Is there objective evidence for quantized spin alignment in superdeformed nuclei?

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(Received 27 March 1992)

It is impossible to determine spins for superdeformed bands by fitting the γ-transition energies alone. It is necessary to introduce a model. By using a variable moment of inertia (VMI) model one can estimate spins but cannot rule out the possibility of a $\pm\hbar$ uncertainty. Therefore, no objective evidence exists requiring the presence of quantized spin alignment in superdeformed nuclei. An alternative hypothesis that all superdeformed nuclei in a given mass region have the same VMI core, and thus essentially no alignment until high-N Coriolis effects become significant, seems to explain the present data.

PACS number(s): 21.60.Ev, 21.10.Re, 21.10.Hw

I. INTRODUCTION

It has been claimed that mass-190 superdeformed nuclei exhibit a *quantized spin alignment* over a wide range of frequencies ω [1,2]. If correct, this result would suggest qualitatively new phenomena not easily explained in the standard theories of nuclear structure. In a published Comment we have presented evidence that the current data do not sustain this conclusion regarding quantized spin alignment [3]. In this paper we would like to discuss this problem in a model-independent way and define more precisely what the current data can and cannot tell us regarding this question.

The conclusion of quantized alignment relies crucially on the assignment of spins for superdeformed bands, but spins for these states are not yet measured quantities. The authors of [1] have proposed a way to determine the spin I for the current experimental situation in which only the transition energies E_γ are known. They first fitted the moment of inertia $\mathcal{J}^{(2)}(\omega)$ by the Harris expansion

$$\mathcal{J}^{(2)}(\omega)=2\alpha+4\beta\omega^2+6\gamma\omega^4 , \tag{1}$$

and then by integration of $dJ_x=\mathcal{J}^{(2)}(\omega)d\omega$ obtained the expression

$$I+\tfrac{1}{2}=2\alpha\omega+\tfrac{4}{3}\beta\omega^3+\tfrac{6}{5}\gamma\omega^5 \tag{2}$$

by assuming $I_x=[I(I+1)]^{1/2}\simeq I+\frac{1}{2}$. However, this procedure is subject to uncertainties associated with untested assumptions such as the arbitrary assignment of an integration constant and the extrapolation required for $\mathcal{J}^{(2)}(\omega)$, as we have pointed out in the previous Comment [3].

II. UNCERTAINTIES IN SPIN DETERMINATION

Let us now ask a general question: is it possible, even in principle, to determine spin in a model-independent manner from fitting $E2$ γ-transition energies alone (for those are the only data available)? The answer is no. Although this should be obvious on logical grounds, there is appreciable confusion on this issue and we now explain in more detail.

A. Expansion in angular momentum

The impossibility can be seen clearly if one makes a general series expansion for $E_\gamma(I)$ around the initial spin I_1 associated with the lowest observed transition energy,

$$E_\gamma(I)=a+b(I-I_1)+c(I-I_1)^2+d(I-I_1)^3+\cdots , \tag{3}$$

which only requires $E_\gamma(I)$ to be a smooth function of spin I (this generally is true in the mercury superdeformed region). By fitting the E_γ's the coefficients $a,b,c,d,\ldots$ can be determined since $I-I_1$ takes the values $0,2,4,\ldots$, if one assumes stretched $E2$ transitions. For the mass-190 superdeformed bands the expansion through quadratic terms gives a good fit, as shown in Table I. However, such a fit cannot determine I_1 and thus can never determine exit spin ($I_{\rm ext}=I_1-2$), since Eq. (3) does not depend on spin but on the *differences* $(I-I_1)$. Thus, any exit spin assignment, set (a) of Table I (which leads to quantized alignment), set (b) [which has the same fitting error as (a) but leads to no alignment], or others are equally plausible mathematically. In general, $I_{\rm ext}$ cannot be determined because the number of unknown variables (I_1 plus $a,b,c,d,\ldots$) is always one more than the number of available equations.

