Alpha widths in deformed nuclei: Microscopic approach

D. S. Delion
Institute of Atomic Physics, Bucharest Magurele, POB MG 6, Romania

A. Insolia
Department of Physics, University of Catania and Istituto Nazionale di Fisica Nucleare, I-95129 Catania, Italy

R. J. Liotta
Manne Siegbahn Institute, S-10405 Stockholm, Sweden
(Received 15 October 1991)

A microscopic approach to the alpha decay problem in deformed nuclei is presented. The nuclear wave functions are calculated in the frame of the Nilsson + BCS approximation, making use of a realistic deformed mean field. A large configuration space has been employed in the calculation of the formation amplitude while the penetration process has been treated within the WKB approximation. The calculated widths agree with the experimental data within a factor of about 3. Effects due to deformation are also discussed. Applications are presented for Ra, Rn, and Th isotopes.

PACS number(s): 23.60.+e

I. INTRODUCTION

Alpha-decay processes are among the oldest branches of microscopic physics. Their analysis, the study and interpretation of the rich amount of data provided by them, has been fundamental since the beginning of this century to build up modern physics. Yet, many questions remain still unanswered in the understanding of the mechanisms that induce the decay of the α cluster. Thus, it is not clear whether the Pauli principle acting between the constituent nucleons in the α particle and those in the daughter nucleons has any importance [1–3]. An estimation of the amount of the correction due to the Pauli principle can be found in Ref. [3]. Within a shell-model basis which includes up to $13\hbar\omega$ excitations an enhancement was found of about a factor of 6 in the reduced width for the ground to ground transition $^{212}\mathrm{Po}\rightarrow{}^{208}\mathrm{Pb}+\alpha$. It is also found that this enhancement decreases as the dimensions of the shell-model basis increases. Another question which has been only recently partially clarified, and one which is relevant for this paper, is the role played by high-lying configurations in α decay [4]. In the study of α decay of spherical nuclei it was found that the mixing of single-particle levels with opposite parity and the use of a pairing interaction strongly clusters pairs of nucleons [5]. In addition, this produces an enhancement in two-particle transfer cross sections [6] as well as in absolute α-decay width [4]. From a microscopic point of view, the pairing collectivity is induced by many two-particle configurations all contributing about equally and coherently to the two-particle transfer form factor [6]. There are many manifestations of the clustering among nucleons produced in this way; double charge exchange reactions [9] is one of them. Even the neutron-proton clustering in the formation of the α particle might proceed through a high-lying collective pairing state in the nuclear spectrum [7], i.e., a "giant pairing resonance" which, however, would be difficult to observe by means of two-particle transfer probes [8].

Both neutrons and protons are affected in the formation of the α particle. Thus, it was not accidental that just the microscopic study of α decay revealed the role played by pairing excitations on clustering of particle pairs.

The absolute values of α-decay widths increase by many orders of magnitude by including a large enough number of configurations in the calculation of the mother nucleus wave function. This is necessary because of the surface of the nucleus, where the α particle is formed, the continuum part of the single-particle representation (or very high-lying shells in a bound representation) is important. But even including up to 13 major harmonic oscillator (h.o.) shells the absolute decay width is smaller than the experimental one in some spherical nuclei [3,4]. This deficiency was ascribed to a deficient treatment of the continuum [4]. To remedy this a complex representation in terms of outgoing single-particle resonances (Gamow resonances [10]) was introduced. This is being used in other processes, like the study of particle decay of giant resonances [11].

The richness of information provided by microscopic studies of α decay has been mainly restricted to spherical nuclei. In deformed nuclei microscopic treatments have been hindered by the formidable task of computing the mother nucleus was function (including high-lying configurations) in terms of a realistic (e.g., Woods-Saxon) single-particle representation. But with the experience gained in the study of spherical nuclei, it may be time to realize such a treatment. Besides the pure academic value of this task, one can try to learn something new about nuclear processes in deformed nuclei, as was the case in spherical nuclei. In particular, it would be interesting to analyze recent works on anisotropic alpha emission [12–14] from a microscopic point of view, in order to obtain information about the structure of nuclei rotating at high spin [15,16]. This is one of the most interesting questions in discussing alpha decay from deformed nuclei.

In this paper we will describe the alpha-decay process in two steps. In the first step we study within the framework of the shell model the behavior of the four nucleons that eventually constitute the alpha particle. This includes their clustering on the nuclear surface. In the second step we describe the penetration of the already formed α particle through the Coulomb barrier by using the WKB approximation [17].

The formalism is in Sec. II, the applications are in Sec. III, and a summary and the conclusions are in Sec. IV.

II. FORMALISM

We will study the alpha decay of deformed nuclei by assuming that the decay proceeds in two steps. First the four nucleons that eventually constitute the alpha particle are clustered at some point close to the nuclear surface. From here the alpha particle thus formed penetrates the Coulomb barrier. Since these two steps correspond to quite different processes, we will analyze them separately.

