Contour in the complex ω plane (at fixed $\mathbf{q}$) used in the derivation of Eq. (4.9) and for the sum-rule integral in Eq. (4.22).

There are two types of matrix elements contributing to each $\Delta\Pi_i$, corresponding to the two time orderings in the correlator [11]. In one case, the intermediate states have baryon number $B=+1$ relative to $|\Psi_0\rangle$ and include quasiparticle states where a quasinucleon is added to the nuclear medium. The corresponding singularities run to the right of the chemical potential (with increasing excitation energy of the state) in the ω plane. In the other case, the intermediate states have baryon number $B=-1$ relative to $|\Psi_0\rangle$ and include quasihole states where a quasinucleon is taken away from the medium. These singularities run to the left of the chemical potential (with increasing energy). The quasiparticle states corresponding to an antinucleon added to the nuclear medium are also in the latter category and are therefore naturally viewed as high-energy excitations of the $B=-1$ system.

At zero density, the spectral weights for nucleon and antinucleon (or, more generally, the $\Delta\Pi_i$'s for $\pm\omega$) are related by the discrete space-time symmetries, because the vacuum is invariant under these operations [26]. In contrast, the finite-density ground state is not invariant under charge conjugation, so the spectral densities for nucleon and antinucleon quasiparticles are not simply related. We note that assuming a sharp quasiparticle pole implies narrow structures for *both* the positive- and negative-energy nucleons. Actually, while we expect a narrow (on hadronic scales) positive-energy quasinucleon, its negative-energy "counterpart" should be very broad. We will use our choice of weighting function to minimize the sensitivity to the negative-energy contribution.

We generalize the usual zero-density ansatz for the spectral functions by assuming a quasiparticle pole for the nucleon, with real self-energies (dependent on $\mathbf{q}$); all higher-energy excitations are included in a continuum contribution. We use the notation of Sec. II. As in Eq. (2.9), Lorentz covariance implies that the quasiparticle-pole contribution to the correlator is

$$\Pi(q)\propto \frac{1}{(q^{\mu}-\tilde{\Sigma}_v^{\mu})\gamma_{\mu}-(M_N+\tilde{\Sigma}_s)} , \tag{4.11}$$

where $\tilde{\Sigma}_v^{\mu}$ and $\tilde{\Sigma}_s$ are the in-medium self-energies. In the language of the hadronic theories discussed in Sec. II, these are the on-shell self-energies for a quasinucleon with three-momentum $\mathbf{q}$. The representations of the individual invariant functions are (in the nuclear matter rest frame)

$$\Pi_s(q_0,|\mathbf{q}|)=-\lambda_N^{*2}\frac{M_N^*}{(q_0-E_q)(q_0-\bar{E}_q)}+\cdots , \tag{4.12}$$

$$\Pi_q(q_0,|\mathbf{q}|)=-\lambda_N^{*2}\frac{1}{(q_0-E_q)(q_0-\bar{E}_q)}+\cdots , \tag{4.13}$$

$$\Pi_u(q_0,|\mathbf{q}|)=\lambda_N^{*2}\frac{\Sigma_v}{(q_0-E_q)(q_0-\bar{E}_q)}+\cdots , \tag{4.14}$$

where we have defined M_N^*, Σ_v, E_q, and $\bar{E}_q$ as in Eqs. (2.7)–(2.11) and introduced an overall residue λ_N^{*2}. The positive- and negative-energy quasinucleon poles are explicit, and the ellipses denote the contribution from higher-energy states, which will be included later.

B. Operator product expansion at finite density

Wilson coefficients for the OPE at finite density can be calculated using the standard background-field techniques [7] applied in vacuum calculations. We present only the basic features of this approach here and refer the reader to the literature and to Ref. [18]. The most important new feature at finite density is that we define the composite QCD operators so that all density dependence appears in the matrix elements; the Wilson coefficients are then independent of density. Then the only substantial difference from the vacuum calculations is that more operator products survive when we take matrix elements in the nuclear matter ground state.

To calculate the coefficients of interest for the present work, we need only an expansion of the coordinate-space quark propagator in the presence of the nonperturbative medium [5,18]:

$$\langle T[q_i^a(x)\bar{q}_j^b(0)]\rangle_{\rho_N} = \frac{i}{2\pi^2}\delta^{ab}\frac{x_\mu}{(x^2)^2}\gamma_{ij}^\mu + \langle q_i^a(x)\bar{q}_j^b(0)\rangle_{\rho_N} + \cdots , \quad (4.15)$$

where q is either an up or down quark field, i and j are Dirac indices, and we have not explicitly written gluonic and other higher-order contributions. We have introduced the notation $\langle\hat{O}\rangle_{\rho_N} \equiv \langle\Psi_0|\hat{O}|\Psi_0\rangle$ to denote the matrix element of an operator $\hat{O}$ taken in the ground state of the nuclear medium with density ρ_N. The current quark masses have been neglected, since their contributions are numerically small for momentum transfers of interest. This expansion is to be used in conjunction with Wick's theorem to evaluate the Wilson coefficients.

The last term in Eq. (4.15) can be decomposed into a piece proportional to δ_{ij} and a piece proportional to γ_{ij}^μ. Working in fixed-point gauge ($x^\mu A_\mu^a = 0$) [7], each piece is expanded in a covariant Taylor series, introducing a set of local condensates with increasing mass dimension (normal ordering with respect to the perturbative vacuum is implied):

$$\begin{aligned}\langle q_i^a(x)\bar{q}_j^b(0)\rangle_{\rho_N} &= -\frac{1}{4N_c}\delta^{ab}[\langle\bar{q}(0)q(x)\rangle_{\rho_N}\delta_{ij} + \langle\bar{q}(0)\gamma_u q(x)\rangle_{\rho_N}\gamma_{ij}^\mu] \\ &= -\frac{1}{4N_c}\delta^{ab}\{[\langle\bar{q}q\rangle_{\rho_N} + \langle\bar{q}D_\nu q\rangle_{\rho_N}x^\nu + \tfrac{1}{2}\langle\bar{q}D_\nu D_\lambda q\rangle_{\rho_N}x^\nu x^\lambda + \cdots]\delta_{ij} \\ &\quad + [\langle\bar{q}\gamma_u q\rangle_{\rho_N} + \langle\bar{q}\gamma_\mu D_\nu q\rangle_{\rho_N}x^\nu + \tfrac{1}{2}\langle\bar{q}\gamma_\mu D_\nu D_\lambda q\rangle_{\rho_N}x^\nu x^\lambda + \cdots]\gamma_{ij}^\mu\} ,\end{aligned} \quad (4.16)$$

where N_c is the number of colors. Many of these condensates vanish in the vacuum but not at finite density. For example, we have a new vector "condensate" $\langle\bar{q}\gamma^\mu q\rangle_{\rho_N}$, which is the rest-frame quark density times u^μ. The modeling of these condensates up through dimension five is discussed in Ref. [18] and involves the use of parton distribution functions [16].