The only way to obtain absolute spin information from fitting the transition energies alone is to use a model adding at least one more relation among these unknowns. It should be obvious that such a spin determination depends entirely on the reliability of the model assumptions and has nothing to do with the quality (χ^2) of the corresponding fit, because the fit to Eq. (3) is indifferent to the model-dependent equation chosen to give the additional constraint.

TABLE I. Exit spin assignment.[a]

	(a) (Quantized alignment)		(b) (No alignment)		Fitting ($E_\gamma = a + b\Delta I + c\Delta I^2$, ~ ~ $\Delta I = I - I_1 = 0,2,4,\ldots$)			
Nucl.	I_{ext} ($\hbar$)	$i(\omega \approx 0.2)$ ($\hbar$)	I_{ext} ($\hbar$)	$i(\omega \approx 0.2)$ ($\hbar$)	a (keV)	b (keV $\hbar^{-1}$)	c (keV $\hbar^{-2}$)	rms error ($\hbar$ MeV^{-1})
^{192}Hg	8	ref	8	ref	214.5	21.88	−0.126	0.43
^{194}Hg($b2$)	8	0.96	7	−0.04	201.1	20.99	−0.088	0.99
^{194}Hg($b3$)	11	0.97	10	−0.03	262.1	20.55	−0.095	1.30
^{191}Hg($b2$)	12.5	0.99	11.5	−0.01	292.1	20.45	−0.087	0.90
^{191}Hg($b3$)	13.5	1.05	12.5	0.05	311.7	20.10	−0.094	1.00
^{193}Hg($b2$)	10.5	0.92	9.5	−0.08	254.0	20.56	−0.089	1.30
^{193}Hg($b3$)	9.5	0.92	8.5	−0.08	233.6	20.62	−0.089	1.20
^{194}Pb($b1$)	6	0.06	6	0.06	169.5	22.17	−0.111	0.98
^{194}Tl($b1$)	12	1.91	10	−0.09	268.1	19.51	−0.053	2.20
^{191}Hg($b1$)	14.5	0.03	14.5	0.03	350.3	20.46	−0.096	0.82
^{194}Hg($b1$)	10	0.16	10	0.16	254.3	21.32	−0.126	0.55
^{193}Hg($b1$)	7.5	1.0	6.5	0.0	191.8	21.23	−0.128	2.70
^{193}Hg($b4$)	13.5	1.8	11.5	−0.2	290.1	18.86	0.057	3.20

[a]In column (a), I_{ext} is the exit spin assigned in [1,2] for 13 superdeformed bands reported in the $A \sim 190$ region. These assignments lead to the conclusion of quantized alignment; i is the spin alignment relative to ^{192}Hg. In column (b), I_{ext} is one of the possible exit spin assignments for the same set of bands which leads to essentially no alignment ($i \approx 0$). In the column marked Fitting, the parameters a,b,c, of the E_γ expansion [see Eq. (3) in the text] obtained by fitting the first nine E_γ values of each band are given, as are the corresponding rms errors for $\mathcal{J}^{(2)}$. The fitting errors are comparable with those given in [4]. However, as explained in the text, the fitting does not depend on the exit spin, so quality of fit cannot ensure a correct spin assignment.

B. Expansion in frequency

The impossibility of determining spins obviously remains if the expansion employed to fit data is changed to I versus ω, as in Ref. [1]:

$$I = A' + B'\omega + C'\omega^2 + D'\omega^3 + E'\omega^4 + F'\omega^5 + \cdots \quad (4)$$

and

$$I - I_1 = B'(\omega - \omega_1) + C'(\omega^2 - \omega_1^2) + D'(\omega^3 - \omega_1^3) + E'(\omega^4 - \omega_1^4) + F'(\omega^5 - \omega_1^5) + \cdots . \quad (5)$$

By fitting E_γ (or ω) one can only determine the coefficients $B', C', D', \ldots$, and not A'. Thus the I's cannot be determined unless a particular value of A' is *assumed.* The expansion in Ref. [1] corresponds to choosing $A' = -\frac{1}{2}$ and neglecting even-power terms: $C' = E' = \cdots = 0$. The uncertainties of this approach originate in the uncertainties of these assumptions.