A. Alpha-formation amplitude

The analysis of a system consisting of two neutrons and two protons moving in an open and deformed core can conveniently be made in the framework of a quasiparticle and deformed shell-model representation. In our case, that is for the decay process

$$B \rightarrow A + \alpha \ , \tag{2.1}$$

we want to describe the wave function of the mother nucleus B in terms of the wave function of the daughter nucleus A times the two-proton and two-neutron quasiparticle states. It is not obvious that such a description would be adequate. It should describe correctly the α clustering around the nuclear surface. A convenient way of performing the analysis of clustering features of the mother nucleus B is by studying the alpha-formation amplitude, i.e., [18],

$$F_L(\mathbf{R}) = \int d\xi_\alpha d\xi_A [\phi_\alpha(\xi_\alpha)\phi_A(\xi_A)Y_L(\hat{\mathbf{R}})]^*_{J_B M_B} \times \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \ , \tag{2.2}$$

where ξ indicates internal coordinates, B (A) labels the mother (daughter) nucleus, and $\mathbf{r}_i$ is the coordinate of the nucleon i measured from the center of the nucleus B. The rest of the notation is standard. From the intrinsic wave function of the α particle we use the standard Gaussian form [3], i.e.,

$$\phi_\alpha(\xi_\alpha) = \left[\frac{2b}{(1/2)!} \right]^{3/2} \exp\left[-\frac{b}{2}(\xi_1^2+\xi_2^2+\xi_3^2) \right] \times (4\pi)^{-3/2} \chi(12)\chi(34) \ , \tag{2.3}$$

where $\xi_1 = (1/\sqrt{2})(\mathbf{r}_1 - \mathbf{r}_2)$, $\xi_2 = (1/\sqrt{2})(\mathbf{r}_3 - \mathbf{r}_4)$, and $\xi_3 = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4)$ and χ is the spin wave function while $b = 0.574$ fm^{-2}. It was further assumed that the particles 1 and 2 (3 and 4) are protons (neutrons). In the case of alpha decay only the singlet component of the spin function contributes. In fact, in spherical nuclei due to the singlet component of the wave functions particles will cluster [4,5]. This is indeed the connecting point between clustering and alpha decay. By increasing the shell-model space the mother wave function shows that the two neutrons and the two protons moving in a singlet state tend to forms a cluster in a region close to the nuclear surface. This increases the overlap integral [Eq. (2.2)] in that region and therefore the formation amplitude increases. Therefore, from the point of view of the formation amplitude one can say that clustering occurs if $F_L(R)$ increases around $R = R_\alpha + R_A$ as the number of configurations is increased. This is the criterion that we will use to define "clustering."

As in the spherical case [4] we write the wave function of the mother nucleus as

$$\phi_B(\xi_B) = \sum_{\pi\nu} X(\pi\nu; B) \times [\{\phi^\pi_{A+2}(\xi_\pi)\}\{\phi^\nu_{A+2}(\xi_\nu)\}]_B \phi_A(\xi_A) \ , \tag{2.4}$$

where π (ν) labels proton (neutron) degrees of freedom and $\phi_A(\xi_A)$ is the BCS vacuum. We assume axially symmetric nuclei. Therefore the BCS vacuum can be labeled by K_A. For simplicity in all our derivations we will also assume $K=0$ bands, i.e., $K_A = K_B = 0$. The two-quasiparticle wave function in Eq. (2.4) is

$$\phi_{A+2}(\xi) = \sum_{\Omega_1 < \Omega_2} v^{(A+2)}_{\Omega_1} u^{(A)}_{\Omega_2} \mathcal{A}\{\varphi_{\Omega_1}(\mathbf{r}_1)\varphi_{\Omega_2}(\mathbf{r}_2)\} \ , \tag{2.5}$$

where Ω_i labels the single quasiparticle states and $\mathcal{A}$ is the antisymmetrization operator. Below we drop the superscripts, although we assume the form (2.5) in all cases.

The philosophy of the Nilsson+BCS wave function for the description of the intrinsic ground state of heavy nuclei has been already used in the study of α decay from deformed nuclei [19]. We have to stress, however, that the limitations of the Mang-Rasmussen calculations [19] have been removed in our procedure. As discussed in more detail below, we use a realistic mean field with no $\Delta N = 0$ restriction in the diagonalization and a very large shell-model space.

In spite of the fact that the wave function is factorized as

$$|(\mathrm{BCS})_\nu\rangle \otimes |(\mathrm{BCS})_\pi\rangle$$

for the parent and the daughter nucleus, the neutron-proton interaction is taken partially into account by fitting the pairing strength to reproduce the experimental pairing gaps. This is, anyway, a minimum requirement to get an acceptable description of the ground state of our nucleus, as good as the BCS approximation can get. Some attempts [7] have been done to include the n-p interaction explicitly in the calculation of the α-decay width. The correction was found to be very small. One may argue that this may be the case because of the approximations entering in Ref. [7]. We are presently investigating how to include within our formalism the effects of the n-p interaction.

The potential that defines our single-particle representation $\{\varphi_\Omega\}$ has a Woods-Saxon plus spin-orbit form [20,21]. The Woods-Saxon potential is

$$V_{\rm WS}(\mathbf{r},\beta)=\frac{V_0}{1+\exp[\mathrm{dist}\Sigma(\mathbf{r},\beta)/a]}\ , \tag{2.6}$$

where $\mathrm{dist}\Sigma(\mathbf{r},\beta)$ is the distance from the point $\mathbf{r}$ to the nuclear surface Σ, which is calculated numerically. The shape parametrization is given by

$$R(\theta,\beta)=c(\beta)r_0A^{1/3}\left[1+\sum_\lambda \beta_\lambda Y_{\lambda 0}(\theta)\right]\ ,$$

where β denotes the set of deformation parameters and $c(\beta)$ is calculated to preserve the constant volume enclosed by the surface Σ. The other parameters in Eq. (2.6) are as in Ref. [21], i.e., $V_0=-49.6[1\pm 0.86(N-Z)/(N+Z)]$ MeV, where the + (−) sign holds for protons (neutrons), $a=0.7$ fm, and $r_0=1.275$ (1.347) fm for protons (neutrons).

The spin-orbit potential has the form

$$V_{\rm s.o.}(\mathbf{r},\beta)=\lambda_{\rm s.o.}\left[\frac{\hbar}{2mc}\right]^2[\Delta V_{\rm WS}(\mathbf{r},\beta)]\times\mathbf{p}\cdot\boldsymbol{\sigma}\ , \tag{2.7}$$

where m is the nucleon mass and $\lambda_{\rm s.o.}$ the coupling constant. The values of r_0 and a in $V_{\rm WS}$ are replaced with $r_{\rm s.o.}$ and $a_{\rm s.o.}$. In the applications below we use $r_{\rm s.o.}=1.32$ fm, $a_{\rm s.o.}=0.7$ fm, and $\lambda_{\rm s.o.}=36.0$ for both neutrons and protons.