We also consider a subset of the possible dimension-six condensates that we expect to be most important. These are four-quark matrix elements of the general form $\langle\bar{q}\Gamma q\bar{q}\Gamma' q\rangle_{\rho_N}$. These are numerically important in the OPE, because the corresponding Wilson coefficients have no loop integrations, which implies anomalously large contributions relative to other terms [6]. By using Eqs. (4.15) and (4.16) as described below, these four-quark matrix elements are automatically included as if a factorization assumption [6] were made explicitly, which leads to simple products of $\langle\bar{q}q\rangle_{\rho_N}$ and $\langle q^\dagger q\rangle_{\rho_N}$ and no new parameters [for example, see the last terms in Eqs. (4.17)–(4.19)]. In the vacuum, only the matrix elements that lead to $\langle\bar{q}q\rangle_{\rm vac}^2$ survive; we refer to this as the "scalar-scalar" four-quark condensate [the a^2 term in Eq. (3.20)]. At finite density, the factorization assumption implies that *only the ground state* is important as an intermediate state. We do not consider this assumption to be reliable. We only adopt the factorized form as the zero-density limit of the scalar-scalar four-quark condensate and consider two different parameterizations of its density dependence. The other four-quark contributions have a much smaller numerical impact on the results [19], so we just use the factorized forms for simplicity. Although these contributions have terms that are explicitly quadratic in the density, they should not be considered the most important source of higher-order density dependence.

The Wilson coefficients are identified by applying Wick's theorem to the coordinate-space time-ordered product in Eq. (4.1), using Eq. (4.15) for each contraction. The Fourier transform is performed using standard formulas that omit divergent polynomials in q^2, which will not contribute to the final sum rules [7]. Finally, the leading contributions to each invariant function are projected using Eqs. (4.4)–(4.6). For convenience, we present the invariant functions of Eq. (4.3) in the rest frame of nuclear matter, where $q\cdot u \to q_0$. For simplicity, we exclude dimension-five terms, which are less important numerically [19]. Also neglected are terms explicitly proportional to light-quark masses and the independent gluon condensate that vanishes in the vacuum. See Ref. [16] for a discussion of the smallness of this and similar condensates at finite density. The complete expressions are given in Ref. [18]. The most important contributions are

$$\Pi_s(q^2,q_0)=\frac{1}{4\pi^2}q^2\ln(-q^2)\langle \bar{d}d\rangle_{\rho_N}-\frac{1}{2\pi^2}q_0\ln(-q^2)\langle \bar{d}iD_0d\rangle_{\rho_N}-\frac{4q_0}{3q^2}\langle \bar{d}d\rangle_{\rho_N}\langle u^\dagger u\rangle_{\rho_N}+\cdots\ , \tag{4.17}$$

$$\begin{aligned}\Pi_q(q^2,q_0)=&-\frac{1}{64\pi^4}(q^2)^2\ln(-q^2)+\frac{1}{6\pi^2}q_0\ln(-q^2)(\langle u^\dagger u\rangle_{\rho_N}+\langle d^\dagger d\rangle_{\rho_N})\\&+\left[\frac{1}{9\pi^2}\ln(-q^2)-\frac{4}{9\pi^2}\frac{q_0^2}{q^2}\right]\langle d^\dagger iD_0d\rangle_{\rho_N}+\left[\frac{4}{9\pi^2}\ln(-q^2)-\frac{4}{9\pi^2}\frac{q_0^2}{q^2}\right]\langle u^\dagger iD_0u\rangle_{\rho_N}\\&-\frac{1}{32\pi^2}\ln(-q^2)\left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}\right\rangle_{\rho_N}-\frac{2}{3q^2}\langle \bar{u}u\rangle^2_{\rho_N}-\frac{4}{3q^2}\langle u^\dagger u\rangle_{\rho_N}\langle d^\dagger d\rangle_{\rho_N}+\cdots\ ,\end{aligned} \tag{4.18}$$

$$\begin{aligned}\Pi_u(q^2,q_0)=&\frac{1}{12\pi^2}q^2\ln(-q^2)(7\langle u^\dagger u\rangle_{\rho_N}+\langle d^\dagger d\rangle_{\rho_N})-\frac{4}{9\pi^2}q_0\ln(-q^2)\langle d^\dagger iD_0d\rangle_{\rho_N}\\&-\frac{16}{9\pi^2}q_0\ln(-q^2)\langle u^\dagger iD_0u\rangle_{\rho_N}-\frac{4q_0}{3q^2}\langle u^\dagger u\rangle_{\rho_N}(\langle u^\dagger u\rangle_{\rho_N}+\langle d^\dagger d\rangle_{\rho_N})+\cdots\ .\end{aligned} \tag{4.19}$$

We have suppressed terms that are simple polynomials in q^2 (including divergent terms), as they will not contribute to the sum rules. Results for the neutron are obtained by interchanging u and d. Since we focus on isoscalar quantities in nuclear matter here, we take

$$\langle \bar{u}u\rangle_{\rho_N}\simeq\langle \bar{d}d\rangle_{\rho_N}\equiv\langle \bar{q}q\rangle_{\rho_N}\ , \tag{4.20}$$

$$\langle u^\dagger u\rangle_{\rho_N}=\langle d^\dagger d\rangle_{\rho_N}\equiv\langle q^\dagger q\rangle_{\rho_N}=\tfrac{3}{2}\rho_N\ , \tag{4.21}$$

and so on, where $\langle q^\dagger q\rangle_{\rho_N}$ is the quark density for one flavor in the nuclear-matter rest frame.

C. Sum-rule equations at finite density

To extend the Borel sum rules of Sec. III to finite density, it is easiest to generalize the approach that starts with an integral over a weighting function. Our goal is to find a weighting function that enhances the positive-energy quasinucleon contribution, suppresses the negative-energy quasinucleon contribution, and reduces to the vacuum sum rule in the zero-density limit. As noted above, the Lehmann representation shows that each invariant function Π_i is analytic in the complex ω plane except for a cut along the entire real axis. Thus we can use Cauchy's theorem to derive sum rules as in Sec. III.

We consider an integral of Π_i times an analytic weighting function $W(\omega)$ on a contour that has two separate pieces: It runs along the real axis above the cut, closing in the upper half plane with a half circle of radius ω_0, and runs along the real axis below the cut, closing in the lower half plane with a half circle of radius ω_0 (see Fig. 2). Thus the total integral is zero:

$$\oint d\omega\, W(\omega)\Pi_i(\omega)=0\ . \tag{4.22}$$

If we require that $W(\omega^*)=[W(\omega)]^*$, then the integrals along the cut combine to an integral for real ω of the discontinuity in $\Pi_i(\omega)$, which is then equal to the integral on a circle of radius ω_0 (with the real axis deleted):

$$\int_{|\omega|=\omega_0}d\omega\, W(\omega)\Pi_i(\omega)=-\int_{-\omega_0}^{\omega_0}d\omega\, W(\omega)\Delta\Pi_i(\omega)\ . \tag{4.23}$$

In the zero-density limit, the discontinuity in ω in Eq. (4.23) reduces to the discontinuity in s in Eq. (3.6) after changing variables.