C. The rotor model

In a recent publication [4], the rotor model has been used to derive the expansion equation (4) by taking

$$E = \frac{I(I+1)}{2\mathcal{J}}, \quad E_\gamma(I) = \frac{2I-1}{\mathcal{J}} , \quad (6)$$

with the moment of inertia $\mathcal{J} = \mathcal{J}(\omega)$ assumed to vary slowly with I. Then by expanding $\mathcal{J}$ with respect to the midterm frequency

$$\omega = \frac{E_\gamma(I+2) + E_\gamma(I)}{4} , \quad (7)$$

one can obtain

$$I + \tfrac{1}{2} = \omega \mathcal{J}(\omega) = \omega(B' + D'\omega^2 + F'\omega^4) . \quad (8)$$

In this way the expansion equation (4) with $A' = -\frac{1}{2}$ seems to be justified.

This new approach is better than the previous one, since the problem of not knowing the integration constant is no longer present and there is no need for $\mathcal{J}^{(2)}(\omega)$ extrapolation, but the problem of spin assignment remains. The obvious and crucial point is that the absolute spin assignment depends on the value of A' in Eq. (4), but the fit Eq. (5) does not. Different models may suggest different values of A' and lead to different spin assignments, but will not change the χ^2 for the fit. Therefore, the spin determined in this way is purely model dependent and *has nothing to do with the fit.* One may believe such a spin determination because of faith in a particular model, but one cannot claim that the spin is uniquely determined by fitting the data.

D. Uncertainty in the fitting minimum

The authors of [4] have insisted that the deep and sharp minima for their plots of χ^2 versus spin imply unique spin assignments. They also use the observation that most spins are found to be within ± 0.1 of an integer or half integer to justify their fitting procedure; in particular, their choice of $A' = -\frac{1}{2}$. These are specious arguments. The location of the χ^2 minimum can be shifted (in principle by an arbitrary amount) without changing the quality of the fit, if the constant A' changes. As discussed above, the value of A' bears no relationship with the fitting error, and any integer number added to $-\frac{1}{2}$ will maintain the spins at integer or half-integer values, but alter their values from those given in [1] and [2] without influencing the χ^2. The fit only depends on the truncation of the expansion Eq. (5) and the assumption that $I-I_1=0,2,4,\ldots$. It does not depend on the spin.

E. Uncertainty in the rotor model

Generally speaking, the rotor model should be a reasonable model for well-deformed nuclei and might be expected to be better for superdeformed bands at low frequency. Therefore, we agree that the spin assignments based on this model could be reasonable *estimations.* However, the question is whether such a model allows a spin determination with an absolute uncertainty of $<0.5\hbar$. It can be estimated that for mass-190 superdeformed bands, where $\mathcal{J}$ is of the order of $100\hbar^2$ MeV^{-1} and $E_\gamma>200$ keV, a 10% model uncertainty in E_γ will cause $1\hbar$ of uncertainty in the determination of spins:

$$\delta I>\tfrac{1}{2}\mathcal{J}\delta E_\gamma=\tfrac{1}{2}\times 100\times 0.10\times 0.2=1\hbar\ . \tag{9}$$

It is unreasonable to expect that the rotor model can be trusted at this accuracy for excited superdeformed bands. Even assuming that $E=I(I+1)/2\mathcal{J}$ is acceptable, the formula $E_\gamma(I)=(2I-1)/\mathcal{J}$ already has as much as a 10% error since there is a term

$$\Delta E_\gamma(I)=-\frac{I(I+1)}{\mathcal{J}^2}\frac{\Delta\mathcal{J}}{\Delta I} \tag{10}$$

from the next order in the expansion of $\mathcal{J}(\omega)$ that has been neglected. For the mass-190 superdeformed bands the rate of change of the moment of inertia is $\Delta\mathcal{J}/\Delta I\simeq 0.5\hbar$ MeV^{-1} for $I=10\sim 40\hbar$; thus

$$\left|\frac{\Delta E_\gamma}{E_\gamma}\right|=\frac{I(I+1)}{(2I-1)\mathcal{J}}\times 0.5=3\%\sim 10\%\ .$$