The Coulomb potential has the form of an uniformly charged system with $(Z-1)$ protons enclosed by the surface Σ of Eq. (2.6). Details of the diagonalization procedure can be found in Ref. [22].

We expand the deformed single-particle wave function in a spherical harmonic oscillator (h.o.) basis with size parameter as the one in the intrinsic wave function of the α particle, Eq. (2.3). This allows us to perform all integrals analytically. Such an expansion requires a large number of h.o. shells because of the large size difference between the alpha particle and the deformed heavy nucleus. But the calculation is feasible, as discussed in the next section.

The single-particle deformed wave function is then expanded as

$$\varphi_\Omega(\mathbf{r})=\sum_{nljm} c(nlj,\Omega)\phi_{nljm}(\mathbf{r})\ , \tag{2.8}$$

where ϕ is the h.o. wave function, i.e.,

$$\phi_{nljm}(\mathbf{r})=R_{nljm}(r)[Y_l(\hat{r})\chi_{1/2}]_{jm}\ . \tag{2.9}$$

Only the singlet part of the two-quasiparticle wave functions in Eq. (2.3) contributes to the α-formation amplitude. Expanding in relative coordinates one then gets

$$\phi_{A+2}(\xi)=\sum_{nlNLJ_{12}} G(nlNL;J_{12})\times[\phi_{nl}(\mathbf{r})\phi_{NL}(\mathbf{R})]_{J_{12}0}(\chi_{1/2}\chi_{1/2})_{00}\ , \tag{2.10}$$

where $\mathbf{r}=(1/\sqrt{2})(\mathbf{r}_1-\mathbf{r}_2)$ [i.e., ξ_1 or ξ_2 in (2.3)], and $\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$ with corresponding quantum numbers nl and NL. The G-expansion coefficients are, with standard notation,

$$G(nlNL;J_{12})=\sum_{n_1l_1j_1}\sum_{n_2l_2j_2} B(n_1l_1j_1,n_2l_2j_2;J_{12})\times\langle((l_1\tfrac{1}{2})j_1(l_2\tfrac{1}{2})j_2;J_{12}|(l_1l_2)J_{12}(\tfrac{1}{2}\tfrac{1}{2})_0;J_{12}\rangle\langle nlNL;J_{12}|n_1l_1n_2l_2;J_{12}\rangle\ , \tag{2.11}$$

where

$$B(n_1l_1j_1,n_2l_2j_2;J_{12})=\sum_{m\Omega>0} v_\Omega u_\Omega c(n_1l_1j_1;\Omega)c(n_2l_2j_2;\Omega)\langle j_1mj_2-m|J_{12}0\rangle\ . \tag{2.12}$$

In Eq. (2.12) we assumed $K_\nu=K_\pi=0$, i.e., $\Omega_1=-\Omega_2=\Omega$ and the c amplitudes from Eq. (2.8). The penetration of the α particle through the centrifugal barrier is strongly hindered as the orbital angular momentum L increases. The α widths for $L=0$, as well as for $L\neq 0$ transitions, have been calculated. The results will be reported in the next section.

Within the BCS approximation the sum in (2.4) contains only one term, namely, π (ν) labels the proton (neutron) two-quasiparticle state and $X(\pi\nu;B)=1$. The formation amplitude then becomes

$$F_0(\mathbf{R})=\sum_{N_\nu L_\nu N_\pi L_\pi}\sum_{N_\alpha L_\alpha} G_\nu(00N_\nu L_\nu;L_\nu)G_\pi(00N_\pi L_\pi;L_\pi)\langle L_\nu 0L_\pi 0|l_\alpha\rangle\rangle\langle N_\nu L_\nu N_\pi L_\pi;L_\alpha|00N_\alpha L_\alpha;L_\alpha\rangle\phi_{N_\alpha L_\alpha 0}(\mathbf{R})\ , \tag{2.13}$$

where α labels the quantum numbers of the α particle, G is as in Eq. (2.11) and ϕ as in Eq. (2.9). All the quantum numbers in Eq. (2.13) are determined by the set of single-particle states. In the next section we will analyze the dependence of F_0 on the number of states includes in the single-particle basis.

B. Barrier penetration and alpha-decay width

We will use the WKB formulation of Ref. [17] (see also Ref. [23]) to describe the penetration of the alpha particle through the Coulomb barrier. In this formulation one calculates the relative wave function of the α particle and the daughter nucleus A at large distances with outgoing boundary conditions. This wave function is uniquely determined, except for an arbitrary constant factor. To adjust this factor it is assumed that the solution inside and outside the nuclear region overlap on a sphere of radius R_1. The value of R_1 is chosen such that the spherical surface lies approximately in the middle of the deformed potential barrier. At this point one matches the wave function in the outside region with the one in the interior, which is assumed to be known. In Ref. [24] this procedure was critized because it requires that the α particle has to go through the whole barrier, while in fact it has a probability of starting the penetration from any point within the barrier. From a point P_0 outside R_1 the penetration is much easier than for a point P_i inside R_1 due to the shorter distance from P_o to the end of the barrier and also to the lower value of the potential at P_o. Although this criticism may be valid, a true check of the validity of the WKB approximation would require a full calculation of the whole α-decay process, a task which is far beyond the scope of this paper. A more sensitive probe would probably be the case of α decay from odd-deformed nuclei. Such a case is not discussed in the present paper. Here we are mainly interested in exploring the relation between the α-decay process and the formation of the α particle in terms of microscopic degrees of freedom. This may eventually even allow one to apply the expressions for the formation amplitudes developed above to the formulation of Ref. [24]. Actually, the formation amplitude F_L in Eq. (2.2) is the shell-model wave function corresponding to the relative motion of the α particle with respect to the daughter nucleus A. Therefore, in principle, F_L should describe the motion of nucleons that constitutes the α particle in the interior as well as in the exterior nuclear region. But to describe the motion of the four nucleons in the exterior region one would need a proper inclusion of the continuum. This is a complex problem which is common to other fields as well, e.g., the particle decay of giant resonances [11]. In the present paper we will use a large enough number of high-lying single-particle shells to describe the α-particle wave function F_L around the surface of the daughter nucleus. This already requires a large basis, as discussed in the next section. With F_L thus calculated we matched the outgoing (α particle$+A$) solution of the Schrödinger equation given by the WKB approximations with F_L at a point $\mathbf{R}=(R,\theta,\varphi)$. Applying the formalism of Refs. [17,23] one finds the absolute value of the decay width to be given by