On the left-hand side of Eq. (4.23), we apply operator product expansions for the Π_i's at $\omega=i\omega_0$ and analytically continue to rest of the circle. (See Ref. [32].) The ω_0 dependence arising from the integral over the circle is equivalent to introducing continuum thresholds at $\pm\omega_0$ with the usual phenomenological ansatz [see Eq. (3.18)]. Because we expect an asymmetry between positive and negative ω_0, we could consider different thresholds; for simplicity we use the same thresholds here. We also take $s_0=\omega_0^2-\mathbf{q}^2$ so that we recover the zero-density ansatz in the limit.

If we choose the weighting function $W(\omega)=\omega e^{-\omega^2/M^2}$, then the vacuum Borel sum rule is reproduced in the zero-density limit up to an overall factor of $e^{-\mathbf{q}^2/M^2}$. This is not an optimal choice at finite density, because it weights positive and negative ω equally. In order to suppress the negative-energy contribution, we use the weighting function

$$W(\omega)=(\omega-\bar{E}_q)e^{-\omega^2/M^2}\ , \tag{4.24}$$

where $\bar{E}_q$ is the energy of the negative-energy pole in our quasiparticle ansatz [see Eq. (2.11)]. This choice suppresses a sharp excitation completely but also strongly suppresses (relative to the positive-energy contribution) a broad excitation in this vicinity. Furthermore, this choice reduces to the usual Borel sum rule in the vacuum: the vacuum spectral densities are odd in ω; thus the $\bar{E}_q$ contribution vanishes.

The same result can be obtained through Borel transforms of the dispersion relation. If we separately consider the real and imaginary parts of Eq. (4.9), with $q_0=i\omega'$ and ω' real, it is easy to show that constructing sum rules by equating the phenomenological and OPE representations of

$$\mathcal{B}\left[\mathrm{Re}\Pi_i(i\omega',\mathbf{q})-\frac{\bar{E}_q}{\omega'}\mathrm{Im}\Pi_i(i\omega',\mathbf{q})\right] \tag{4.25}$$

is equivalent to using the weighting function of Eq. (4.24) in Eq. (4.23). Here the Borel transform is with respect to ω'^2. The difference from a Borel transform with respect to $Q^2=-q^2$ is a factor of $e^{-\mathbf{q}^2/M^2}$ common to all terms. In the zero-density limit, the second term in Eq. (4.25) vanishes, and we once again recover the usual vacuum sum rules.

Evaluating the integrals in Eq. (4.23) with the ansatz in Eqs. (4.12)–(4.14) and the expansions in Eqs. (4.17)–(4.19) yields three sum rules—one for each invariant function:

$$\lambda_N^{*2}M_N^{*}e^{-(E_q^2-\mathbf{q}^2)/M^2}=-\frac{1}{4\pi^2}M^4\langle\bar{q}q\rangle_{\rho_N}-\frac{1}{2\pi^2}\bar{E}_qM^2\langle\bar{q}iD_0q\rangle_{\rho_N}-\frac{4}{3}\bar{E}_q\langle\bar{q}q\rangle_{\rho_N}\langle q^\dagger q\rangle_{\rho_N}, \tag{4.26}$$

$$\lambda_N^{*2}e^{-(E_q^2-\mathbf{q}^2)/M^2}=\frac{1}{32\pi^4}M^6+\frac{1}{3\pi^2}\bar{E}_qM^2\langle q^\dagger q\rangle_{\rho_N}-\left[\frac{5}{9\pi^2}-\frac{8}{9\pi^2}\frac{\mathbf{q}^2}{M^2}\right]M^2\langle q^\dagger iD_0q\rangle_{\rho_N}$$
$$+\frac{1}{32\pi^2}M^2\left\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}\right\rangle_{\rho_N}+\frac{2}{3}\langle\bar{q}q\rangle^2_{\rho_N}+\frac{4}{3}\langle q^\dagger q\rangle^2_{\rho_N}, \tag{4.27}$$

$$\lambda_N^{*2}\Sigma_v e^{-(E_q^2-\mathbf{q}^2)/M^2}=\frac{2}{3\pi^2}M^4\langle q^\dagger q\rangle_{\rho_N}+\frac{20}{9\pi^2}\bar{E}_qM^2\langle q^\dagger iD_0q\rangle_{\rho_N}+\frac{8}{3}\bar{E}_q\langle q^\dagger q\rangle^2_{\rho_N}. \tag{4.28}$$

Our prescription for the weighting function implies that linear factors of q_0 in Eqs. (4.17)–(4.19) become minus the energy of the negative-energy pole in Eqs. (4.26)–(4.28); that is,

$$q_0\to-\bar{E}_q=-\Sigma_v+(\mathbf{q}^2+M_N^{*2})^{1/2}. \tag{4.29}$$

Some higher-order terms from the expansion in Eq. (4.16) are not considered explicitly here. They are accompanied by factors of $\mathbf{q}^2/M^2$, which suppresses these terms for bound or low-energy nucleons. Continuum factors multiplying M^2, M^4, and M^6 terms are the same as in Eqs. (3.25)–(3.27), with $s_0=\omega_0^2-\mathbf{q}^2$. As noted above, the thresholds could be different for positive and negative ω_0. In practice, this does not seem to be a major effect (see Ref. [19] for numerical details).

The exponents on the left-hand sides contain the squared four-momentum of the positive-energy quasinucleon. This was called μ^2 in Ref. [5] and can be written as $\mu^2\equiv M_N^{*2}+\Sigma_v^2+2E_q^*\Sigma_v$. In the vacuum, $\mu^2=M_N^2$ for all $\mathbf{q}$, and terms proportional to $\langle q^\dagger q\rangle_{\rho_N}$ vanish, so we recover Eqs. (3.19) and (3.20) up to the dimension-six condensates in the zero-density limit. At nuclear matter saturation density, the empirical value of μ^2 is around 1 GeV^2 for low- and intermediate-energy nucleons; i.e., it is essentially unchanged from the free-space value. Note that the generalization from zero to finite density involves more than just putting stars on quantities, as in $M_N\to M_N^*$.

These sum rules explicitly involve the Borel mass M. If both the theoretical and phenomenological sides were calculated to high accuracy, predictions for the spectral parameters M_N^*, Σ_v, λ_N^{*2}, and the threshold ω_0 (or s_0) should become independent of M^2. In practice, both sides are represented imperfectly, so we expect (at best) only a range of M^2 with good overlap. Based on experience with zero-density sum rules [8,25], we anticipate the optimal values of M^2 for this overlap to be around μ^2 (~ 1 GeV^2), which should ensure reasonable convergence of the OPE while suppressing contributions from higher-mass singularities.

We anticipate a significant asymmetry between the positive- and negative-energy spectral functions. At zero density, the charge conjugation invariance of the vacuum guarantees that the functions are identical (up to a sign, depending on conventions). At nuclear matter saturation density, the ground state is not invariant under charge conjugation, and we expect large differences on physical grounds between the propagation of an antinucleon and a nucleon in the infinite medium. Thus the antinucleon-pole contribution in the vacuum will spread significantly in the finite-density medium. We also predict *a shift* in the "pole" position from the quasinucleon ansatz. (This in-medium shift would be difficult to recognize in an experiment with antinucleons: by the time they leave the nucleus and are detected, the antinucleons would have reverted to their free-space properties.)