In addition, the formula $E=I(I+1)/2\mathcal{J}$ is just a simple VMI (variable moment of inertia) model. Many effects have yet to be included. For example, if Coriolis alignment i_0 and K quantum numbers are taken into account, a more general energy formula

$$E=\frac{(I-i_0)(I-i_0+1)-K^2}{2\mathcal{J}} \tag{11}$$

should be used. Then A' is no longer $-\frac{1}{2}$ but $i_0-\frac{1}{2}$.

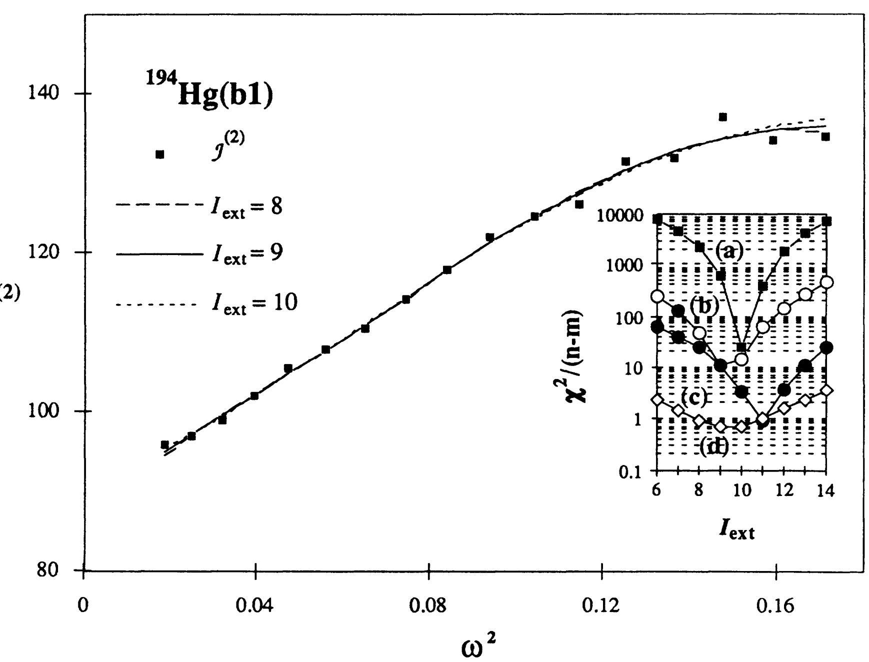

FIG. 1. The insensitivity of spin determination by fitting E_γ alone. The inset figure is $\chi^2/(n-m)$ vs exit spin I_{ext}, where n is the number of data points employed in fitting (17 in this example), and m is the number of parameters. χ^2 is defined as the squared sum of differences between the $I-I_1$ values for a given I_{ext} calculated according to Eq. (4) with the assumption that $A'=-\frac{1}{2}$ and the experimental values $(0,2,4,\ldots)$ for the spin differences. Case (a) is a 2-parameter fit (B' and D', $m=2$); case (b) is a 3-parameter fit (B', C', and D' ; $m=3$); case (c) is a 4-parameter fit (B', C', D' and F'; $m=4$); case (d) is a 5-parameter fit (B', C', D', E', and F'; $m=5$). The $\mathcal{J}^{(2)}$ vs ω^2 plot compares calculated moments of inertia for three different exit spin assignments ($I_{ext}=8$, 9, and 10) for case (c) (lines) and the experimental values (dots). The calculation is based on the formula $\mathcal{J}^{(2)}=dI/d\omega=B'+2C'\omega+3D'\omega^2+4E'\omega^3+5F'\omega^4$. The parameters in order from B' to F' are 41.97, 425.1, -1775, 4095, -3460 for $I_{ext}=8$; 63.27, 250.7, -1087, 2786, -2496 for $I_{ext}=9$; and 84.92, 70.46, -365.7, 1392, -1455 for $I_{ext}=10$. Three different exit spin assignments result in almost the same quality of fit to the dynamical moment of inertia $\mathcal{J}^{(2)}$, indicating that the fitting procedure is highly insensitive to the spin.