$$\Gamma(R)=\frac{\hbar}{T_{1/2}(R)}=\hbar v\left[\frac{R}{G_0(E,R)}\right]^2\sum_l \exp\left[-\frac{2l(l+l)}{\chi}\left[\frac{\chi}{kR}-1\right]^{1/2}\right]$$
$$\times\left|\sum_\Omega(-1)^\Omega\langle I_iK_il-\Omega|I_fK_f\rangle\sum_{l'}K^\Omega_{ll'}(B)a_{l'\Omega}(R)\right|^2 , \qquad (2.14)$$

where k is the wave number and $\hbar v=\hbar c\sqrt{2E/\mu}$, E is the α-particle kinetic energy in MeV, and μ is the reduced mass. The dimensionless quantity χ is $\chi=e^2 4(Z-2)/\hbar v$. In this formalism [17,23] the quadrupole deformation is separated from the rest. The quadrupole contribution is given by the matrix K, i.e.,

$$K^\Omega_{ll'}(B)=\int_0^\pi\Theta_{l\Omega}(\vartheta)\exp\{BP_2(\cos\vartheta)\}\Theta_{l'\Omega}(\vartheta)\sin\vartheta\, d\vartheta , \qquad (2.15a)$$

where $\Theta_{l\Omega}(\vartheta)$ is the normalized ϑ-dependent function in the spherical harmonic $Y_{l\Omega}$ and P_2 is the quadrupole Legendre polynomial, while

$$B=\chi\beta_2\left[\frac{5}{4\pi}\frac{kR}{\chi}\left[1-\frac{kR}{\chi}\right]\right]^{1/2}\left[1-\frac{1}{5}q_0-\frac{2}{5}q_0\frac{kR}{\chi}\right] . \qquad (2.15b)$$

Here q_0 is a dimensionless quantity which depends on the charge distribution of the nucleus. By assuming the nucleus to have ellipsoidal shape and uniform charge density q_0 is equal to 1. The contribution of the other values of β (β_λ with $\lambda\neq 2$) is given by the matrix $a_{l\Omega}$ in Eq. (2.14). It is

$$a_{l\Omega}(R)=\int_0^{2\pi}d\varphi\int_0^\pi\sin\vartheta\, d\vartheta\, Y^*_{l\Omega}(\vartheta\varphi)\Psi_1(R\vartheta\varphi) \qquad (2.16a)$$

with

$$\Psi_1(R\vartheta\varphi)=F_0(R\vartheta\varphi)\exp\left[\chi\sum_{\substack{\lambda>0\\ \lambda\neq 2}}\beta_\lambda\left\{\left[\frac{R}{r_0}\left[1-\frac{R}{r_0}\right]\right]^{1/2}-f_\lambda\right\}Y_{\lambda 0}(\vartheta)\right] , \qquad (2.16b)$$

where $F_0(\mathbf{R})$ is the formation amplitude (2.13), $r_0=\chi/k$, and

$$f_\lambda=\frac{1.5}{2\lambda+1}\left[\frac{R}{r_0}\right]^\lambda \times\sum_{m=0}^{\lambda-1}\frac{(\lambda-1)!}{m!(\lambda-1-m)!}\frac{\left[\frac{r_0}{R}-1\right]^{m+1/2}}{(m+\frac{1}{2})} . \quad (2.16c)$$

The expression for the decay width $\Gamma(R)$ thus obtained may be strongly dependent upon the distance R. This actually provides a test of the reliability of the formalism. If Γ is indeed strongly dependent upon R on the nuclear surface (where we assumed the validity of the shell model as well as of the semiclassical description) then the theory is incorrect. This will be an important theme in the applications below.

The angular distribution of alpha emission is also given by (2.14) but without integrating on the angle ϑ in Eq. (2.16a).

III. APPLICATIONS TO Ra, Rn, AND Th ISOTOPES

In this section we will apply the formalism described above to analyze α decay of deformed nuclei and, specifically, to Ra, Rn, and Th isotopes. The single-particle states for these nuclei have been calculated using the deformation parameters [25] of Table I and the deformed WS potential [22] of Eqs. (2.6) and (2.7). We expanded the corresponding single-particle wave functions in terms of a h.o. basis with size parameter b as the one corresponding to the α particle in Eq. (2.3). The b value used corresponds to the size parameter for α particle according to the experimentally measured rms radius. Therefore this number comes from independent nuclear structure calculations and cannot be considered as a parameter in our calculation. This choice, in the expansion in terms of h.o. single-particle wave functions, is only a matter of convenience from the mathematical point of view. A small variation, in Eq. (2.3), around this value does not change the calculated width in an appreciable way. A drastic reduction (i.e., a factor of 2) of the b value used would increase the width: this would be a totally unphysical effect. With such a value of b one may think that the convergence of the expansion of the radial wave function would be very poor. Using $N=18$ shells in this expansion, this happens to be indeed the case for $r>12$ fm, which corresponds to a value behind the matching point with the Coulomb wave function. Therefore, our truncation should not dramatically affect the calculated widths. A preliminary Brief Report about our microscopic approach has already been published [26].