By considering a weighting function that cancels the positive- and negative-energy spectra, we might hope to learn directly about the anticipated asymmetry. For example, $W(\omega)=e^{-\omega^2/M^2}$ could be chosen. Applied to the sum-rule equations (4.17)–(4.19), we see that only terms with odd powers of q_0 in the OPE will survive. Note that these all vanish in the zero-density limit. On the phenomenological side, we obtain a weighted difference of the positive- and negative-energy spectra.

If we use the simple pole model with this new sum rule, we find a significantly smaller prediction for the vector self-energy than we found in our other analysis. However, this sum rule is very sensitive to the phenomenological ansatz for the spectra, since it involves differences of spectra, each of which is only known crudely. This is in contrast to our usual analysis, where we rely most heavily on phenomenological insight into the positive-energy spectrum. Thus, a direct study of the asymmetry is inconclusive.

V. RESULTS

In this section, we present results from the nucleon sum rules at finite density. We start with the simplified version of the sum rules introduced in Ref. [5], which requires only the $\langle \bar{q}q \rangle_{\rho_N}$ and $\langle q^\dagger q \rangle_{\rho_N}$ condensates as a function of density. As described in Ref. [5], this calculation predicts nucleon self-energies in qualitative agreement with relativistic phenomenology. Then we test the stability of the results by including higher-dimensional condensates in the OPE, accounting for the contribution of higher-energy states, and considering a more sophisticated sum-rule analysis. Two different methods of analysis are used, as described below, and we rely on results from Ref. [18] to model some of the new condensates.

A. Finite-density condensates

To extract values for the self-energies from the formulas of Sec. IV, we need to estimate the various matrix elements (condensates) at finite density. For the simplified sum rule, however, we need only the scalar and vector quark condensates in the nuclear medium, $\langle \bar{q}q \rangle_{\rho_N}$ and $\langle q^\dagger q \rangle_{\rho_N}$, so we focus on them first. We will also find that they remain the most important elements in the full sum-rule calculations. As noted earlier, the vector condensate is independent of dynamics. Since the baryon current is conserved, $\langle q^\dagger q \rangle_{\rho_N}$ is simply related to the nucleon density: $\langle q^\dagger q \rangle_{\rho_N} = \frac{3}{2}\rho_N$. The in-medium scalar condensate was considered in detail in Ref. [13], and we briefly summarize the results here.

To estimate the scalar condensate, we work to leading order in a density expansion. At this level, the *change* in the scalar condensate is *model independent*. It is related to the nucleon σ term σ_N [12,13]:

$$2m_q(\langle \bar{q}q \rangle_{\rho_N} - \langle \bar{q}q \rangle_{\text{vac}}) = \sigma_N \rho_N + \cdots \ , \tag{5.1}$$

where $m_q \equiv \frac{1}{2}(m_u + m_d)$ is the average of the up and down current quark masses. (The same normalization scale μ is applied everywhere to consistently define $\langle \bar{q}q \rangle_{\rho_N}$ at that scale.)

This result can be understood in terms of the Hellmann-Feynman theorem [13]. In particular, the theorem, when applied to matrix elements of the QCD Hamiltonian, implies that

$$2m_q(\langle \bar{q}q \rangle_{\rho_N} - \langle \bar{q}q \rangle_{\text{vac}}) = m_q \frac{d\mathcal{E}}{dm_q} \ , \tag{5.2}$$

where the derivative is taken at fixed density. The energy density of nuclear matter $\mathcal{E}$ is given by

$$\mathcal{E} = M_n \rho_N + \delta\mathcal{E} \ , \tag{5.3}$$

where $\delta\mathcal{E}$ is the contribution to the energy density from the nucleon kinetic energies and NN interactions. $\delta\mathcal{E}$ is of higher order in the nucleon density, and its contribution to the density dependence $\langle \bar{q}q \rangle_{\rho_N}$ is model dependent.

A further application of the Hellmann-Feynman theorem to the nucleon matrix element of the QCD Hamiltonian yields

$$\sigma_N = 2m_q \int d^3x (\langle N|\bar{q}q|N\rangle - \langle 0|\bar{q}q|0\rangle) \tag{5.4}$$

$$= m_q \frac{dM_N}{dm_q} \ , \tag{5.5}$$

where Eq. (5.4) follows from the definition of σ_N. Combining Eqs. (5.2) and (5.5), we obtain Eq. (5.1). Corrections due to higher-order terms in the density expansion (from $\delta\mathcal{E}$) were estimated in a variety of models in Ref. [13] and were found to be small ($\sim 10\%$) at nuclear matter saturation density and below. We will assume that these estimates are reliable and use only the leading contribution from Eq. (5.1). The value of σ_N remains a subject of some controversy; values from 30 to 60 MeV have been cited and vigorously defended [38–40]. In this paper we show results only for $\sigma_N = 45$ MeV, which is the value from a recent analysis [39], and defer to Ref. [19] a study of how results depend on the magnitude of the σ term.

To relate the current quark mass and the quark condensate in vacuum, we use the Gell-Mann–Oakes–Renner relation

$$2m_q \langle \bar{q}q \rangle_{\text{vac}} = -m_\pi^2 f_\pi^2 \ . \tag{5.6}$$

We take $m_\pi = 138$ MeV and $f_\pi = 93$ MeV. The product on the left-hand side is renormalization-group invariant, so fixing a value for the quark mass at the scale of interest fixes the consistent value of the quark condensate in vacuum. Here we will follow Ref. [25] and take $m_q \simeq 5.5$ MeV, which implies that $\langle \bar{q}q \rangle_{\text{vac}} \simeq -(245 \text{ MeV})^3$. Other values [such as $m_q \simeq 7$ MeV, $\langle \bar{q}q \rangle_{\text{vac}} \simeq -(225 \text{ MeV})^3$] will be considered explicitly in Ref. [19]; the qualitative results from the full sum-rule analysis are not sensitive to this difference.

The in-medium gluon condensate $\langle (\alpha_s/\pi) G^a_{\mu\nu} G^{a\mu\nu} \rangle_{\rho_N}$ is estimated in Ref. [13]. Other condensates, $\langle q^\dagger i D_0 q \rangle_{\rho_N}$ and $\langle \bar{q} i D_0 q \rangle_{\rho_N}$, that appear in Eqs. (4.26)–(4.28) are estimated in Ref. [18] along with other higher-dimensional condensates. These condensates are given to leading order in the density in terms of nucleon matrix elements. We simply quote the values used here:

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_{\rho_N} = (330 \text{ MeV})^4 - (700 \text{ MeV})\rho_N \ , \tag{5.7}$$

$$\langle q^\dagger i D_0 q \rangle_{\rho_N} = (150 \text{ MeV})\rho_N \ , \tag{5.8}$$

$$\langle \bar{q} i D_0 q \rangle_{\rho_N} \sim m_q \rho_N \simeq 0 \ , \tag{5.9}$$

where ρ_N is expressed in MeV^3.