Furthermore, other effects such as band mixing, which will change the level spacing and thus E_γ, will influence the effective value of A'. The cumulative uncertainty in E_γ caused by considerations such as these can easily reach 10%, and such an uncertainty in the E_γ formula of the assumed model is sufficient to produce at least a $1\hbar$ uncertainty in the spin determination. This translates into a $\geq 100\%$ uncertainty in the proposed quantized spin alignment of [1,2].

F. Is the fitting minimum sharp?

The sharpness of the χ^2 minimum in [4] is also a consequence of the model assumptions. In addition to the assumption that $A'=-\frac{1}{2}$, in the fitting procedure of [4] it has been assumed that the expansion equation (4) has no higher power than the cubic term. The validity of such a truncation should be tested by adding higher-order terms to check for convergence. We have found that if higher powers of frequency (say up to fifth power) are included in the expansion, χ^2 can be decreased by an order of magnitude. However, the corresponding χ^2 distribution broadens significantly with respect to the exit spin. A typical example is shown in Fig. 1 for band 1 in ^{194}Hg. This indicates clearly that even if one believes that $A'=-\frac{1}{2}$, one still cannot determine spin with confidence because the fit is insensitive to the exit spin if higher-power terms are taken into account. Therefore, the sharpness of the χ^2 minimum in [4] is an artifact resulting from limiting the number of parameters in the expansion; similar conclusions have been reported to us by others [5]. Since one is attempting to determine simultaneously the exit spin and the other fitting parameters in the procedure of [4], it is no surprise that restricting the number of additional parameters in the expansion constrains the allowed variation of the exit spin. In any case, once fits are found like those in Fig. 1 that show almost no change for exit spin varying by 2 units, one is less confident of the spin determination suggested by [1,2,4].

Finally, let us close this discussion of uncertainties associated with spin estimates by noting that although application of the spin determination method of [1,2,4] in well-deformed normal nuclei often works for yrast bands, there are a number of instances where for excited bands with known spins the method fails [6,7]. This finding is particularly significant given that many superdeformed bands that are claimed to exhibit quantized alignment correspond to excited superdeformed states.

III. INCREMENTAL ALIGNMENT

In [2] an attempt was made to find a spin-independent manifestation of quantized alignment. The authors defined an *incremental alignment*, Δi, that depends only on γ-ray energies E_γ and is related to the total alignment i through

$$i=\Delta I+\Delta i \ , \tag{12}$$

where ΔI is the spin difference between a state of the nucleus in question and the corresponding state of the reference nucleus (^{192}Hg) that is associated with the closest E_γ. The data indicate that Δi is quantized over intermediate frequency ranges for many (though certainly not all) of the superdeformed bands in the mercury region, and this seems to provide a direct test of the quantized alignment hypothesis, independent of knowledge about the spins: because ΔI is quantized; if Δi is quantized then i must be quantized. However, if there is neglible alignment ($i=0$), then $\Delta i=-\Delta I$ is certainly quantized since it is just the spin difference of quantized angular momentum states in two nuclei. Hence, the empirical evidence that Δi is quantized is no proof of the existence of a quantized alignment, unless the spin is measured and i is shown to have nonzero values.

At this point one might argue that $i=0$ is a special case of the quantized alignment hypothesis, since zero is an integer. This is incorrect. According to the definition of [1], the condition $i=0$ only implies that a superdeformed band has the same alignment as a given reference band. Although $i=0$ is itself nontrivial, this need have nothing to do with a quantized alignment. In elementary quantum mechanics, the concept of quantization deals with a physical quantity that can take *more than one discrete value*, and only these values. It is meaningless to invoke the concept of quantization if a quantity has only one value.