TABLE I. Deformation parameters β_λ used in the alpha width calculation.

X	A	β_2	β_3
Rn	222	0.116	0.094
Rn	220	0.102	0.096
Rn	218	0.085	0.092
Ra	226	0.139	0.101
Ra	224	0.128	0.105
Ra	222	0.119	0.095
Th	232	0.250	0.0

A. Some features of the formation amplitude

The clustering of four nucleons that eventually constitute the alpha particle is reflected in the formation amplitude $F_0(R)$. The more the clustering features are pronounced in the mother nucleus wave function the larger the value of F_0, as can be seen from Eq. (2.2), and the better the assumptions of our formulation will be fulfilled. In what follows most of the calculations refer for simplicity to the ^{222}Ra case. In the other considered nuclei the results are very similar. The formation amplitude in Eq. (2.13) can be written as

$$F_0(\mathbf{R})=\sum_{N_\alpha L_\alpha} W_{N_\alpha L_\alpha}\phi_{N_\alpha L_\alpha}(\mathbf{R}) , \quad (3.1)$$

where α labels the quantum numbers of the α particle and the W coefficients can be obtained comparing with Eq. (2.13). They are thus expressed in terms of the transformation coefficients from the individual nucleon coordinates to center-of-mass and relative coordinates (including the BCS occupation amplitudes and the corresponding coefficients of the expansion in the spherical basis).

One finds the following results.

(i) The component $L_\alpha=0$ in Eq. (3.1) is much larger than the other components, as can be seen in Fig. 1, where the formation amplitudes, corresponding to $L_\alpha=0,1,2$ and summed over N_α, are plotted as a function of the distance. In alpha-decay calculations one usually assumes the component $L_\alpha=0$ to be dominant because the penetration of the alpha particle through the centrifugal barrier is hindered by the decay angular momentum. What we show here is that even the formation amplitude of the alpha particle is the nuclear surface proceeds mainly through s channels, even if the remaining $L_\alpha\neq 0$ cannot be neglected. The calculated value for the $L_\alpha=0$ component is about 4.2×10^{-4} fm$^{-3/2}$. In comparing this value with the phenomenological model, one should keep in mind that this is a result of a microscopically calculated formation amplitude. We do not attempt to extract spectroscopic factors, because its meaning within a microscopic model would not be so clear as in a cluster model.

(ii) All the terms in the sum (3.1) contribute coherently to the formation amplitude in the region outside the nuclear surface, as shown in Fig. 2. This effect is responsible for the strong enhancement of the calculated absolute decay width as the number of configurations in the nuclear wave function is increased. The physical meaning of this enhancement is that the alpha particle is formed outside the nuclear surface. As seen from Eq. (2.2), an in-

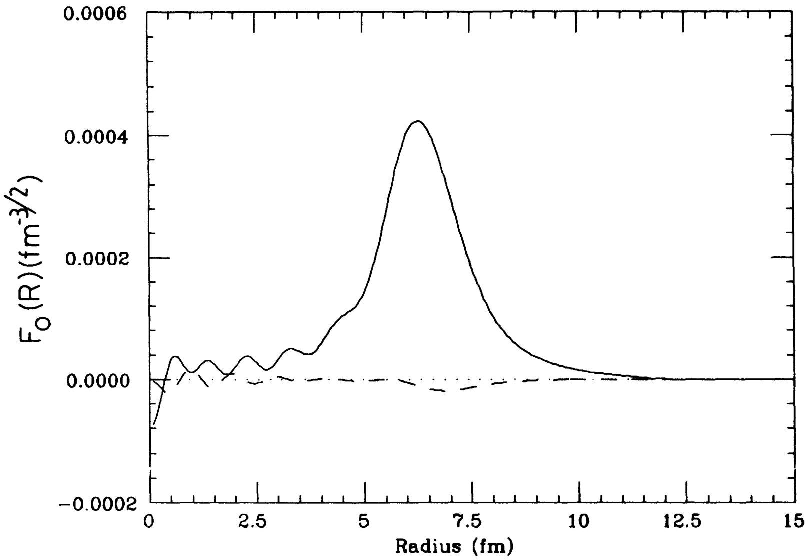

FIG. 1. Alpha formation amplitude (in $fm^{-3/2}$), summed over N_α, for $L_\alpha=0$ (solid line), $L_\alpha=1$ (dashed line), and $L_\alpha=2$ (dotted line).

crease in the formation amplitude implies a better overlap between the mother nucleus wave function and the fragments produced in the decay. Since this occurs at large distances, the continuum part of the single-particle representation should become important and, therefore, a large configuration space is required to describe clustering processes.