B. Simplest finite-density sum rules

The simplified finite-density sum rules follow by keeping in each of the three sum rules of Eqs. (4.26)–(4.28) only the quasinucleon pole contribution to the phenomenological side (i.e., no continuum factors), and only the leading term in the operator product expansion on the

OPE side:

$$\lambda_N^{*2} M_N^* e^{-(E_q^2-\mathbf{q}^2)/M^2} = -\frac{1}{4\pi^2} M^4 \langle \bar{q}q \rangle_{\rho_N} , \tag{5.10}$$

$$\lambda_N^{*2} e^{-(E_q^2-\mathbf{q}^2)/M^2} = \frac{1}{32\pi^4} M^6 , \tag{5.11}$$

$$\lambda_N^{*2} \Sigma_v e^{-(E_q^2-\mathbf{q}^2)/M^2} = \frac{2}{3\pi^2} M^4 \langle q^\dagger q \rangle_{\rho_N} . \tag{5.12}$$

By considering ratios of these sum rules evaluated at a value of the Borel mass in the middle of the range where the full sum rules will be considered, we hope to extract the basic physics. As we shall see later, this truncation is qualitatively reasonable *except* for the large contribution of the scalar-scalar four-quark condensate.

Taking ratios of Eqs. (5.10)–(5.12), we obtain simple expressions for the scalar and vector self-energies,

$$\Sigma_s = -\frac{8\pi^2}{M^2}(\langle \bar{q}q \rangle_{\rho_N} - \langle \bar{q}q \rangle_{\text{vac}}) = -\frac{4\pi^2}{M^2}\frac{\sigma_N \rho_N}{m_q} , \tag{5.13}$$

$$\Sigma_v = \frac{64\pi^2}{3M^2} \langle q^\dagger q \rangle_{\rho_N} = \frac{32\pi^2}{M^2} \rho_N , \tag{5.14}$$

which we expect to apply up to nuclear matter saturation density. To derive Eq. (5.13) we have subtracted the zero-density result [Eq. (3.30)] adopting the same Borel mass. (Recall that $\Sigma_s = M_N^* - M_N$.)

We can directly compare the magnitudes of the scalar and vector self-energies by taking the ratio of Eqs. (5.13) and (5.14). The explicit dependence on the Borel mass and the density drops out, yielding

$$\frac{\Sigma_s}{\Sigma_v} = -\frac{\sigma_N}{8m_q} . \tag{5.15}$$

For typical values of σ_N and the light quark masses, this ratio is close to -1, indicating a substantial cancellation of Σ_s and Σ_v in the medium. Thus we have qualitative agreement with several features of relativistic phenomenology: the self-energies scale with the density, they are weakly dependent on the nucleon state (three-momentum), and scalar and vector self-energies cancel.

Alternatively, we can normalize the self-energies to the zero-density mass as calculated via the sum rules. This is meant to reduce the sensitivity to particular details of the sum rules and to the level of truncation, provided we work to the same level of approximation at finite and zero density. Taking ratios of Eqs. (5.10)–(5.12) at both zero and finite density, we obtain results independent of M^2:

$$\frac{M_N^*}{M_N} = 1 + \frac{\Sigma_s}{M_N} = \frac{\langle \bar{q}q \rangle_{\rho_N}}{\langle \bar{q}q \rangle_{\text{vac}}} = 1 - \frac{\sigma_N \rho_N}{m_\pi^2 f_\pi^2} , \tag{5.16}$$

$$\frac{\Sigma_v}{M_N} = -\frac{8}{3}\frac{\langle q^\dagger q \rangle_{\rho_N}}{\langle \bar{q}q \rangle_{\text{vac}}} = \frac{8 m_q \rho_N}{m_\pi^2 f_\pi^2} . \tag{5.17}$$

The last equalities in Eqs. (5.16) and (5.17) follow from Eqs. (5.1) and (5.6). [Note that the independence of M^2 in the ratios in Eqs. (5.16) and (5.17) should not be interpreted as evidence that the *individual* sum rules are weakly dependent on M^2.] For typical values of the relevant condensates and other parameters, $M_N^*/M_N \sim 0.6$–0.7 and $\Sigma_v/M_N \sim 0.3$–0.4. This is in good agreement with the values used in relativistic mean-field models that provide good fits to bulk properties of finite nuclei [4].

The key feature that assures qualitative agreement with relativistic phenomenology is that Eq. (5.11) is density independent to leading order. In the simple sum rule, this implies that the pole position and residue do not vary much with density. This in turn implies the results of Eqs. (5.13) and (5.14), in which the effective mass naturally follows $\langle \bar{q}q \rangle_{\rho_N}$ and the vector self-energy follows $\langle q^\dagger q \rangle_{\rho_N}$. In the more complete sum rules considered below, these basic results survive if the correction terms to Eqs. (5.10) and (5.12) are not overly large *and* if Eq. (5.11) remains weakly density dependent. This latter condition turns out to be problematic.

C. Detailed sum-rule analysis

We use two approaches to analyze the complete sum rules. The first is a ratio method, which generalizes the simple analysis of Ref. [5] and the previous section. In each of the three sum rules in Eqs. (4.26)–(4.28), the contribution from the quasiparticle pole is isolated on one side of the equation. As in Sec. III, the continuum dependence is included with the operator product expansions of the invariant functions, which we denote as Π'_q, Π'_s, and Π'_u. Then the ratios Π'_s/Π'_q and Π'_u/Π'_q predict M_N^* and Σ_v for each value of the Borel M^2. In order to focus on the change from the vacuum values and to decrease the sensitivity to details of the calculation that affect the vacuum predictions as well as the finite-density values, we consider the normalized ratios M_N^*/M_N and Σ_v/M_N. The value of M_N is obtained from the zero-density limit of the same sum rules.

Detailed comparisons of how various contributions affect results and studies of the uncertainties due to various assumptions about the density dependence of condensates will be documented and discussed in a separate paper [19]. (Examples include anomalous dimensions, choice of the range in Borel mass, treatment of continuum thresholds, and variation of input parameters.) In general, the normalized ratios are not very sensitive to these details. As noted earlier, the one exception is that the scalar-scalar four-quark condensate is found to play a crucial role, and qualitatively different results are found from different assumptions about its density dependence. Therefore, we will consider two extreme assumptions as to how it varies with density in our discussion here.

In Fig. 3, we plot the nucleon mass predicted by the ratio sum rule in vacuum as a function of Borel M^2. Both sets of curves neglect corrections from anomalous dimensions. The lower set of three curves corresponds to the zero-density sum rule from Sec. III, including only up to dimension-six condensates. This is the underlying sum rule for M_N used in our finite-density calculations of M_N^*/M_N and Σ_v/M_N.