Nevertheless, the observation of quantized incremental alignment is important, because it implies that there are *only two possibilities* to account for the $\mathcal{J}^{(2)}$ behavior in the mass-190 region: either there is no relative alignment between bands in the mass-190 region, or the alignment differences between these bands is often quantized. The second alternative corresponds to that proposed in [1]; in the remainder of this paper we discuss the first alternative. Although the present data cannot provide definitive evidence for either alternative, the subsequent discussion will show that the quantized alignment hypothesis is not a unique way, nor is it the simplest way, to account for the data.

IV. ZERO RELATIVE ALIGNMENT

Given alternative possibilities to account for a set of data, it is more economical to frame simple hypotheses (Occam's Razor). One simple hypothesis is to assume that *all superdeformed nuclei in a given mass region have the same VMI-like core,* and thus there is essentially no alignment below the crossing frequency of high-N orbitals. Only when the freuency is beyond the crossing frequency where alignment of high-N orbitals becomes significant can two nuclei begin to show differences that lead to $i\neq 0$. (Note that the alignment i that we are discussing is the alignment between two nuclei and not the alignment of a nucleus relative to its own core.) This conjecture appears to be consistent with all present data, as we now demonstrate.

A. Identical bands

In the subsequent discussion we employ the following terminology. *Identical bands* are bands that have the same values of the dynamical moment of inertia $\mathcal{J}^{(2)}$. We will distinguish two types of identical bands: the strong-coupled identical band (SIB) and the decoupled identical band (DIB). Strong-coupled identical bands not only have the same values of $\mathcal{J}^{(2)}$, but identical values of $\mathcal{J}^{(1)}$ as well, which implies that $i=0$ with no signature splitting; decoupled identical bands have identical values of $\mathcal{J}^{(2)}$ but different values of $\mathcal{J}^{(1)}$, which implies that $i \neq 0$. As we shall see in the following, all superdeformed bands in the mass-190 region appear to be SIB's within an intermediate range of frequencies, while very few identical bands are found in the mass-150 region for any observed frequency ranges (one pair of SIB's and five pairs of DIB's have been reported in this region).

In Fig. 2 we show alignment as a function of frequency in the mass-190 region according to the formula $i=\Delta I+\Delta i$. Figure 2(a) is the i versus ω plot for 8 bands belonging to the "^{192}Hg family" [2], which has been interpreted as evidence for quantized spin alignment if the spin assignment is chosen according to set (a) of Table I. According to this assignment, only band 1 in ^{194}Pb is a SIB relative to the ^{192}Hg reference; all other bands are DIB's. This is difficult to understand. An odd-A band can be a DIB if $K=\frac{1}{2}$ and $a \neq 0$; in this situation a constant alignment $i=a/2$ can be obtained. Five pairs of such bands have been found in the mass-150 region with a decoupling constant $a \approx 1$, as we will discuss below. In the mass-190 region, however, no $K=\frac{1}{2}$ band has been identified; furthermore, the absence of signature splitting suggests strong coupling ($a \simeq 0$) and appears to argue against this possibility. For even-A nuclei it is difficult to produce a quantized alignment in any standard theory. Moreover, if we accept this picture, there are additional questions to face. For example, how is it that in ^{193}Hg (^{191}Hg) the ^{192}Hg even-even core has a half-integer spin and the single odd neutron (odd-neutron hole) carries a spin alignment of $1\hbar$; how is it that in the ^{194}Hg even-spin excited band (b2) the ^{192}Hg core has an odd spin if one believes the ^{192}Hg superdeformed band to be a $K=0$ band;. . .?