(iii) The coefficients $W_{N_\alpha L_\alpha}$ are also strongly dependent upon N_α with the maximum centered around $N_\alpha=10$, as seen in Fig. 3 for the cases $L_\alpha=0,2,4$. This means that the largest W coefficient correspond to the N values for which, in a pure cluster approximation, all the relative motions, for the four nucleons constituting the α particle, would be taken as 0*s*. But it should be noticed that the

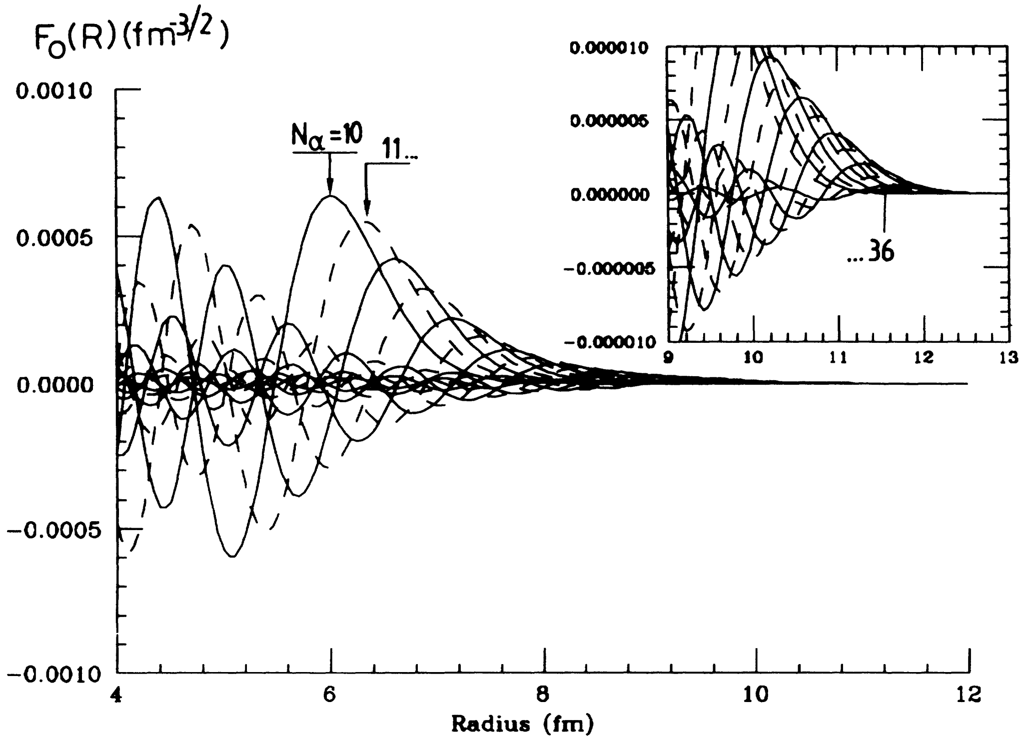

FIG. 2. Some of the contributions to the total formation amplitude (in $fm^{-3/2}$) for large N_α values and $L_\alpha=0$. The coherence in the tail region can be clearly seen. The contributions with N_α ranging from 28 to 36.

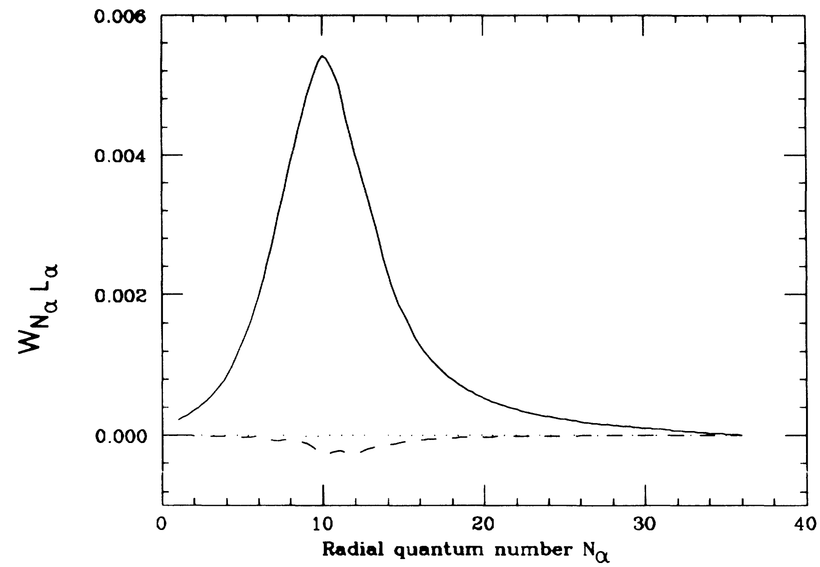

FIG. 3. $W_{N_\alpha L_\alpha}$ coefficients for $L_\alpha=0$ (solid line), $L_\alpha=2$ (dashed line), and $L_\alpha=4$ (dotted line). The largest contribution comes from $N_\alpha=10$, which would correspond to a pure cluster approximation with all the relative motions in a 0s state.

contribution of N_α is the range 28–36 (see Fig. 2) plays a crucial role. As mentioned above, they determine the behavior of the formation amplitude at large distances, where the four nucleons that eventually constitute the alpha particle will cluster. In this region the Pauli principle acting upon the nucleons in the alpha particle and those in the daughter nucleus should not play a crucial role.

(iv) The formation amplitude shows an intrinsic dependence on the deformation parameter which could be used as evidence of anisotropy in α decay. As discussed in the Introduction, some experimental evidence has recently been published [12,13]. It has been found [12,13] in the case of well-deformed odd At isotopes that when the neutron number decreases the alpha particles are more preferentially emitted perpendicularly to the nuclear spin direction. This fact has been interpreted in terms of the high sensitivity of the α emission probability to changes in the nuclear shapes. Our calculation has been limited to $K=0$ transitions in even-even nuclei. Therefore, we can only make a very general type of prediction about the β_λ dependence in the formation amplitude. Just assuming only quadrupole deformation we found, at the matching point, that the square of the ratio of the formation amplitude at 0° and 90° ranges from 1.05, at the realistic $\beta_2=0.119$ value for ^{222}Ra, to 1.15 for the rather large $\beta_2=0.4$ value.

(v) As far as the radial dependence of the formation amplitude is concerned, one should stress the large enhancement around the surface of the system. The square of the formation amplitude versus R is reported in Fig. 4, comparing the results obtained using only quadrupole deformation (solid line) with the quadrupole+octupole case (dashed line). This difference will produce an increase of about 30% in the total width.