We expect that the sum rule should be valid in a region near $M^2 = 1\ \text{GeV}^2$ [8]. That is to say, the overlap be-

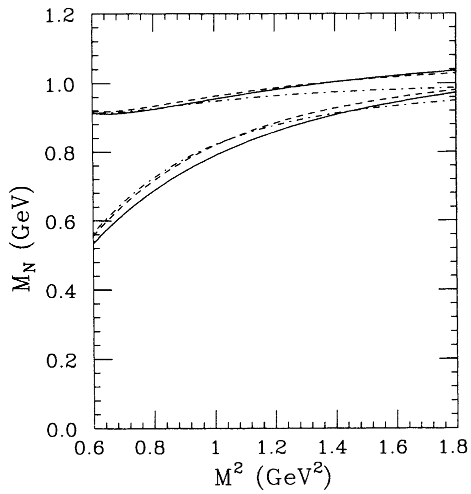

FIG. 3. Ratio sum-rule predictions for the nucleon mass M_N at zero density [see Eqs. (3.19) and (3.20)] as a function of Borel M^2. Anomalous dimension corrections are not included here. The upper group of curves show the predictions of the sum rule from Ref. [25], and the lower curves are from the ratio sum rule including up to dimension-six condensates only. All curves use the parameters described in the text. In both cases, the three curves correspond to continuum thresholds $s_0=2.0$ GeV^2 (solid), $s_0=2.5$ GeV^2 (dashes), and $s_0=3.0$ GeV^2 (dot-dashes).

tween the region where the sum rule is dominated by the nucleon contribution and the region where the truncated operator product expansion is reliable is around 1 GeV^2. In this overlap region, the prediction is only weakly dependent on the continuum threshold, but the curves are not flat. If the sum rule were perfect, we would expect to see horizontal lines, since the mass should be independent of the auxiliary parameter M^2. It is evident that the nucleon sum rule truncated at dimension-six condensates does not provide a convincing "plateau." One interpretation of the variation of the curve in the fiducial region near 1 GeV^2 is that this is a measure of the uncertainty in the prediction. Another interpretation would be that the sum rule is not working. We will assume that the sum rule actually has a region of overlap, although imperfect.

We note that the inclusion of higher-dimensional condensates tends to bring up the curve at lower M^2. This is evident in the top set of curves in Fig. 3, which corresponds to the sum rule given by Ioffe and Smilga in Ref. [25] [see Eqs. (3.19) and (3.20)], which we take as an example of the more complete treatments. Ioffe and Smilga identify the fiducial interval in M^2 as roughly 0.8 GeV$^2<M^2<1.4$ GeV^2. We will adopt these boundaries as the maximal limits of applicability of our sum rules at finite density.

It is clear that even an optimistic interpretation of the nucleon sum rule implies a significant uncertainty in the predicted mass. One might worry that the uncertainties are as large as the changes we expect to see at finite density. However, we have some advantages that improve the situation. We rely on the cancellation of systematic discrepancies by normalizing all finite-density self-energies to the zero-density prediction for the mass. This is designed to compensate for general deficiencies of the nucleon sum rule. While the individual curves can be shifted up and down by varying parameters and assumptions, the ratios do not vary much. We also note that values determined by optimizing the agreement of left- and right-hand sides of the sum-rule equations over a region in M^2 are much less sensitive to details. Furthermore, we can tolerate a large error (say $\sim 50\%$) and still find useful results.

Now we turn to results from the finite-density sum rules. We first consider the case with the scalar-scalar four-quark condensate *fixed* at the zero-density factorized value $\langle \bar{q}q\rangle^2_{\text{vac}}$. In Fig. 4, the ratios M_N^*/M_N and Σ_v/M_N are plotted at nuclear matter saturation density ($\rho_N^{\text{sat}}=0.17$ fm^{-3}) as a function of Borel M^2. As before, the three curves are for three different continuum thresholds, which are the *same* for finite density and zero density. The curves are quite flat over the relevant region even though the individual sum rules are not. The predictions from this analysis are $M_N^*/M_N \sim 0.65-0.70$ and $\Sigma_v/M_N \sim 0.25-0.30$, which are comparable to typical values from relativistic phenomenology [4]. The results are insensitive to the variation of parameters within the usual ranges, the inclusion of anomalous dimensions, the treatment of dimension-four and -five condensates, and the choice of three momentum (if below 1 GeV) [19]. The three-momentum enters only through $\bar{E}_q$, the combination $E_q^2-\mathbf{q}^2$, and in factors of $\mathbf{q}^2/M^2$ that accompany higher-dimensional condensates. If $|\mathbf{q}|$ is much less than M_N, the sum rules are little affected by the actual value. All results plotted here use $|\mathbf{q}|=270$ MeV.

Predictions for M_N^*/M_N and Σ_v/M_N as a function of density are shown in Fig. 5 with the same input parameters as above. These results are from an optimization

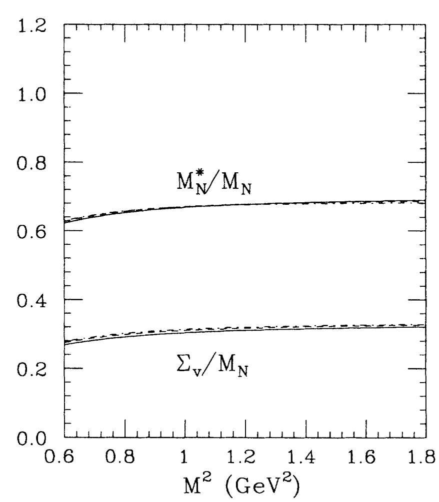

FIG. 4. Ratio sum rules for M_N^*/M_N and Σ_v/M_N as a function of Borel M^2, evaluated at nuclear matter saturation density. The input parameters are described in the text. The scalar-scalar four-quark condensate (see text) is held fixed at the zero-density factorized value. In both cases, the three curves correspond to continuum thresholds $s_0=2.0$ GeV^2 (solid), $s_0=2.5$ GeV^2 (dashes), and $s_0=3.0$ GeV^2 (dot-dashes).

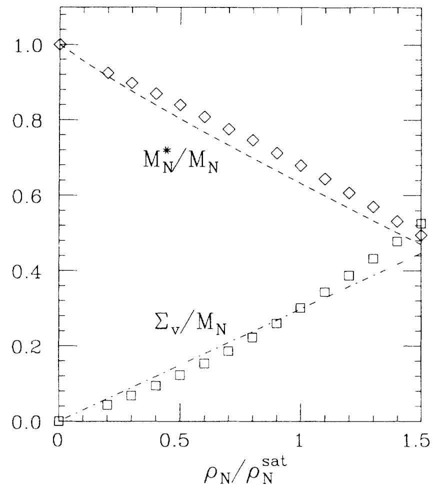

FIG. 5. Optimized sum-rule predictions for M_N^*/M_N (diamonds) and Σ_v/M_N (squares) as a function of density. The predictions from a relativistic mean-field model [41] are shown for comparison (dashed and dot-dashed curves). The scalar-scalar four-quark condensate is held fixed at the zero-density factorized value.

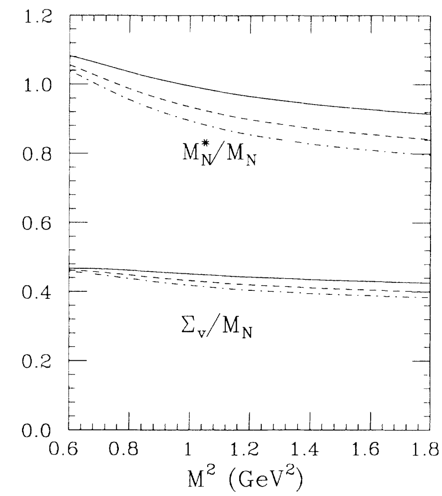

FIG. 6. Ratio sum rules for M_N^*/M_N and Σ_v/M_N as a function of Borel M^2, evaluated at nuclear matter saturation density. The input parameters are described in the text. The scalar-scalar four-quark condensate varies with density according to the factorized form ($\propto \langle \bar{q}q \rangle_{\rho_N}^2$). In both cases, the three curves correspond to continuum thresholds $s_0=2.0$ GeV^2 (solid), $s_0=2.5$ GeV^2 (dashes), and $s_0=3.0$ GeV^2 (dot-dashes).

procedure applied to obtain a "best fit" of the sum-rule equations. To perform the optimization, a range in Borel M^2 is chosen (here 0.8 GeV$^2 < M^2 < 1.4$ GeV^2) and the left- and right-hand sides of the sum rules [Eqs. (4.26)–(4.28)] are evaluated at each density for a large number (e.g., 100) of equally spaced points in this range. Then we minimize the sum over M^2 points of

$$\ln\left[\frac{\max\{\lambda_N^{*2}e^{-\mu^2/M^2},\Pi_q',\Pi_s'/M_N^*,\Pi_u'/\Sigma_v\}}{\min\{\lambda_N^{*2}e^{-\mu^2/M^2},\Pi_q',\Pi_s'/M_N^*,\Pi_u'/\Sigma_v\}}\right] \qquad (5.18)$$

as a function of λ_N^{*2}, M_N^*, Σ_v, and s_0. This procedure reduces the sensitivity to the end points of the range and removes the scales of the curves being fit [25].

The dashed and dot-dashed lines in Fig. 5 are the values of M_N^*/M_N and Σ_v/M_N for a relativistic mean-field (σ-ω) model (see Ref. [41]) that is typical of models that fit a wide range of bulk properties of finite nuclei. (It is a nonlinear model that is quantitatively more successful than the original Walecka model.) The sum-rule self-energies seem to be similar to those of successful relativistic phenomenology. The cancellations between the scalar and vector self-energies are manifest in the figure; the implied energy of the quasinucleon, $E_q=\Sigma_v+(\mathbf{q}^2+M_N^{*2})^{1/2}$, is almost independent of density. The continuum threshold is also essentially constant at 2.1 GeV^2, and the residue drops only about 15% at saturation density relative to the vacuum value. The precise values and systematics of the threshold and residue, however, are much more sensitive to the details of the sum rule (parameters, choice of Borel range) than are the self-energy ratios.

Next we consider another extreme assumption for the scalar-scalar four-quark condensate. We assume it factorizes at finite density, so that it is proportional to $\langle \bar{q}q \rangle_{\rho_N}^2$ for all densities. Its density dependence is then fixed by $\langle \bar{q}q \rangle_{\rho_N}$. The results are shown for the ratio method in Fig. 6 and for the optimization method in Fig. 7. In the ratios, the Σ_v/M_N curve is relatively flat, with somewhat higher values than before (~ 0.4). The ratio is not particularly sensitive to the continuum threshold. In contrast, M_N^*/M_N is not well determined, with the curves varying over a much larger range from 0.85 to greater than one, with significant dependence on the threshold.

The optimized values for this case show Σ_v/M_N to be similar to the relativistic mean-field curve up to saturation density and not very different from the curve in Fig. 5. However, within the uncertainties, M_N^*/M_N is flat and equal to unity up to saturation density. The net result is

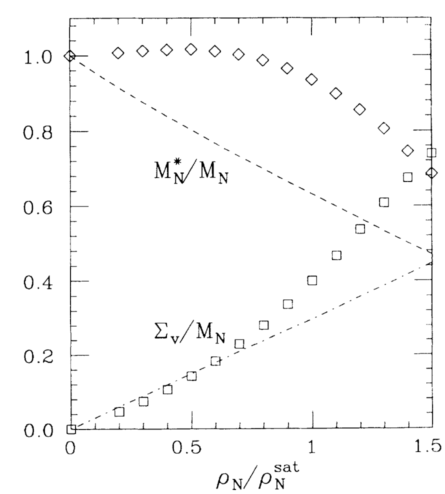

FIG. 7. Optimized sum-rule predictions for M_N^*/M_N (diamonds) and Σ_v/M_N (squares) as a function of density. The predictions from a relativistic mean-field model [41] are shown for comparison (dashed and dot-dashed curves). The scalar-scalar four-quark condensate is varied with density according to the factorized form ($\propto \langle \bar{q}q \rangle_{\rho_N}^2$).

an increasing value for the pole energy E_q, which becomes 30% greater than the free nucleon mass. In addition, the optimized continuum threshold and residue *increase* steadily and are each about 30% larger in nuclear matter, implying a significant rearrangement of the spectrum with unexpected systematics. Clearly the sum rules are problematic with this set of assumptions.

VI. DISCUSSION

The sensitivity of our results to the unknown density dependence of the scalar-scalar four-quark condensate means that our conclusions must still be somewhat indefinite. Nevertheless, we emphasize that the results obtained with the two different assumed forms of this condensate are not equally compatible with known nuclear phenomenology.

In particular, when the four-quark condensate varies slowly with density, our predictions are very similar to the characteristics of self-energies found in relativistic nuclear physics phenomenology (see Sec. II). For example, the change in the scalar self-energy is negative with increasing density (decreasing effective mass), the change in the vector self-energy is positive, and there is a significant degree of cancellation between them. This is particularly important, because the empirical observation is that the nucleon energy (the quasiparticle pole position) is shifted only slightly in nuclear matter relative to its free-space mass. When normalized to the predicted nucleon mass at zero density, the magnitudes of the predicted self-energies are comparable to those of relativistic phenomenology and are weakly dependent on the three-momentum (at least for relatively small values, less than 1 GeV). The continuum thresholds and residues predicted by the optimization of the sum rules do not vary significantly with density up to nuclear matter saturation density (not at all within reasonable error estimates). This is compatible with experiment; there is no evidence for a strong rearrangement of the spectrum at these densities, merely a spreading of strength over energy scales too small to be resolved by the sum rules.

If we assume that the scalar-scalar four-quark condensate in nuclear matter is essentially unchanged from its vacuum value, the basic physics is dictated by the dominant behavior of the QCD side of the three sum rules: (1) The Π_q expansion is largely density independent, leading to essentially constant values of the effective pole position, residue, and continuum threshold. (2) The Π_s expansion is dominated by the quark condensate $\langle \bar{q}q \rangle_{\rho_N}$. Thus, given the previous constraint, changes in the condensate with density are directly reflected in changes in the nucleon effective mass M_N^*. We have estimated the rate of change through the empirical value of the nucleon σ term σ_N and the nucleon density ρ_N. (3) The Π_u expansion is dominated by the vector quark condensate, that is, the baryon density. Given the constraint from the Π_q rule, the nucleon vector self-energy Σ_v simply follows the density. Higher-order terms in each of the sum rules are not negligible, but they do not change the qualitative features outlined above.