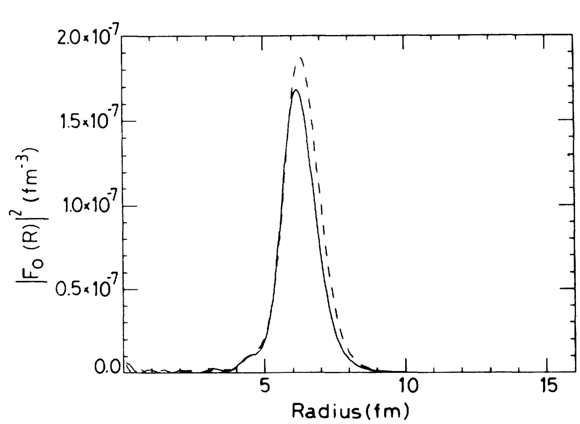

FIG. 4. Square of the alpha-formation amplitude (in fm^{-3}) vs R. The solid line corresponds to the inclusion of quadrupole deformation; the dashed line to the quadrupole+octupole case.

B. The α widths

A systematic analysis has been performed in the Rn, Ra, and Th isotopes. For all those nuclei known deformation parameters have been used [22,24]. The pairing strengths have been adjusted to reproduce the experimental pairing gaps. In all considered cases $N=18$ major shells have been used. The calculated total widths are reported in Table II. The agreement with the experimental data is quite remarkable. The dramatic reduction of the width is well reproduced and the absolute values agree with data within a factor ranging from 1.7 (in ^{232}Th) to 5.2 (in ^{218}Rn).

Transitions to $I_f\neq 0$ states have also been calculated. Our relative widths are compared with the relative probabilities (from the branching ratios) for $I_f=2$ in Table III. The agreement with the experimental data is as good as it was for the ground to ground transitions. The experimental data are taken from the compilation in Ref. [24]

TABLE II. Calculated α-decay widths $\Gamma(R)$ at the matching point. Comparison with experimental data is shown.

X	A	Γ_{exp}	Γ_{th}
Rn	222	1.38×10^{-27}	0.70×10^{-27}
Rn	220	8.2×10^{-24}	2.7×10^{-24}
Rn	218	1.30×10^{-20}	0.25×10^{-20}
Ra	226	0.90×10^{-32}	0.66×10^{-32}
Ra	224	1.44×10^{-27}	0.83×10^{-27}
Ra	222	1.20×10^{-23}	0.52×10^{-23}
Th	232	1.0×10^{-39}	0.60×10^{-39}

TABLE III. Experimental and calculated relative probabilities (R_{exp} and R_{th}, respectively) for transitions to $I_f\neq0$. The comparison with available data shows the same type of agreement as for the ground to ground transitions (in Table II).

X	A	I_f	R_{th}	R_{exp}
Rn	222	2	4.0×10^{-4}	7.9×10^{-4}
Rn	220	2	4.8×10^{-4}	7.1×10^{-4}
Rn	218	2	0.8×10^{-3}	2.0×10^{-3}
Ra	226	2	2.3×10^{-2}	5.7×10^{-2}
Ra	224	2	1.9×10^{-2}	5.5×10^{-2}
Ra	222	2	1.2×10^{-2}	4.1×10^{-2}
Th	232	2	0.25	0.29

and from the Table of Isotopes [27].

An important comment has to be made: the question of whether the value of $\Gamma(R)$ is strongly dependent upon R, in which case one could choose R conveniently to fit the experimental data (thus making the calculation irrelevant). We found that the calculated value of the absolute decay width, Γ^{th}, is practically constant within the large interval of 1 fm behind the touching points of the α particle and the daughter nucleus (note that $R_\alpha+R_A\approx9.6$ fm). This value could seem too large with respect to the nuclear surface, but one should, indeed, recall two points: (a) the location of the Coulomb barrier for our Rn, Ra, and Th isotopes is around 11 fm; and (b) according to the Fröman method, which has been used to solve the penetration problem through the barrier, the matching point should be just in the middle of the Coulomb barrier.

IV. SUMMARY AND CONCLUSIONS

We have presented in this paper a realistic microscopic approach to the calculation of the formation amplitude for the alpha-decay problem in axially symmetric deformed nuclei, within the well-known approach by Mang and Rasmussen [19]. It is worthwhile to stress the main ingredients of the calculations: (i) the use of a realistic deformed mean field, (ii) the large shell-model space, (iii) exact diagonalization of the deformed mean field, and (iv) no cut (within the selected large basis) in the calculation of the alpha formation amplitude. Within our semiclassical approach the antisymmetrization between nucleons in the alpha particle and in the daughter nucleus have been neglected. We plan to analyze this complicated question in the future. We also contemplate the possibility of generalizing the calculations to odd nuclei.

The predicted absolute values of the alpha total widths are smaller by a factor of about 2 with respect to the experimental widths. This may indicate that our treatment of the continuum is still deficient, as it seems to happen in spherical nuclei [4]. It is also possible that the treatment of the barrier penetration used here is responsible, at least partially, for this deficiency [10,16]. An open question is the role of quarteting of nucleons which should increase the formation amplitude. We are planning to analyze some possibilities to take into account these type of correlations. Finally, one may think that the inclusion of the other deformations (with multipolarities larger than 3) would improve the agreement between theory and experiment.

We have not attempted in this paper a comparison with other, more heuristic, spherical models [28–31]. Within those models the agreement with experimental data is of the same order as we get or even better. According to our view this comparison cannot be done by just comparing a single piece of the ingredients of a microscopic calculation, but using the global results and the understanding of the process: availability of experimental branching ratios of the 4^+ states, description of α anisotropy in odd deformed nuclei, consistent microscopic description of the α scattering from deformed nuclei, and so on. Such a planned work on the subject is not yet fully accomplished.

The main question in starting this project was: Do we need to understand α decay on more microscopic levels than current heuristic models for nuclear decays? Our answer is: Yes. The comparison between the two approaches is useful and important, but it does not seem to us to be necessary to justify a microscopic approach to the problem. When the microscopic approach is fully tested with respect to the available experimental data a better level of understanding will arise from comparison with other models [28–31].

[1] T. Fliessback, H. J. Mang, and J. O. Rasmussen, Phys. Rev. C **13**, 1318 (1976).

[2] D. F. Jackson and M. Rhoades-Brown, J. Phys. G **4**, 1441 (1978).

[3] A. Arima and I. Tonozuka, Nucl. Phys. **A323**, 45 (1979).

[4] G. Dodig-Crnkovic, F. A. Janouch, and R. J. Liotta, Nucl. Phys. **A501**, 533 (1989); A. Insolia, R. J. Liotta, and E. Maglione, Europhys. Lett. **7**, 209 (1988).

[5] F. A. Janouch and R. J. Liotta, Phys. Rev. C **27**, 896 (1983); F. Catara, A. Insolia, E. Maglione, and A. Vitturi, *ibid.* **29**, 1091 (1984); W. T. Pinkston, *ibid.* **29**, 1123 (1984); A. Insolia, R. J. Liotta, and E. Maglione, J. Phys. G **15**, 1249 (1989).

[6] M. W. Herzog, O. Civitarese, L. Ferreira, R. J. Liotta, T. Vertse, and L. J. Sibanda, Nucl. Phys. **A448**, 441 (1986).

[7] G. Dodig-Crnkovic, F. A. Janouch, R. J. Liotta, and L. J.

Sibanda, Nucl. Phys. **A444**, 419 (1985); G. Dodig-Crnkovic *et al.*, Phys. Scr. **37**, 523 (1988); G. Dodik-Crnkovic *et al.*, Phys. Lett. **139B**, 143 (1984).
[8] C. H. Dasso and R. J. Liotta, Phys. Rev. C **36**, 448 (1987).
[9] D. R. Bes, O. Dragun, and E. E. Maqueda, Nucl. Phys. **405**, 313 (1983).
[10] T. Berggren, Nucl. Phys. **A109**, 265 (1968).
[11] P. Curutchet, R. J. Liotta, and T. Vertse, Phys. Rev. C **39**, 1020 (1989).
[12] J. Wouters, D. Vandenplassche, E. van Walle, N. Severijns, and L. Vanneste, Phys. Rev. Lett. **56**, 1901 (1986).
[13] J. Wouters, D. Vandenplassche, E. van Walle, N. Severijns, and L. Vanneste, Nucl. Instrum. Methods **B26**, 463 (1987).
[14] T. Berggren, Phys. Lett. B **197**, 1 (1987).
[15] T. Berggren, Hyperfine Interact. **43**, 407 (1988).
[16] F. A. Dilmanian, D. G. Sarantites, M. Jaaskelainen, H. Puchta, R. Woodward, J. R. Beene, D. C. Hensley, M. L. Halbert, R. Novotny, L. Adler, R. Chawdhury, M. N. Namboodiri, R. P. Schmitt, and J. B. Natowitz, Phys. Rev. Lett. **49**, 1909 (1982).
[17] P. O. Fröman, Mat. Fys. Skr. Dan. Vidensk. Selsk. **1**, no. 3 (1957).
[18] R. G. Thomas, Prog. Theor. Phys. **12**, 253 (1954).
[19] H. J. Mang, Annu. Rev. Nucl. Sci. **14**, 1 (1964); H. J. Mang and J. O. Rasmussen, Mat. Fys. Skr. Dan Vidensk. Selsk. **2**, no. 3 (1962); J. K. Poggenburg, H. J. Mang, and J. O. Rasmussen, Phys. Rev. **181**, 1697 (1969).
[20] J. Dudek, A. Majhofer, J. Skalski, T. Werner, S. Cwiok, and W. Nazarewicz, J. Phys. G **5**, 1359 (1979).
[21] R. Bengtsson, J. Dudek, W. Nazarewicz, and P. Olanders, Phys. Scr. **39**, 196 (1989).
[22] S. Cwiok, J. Dudek, W. Nazarewicz, J. Skalski, and T. Werner, Comput. Phys. Commun. **46**, 379 (1987).
[23] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2.
[24] T. Berggren and P. Olanders, Nucl Phys. **A473**, 189 (1987); **A473**, 221 (1987).
[25] W. Nazarewicz, private communication.
[26] A. Insolia, P. Curutchet, R. J. Liotta, and D. S. Delion, Phys. Rev. C **44**, 545 (1991).
[27] *Table of Isotopes,* 7th ed., edited by C. M. Lederer and V. S. Shirley (Wiley-Interscience, New York, 1979).
[28] R. Blendowske, T. Fliessbach, and H. Walliser, Z. Phys. A **339**, 121 (1991); R. Blendowske and H. Walliser, Phys. Rev. Lett. **61**, 1930 (1988).
[29] D. N. Poenaru, W. Greiner, K. Depta, M. Ivascu, D. Mazilu, and A. Sandulescu, At. Data Nucl. Data Tables. **34**, 423 (1986); D. N. Poenaru, D. Schnabel, W. Greiner, D. Mazilu, and I. Cata, *ibid.* **48**, 231 (1991).
[30] P. B. Price, J. D. Stefenson, S. W. Barwick, and H. L. Ravn, Phys. Rev. Lett. **54**, 297 (1985); P. B. Price, Annu. Rev. Part. Sci. **39**, 19 (1989).
[31] B. Buck, A. C. Merchant, and S. M. Perez, Phys. Rev. Lett. **65**, 2975 (1990); J. Phys. G **17**, 1223 (1991).