In contrast, if we assume that the scalar-scalar four-quark condensate varies significantly with density, as implied by a finite-density factorization assumption, we find results that are not compatible with nuclear phenomenology. That is to say, the choice is not between a relativistic and nonrelativistic picture of a nucleon in the medium but, rather, between the relativistic picture and a failure of our sum-rule calculations. Specifically, we find a large change in the vector self-energy with density (larger than in the case described above) with almost no change in the scalar self-energy up to the empirical nuclear matter saturation density. So the effective mass stays at the free-space mass, while we inevitably find a large vector self-energy. The resulting pole position shifts significantly, in contradiction with experiment.

If we assume that the first scenario is correct (i.e., so that the sum-rule calculations work for the quasinucleon), there are still some important open questions. The cancellation of scalar and vector contributions found in the nucleon case may not be predicted elsewhere, leading to contradictions with experiment. We must test sum rules for other baryons as well as other nucleon properties such as isovector self-energies and magnetic moments. The Δ and the Λ should be particularly informative. There is useful information on the Δ in the nuclear medium from both electron and proton scattering from nuclei. Since the Δ sum rule is especially sensitive to the scalar-scalar four-quark condensate [7,8], we may obtain some phenomenological constraints on its density dependence.

In the simplest sum rule, the scalar self-energy depends on the degree of chiral restoration in the nuclear medium, while the vector self-energy is simply proportional to the baryon density. Nevertheless, they tend to cancel one another at all densities. This cancellation follows in leading order because the ratio $\sigma_N/8m_q$ is approximately equal to unity [see Eq. (5.15)]. How do we understand this?

Finally, there is the role of correlations to consider. Because of confinement and the strongly repulsive short-range *NN* interaction, correlations of quarks, clustered into hadrons, and correlations of the nucleons themselves might play an important role in determining nucleon properties at finite density. Yet the basic physics of the simplest sum rule indicates that correlations play a secondary role. In particular, the uniform condensates $\langle \bar{q}q \rangle_{\rho_N}$ and $\langle q^\dagger q \rangle_{\rho_N}$ seem to be most important while the higher-dimensional terms with derivatives, which could reflect the "lumpiness" of nuclear matter, are numerically less important. This is an important topic for further study, since underestimating correlation effects might overemphasize changes in the self-energies with density.

VII. SUMMARY

In this paper, we apply QCD sum-rule methods to study the self-energies of a quasinucleon propagating in nuclear matter. A Fourier-transformed correlator of nucleon interpolating fields, evaluated in the finite-density ground state, is the basic object of study [see Eq. (4.1)]. Covariance and the (assumed) properties of the ground state under parity and time reversal imply that the correlator is specified by three independent invariant functions [Eq. (4.3)]. We derive sum rules for each of these functions.

We consider a Lehmann representation for the correlator in the energy variable q_0, with fixed three-momentum. (We work in the rest frame of nuclear matter for convenience; generalizing to arbitrary frames is straightforward but unnecessary for our purposes). This representation manifests the analytic structure of each invariant function, enabling us to relate the value of the function for complex q_0 to an integral over the spectral density, which is defined by the discontinuity across the real q_0 axis. A simple spectral ansatz introduces the effective Lorentz scalar and vector self-energies for the quasinucleon, Σ_s and Σ_v. In free space, the nucleon contribution to the spectral function is a delta function that is reasonably well isolated (on hadronic scales) from higher-energies excitations. We assume, based on nuclear physics phenomenology, that it is reasonable in the sum rule to approximate the low-energy spectrum at finite density with an effective quasinucleon pole. That is, we assume that there are no major rearrangements of the spectrum and that the major spreading of strength with density is on nuclear (tens of MeV) rather than hadronic (hundreds of MeV) energy scales. The contribution of higher-energy states is roughly approximated, starting at an effective threshold.

Each dispersion integral is equated to the corresponding invariant function evaluated at large imaginary q_0. In the latter region, we can apply an operator product expansion (OPE) to each function, which involves finite-density matrix elements of QCD composite operators (condensates) and coefficient functions evaluated in QCD perturbation theory. This equivalence of the OPE and the spectral representation is a QCD sum rule. In the vacuum, a Borel transform with respect to q^2 is applied to improve the convergence of the OPE while concentrating the dispersion integral on the nucleon contribution (within an appropriate range of Borel M^2). At finite density, we apply the Borel transform to a particular combination of the real and imaginary parts of the sum rule chosen to suppress the contribution from the antinucleon "pole." This approach is equivalent to considering an integral over a contour in the complex q_0 plane (see Fig. 2) of the correlator times an optimal weighting function [see Eq. (4.22)].

A truncated version of the resulting sum rules indicates that the key phenomenological inputs are the baryon density and the value of the in-medium scalar quark condensate, which we relate to the density and the nucleon σ term divided by the average current mass of the light quarks [Eq. (5.1)]. Using standard values, we see that the scalar condensate has a strong density dependence: it is reduced by 30–50 % from its vacuum value at nuclear matter density [13]. This implies a large change in the nucleon scalar self-energy (so the nucleons experience a large scalar potential). Given the empirical fact that the total potential, which is roughly equal to scalar plus vector, is known to be small (of order tens of MeV), we expect a strong vector potential with the opposite sign. We find that this emerges naturally from the sum rule, depending in leading order only on the total quark density. Scalar attraction and vector repulsion of this sort are the essential ingredients of relativistic nuclear physics.

We observe that the strong cancellation between scalar and vector components occurs to a large degree because the ratio of σ_N to m_u+m_d is approximately four. It remains an open question as to whether this is a consequence of some deep principle in QCD or if it is accidental. One can explore this issue through a calculation of σ_N using the same techniques used in our calculation of the nucleon self-energies, i.e., via QCD sum rules. Work in this direction is in progress.

We find that the basic features of the simplified sum rules survive in a more detailed analysis that incorporates the effects of higher-energy states and higher-dimensional condensate contributions. The qualitative results are insensitive to most details of the corrections considered here. The exception is the influence of the "scalar-scalar" four-quark condensate, which plays an important role in the vacuum sum rules for the nucleon. Upon considering two possible scenarios for the density dependence of this condensate, we find either good qualitative agreement with relativistic phenomenology or else a basic disagreement with the empirical situation. We note that a large vector self-energy is implied in either case. In the second case, this large vector piece is coupled with an essentially constant effective mass, so that the nucleon energy is predicted to shift by hundreds of MeV, in contradiction with experiment. Clearly, further study of the four-quark condensates in the medium is a high priority, along with analyses of the higher-order density dependence of other condensates and the convergence of the OPE.

Many related areas of investigation are also possible within the same framework we have described here. Other nucleon properties, such as magnetic moments, form factors, and isovector self-energies, can be predicted. In addition, one is naturally led to consider the spectral properties of other baryons in nuclear matter, such as Δ's and Λ hyperons. In these cases there are both experimental data and phenomenological models to confront with the QCD sum-rule predictions. Finally, the properties of mesons in medium can be studied (see Ref. [16]). These topics will be discussed in forthcoming publications.

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