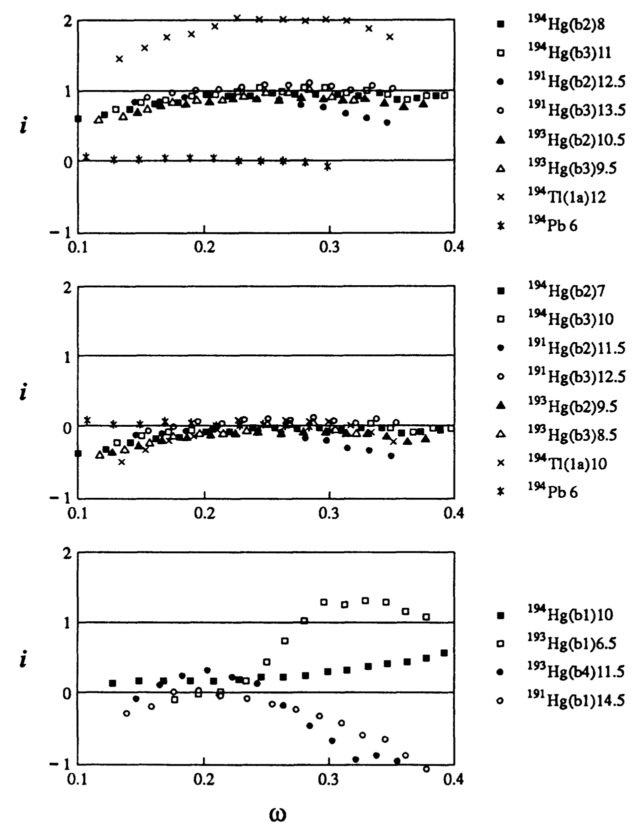

FIG. 2. Relative alignments in $A \sim 190$ superdeformed nuclei. (a) Quantized alignments as in [1,2], with exit spin assignment according to (a) in Table 1; (b) the same bands with essentially no alignment if exit spin assignment is made according to (b) in Table I; (c) the bands that show significant alignment at high frequency. Exit spin assignments in each case are shown on the right side of the legend. Data are taken from E. F. Moore *et al.*, Phys. Rev. Lett. **63**, 360 (1990); M. P. Carpenter *et al.*, Phys. Lett. B **240**, 44 (1990); J. A. Becker *et al.*, Phys. Rev. C **41**, 9 (1990); D. Ye *et al.*, *ibid.* **41**, 13 (1990); D. M. Cullen *et al.*, Phys. Rev. Lett. **65**, 1547 (1990); M. A. Riley *et al.*, Nucl. Phys. **A512**, 178 (1990); F. Azaiez *et al.* (unpublished); M. J. Brinkman *et al.*, Z. Phys. A **336**, 115 (1990); and K. Theine *et al.*, *ibid.* **336**, 113 (1990).

Such questions do not mean that the quantized alignment hypothesis is incorrect; the failure of standard ideas described above could be a genuine signal of new physics. However, these examples suggest that quantized alignment, which only works if applied to a selected subset of available superdeformation data, raises questions comparable in number to the ones that it purports to answer. Moreover, there is accumulating evidence that nearly identical bands are more common in normally deformed nuclei than previously thought, and that in such bands (where spins are known) there is no support for quantization of pseudospin alignment [7]. Although it is always possible to argue that some feature of superdeformation is special relative to large normal deformation, this finding appears to reduce the plausibility of quantized alignment for superdeformation.

B. An alternative interpretation

However, spins for these bands are uncertain; if the spin assignment is chosen according to set (b) of Table I for the same bands as shown in Fig. 2(a), the quality of fit is equal to that for the assignments in set (a), there is essentially no alignment [see Fig. 2(b)], and all the difficulties mentioned above vanish. Alignment $i \approx 0$ implies that these bands are SIB's, and that the quantized incremental alignment Δi reduces to the angular momentum difference between a state in the ^{192}Hg family and the one in the reference band that has the closest transition energy ($\Delta i \approx -\Delta I$, because $i=\Delta I+\Delta i$, and $i \approx 0$). It is an elementary observation that this difference must be quantized. The incremental alignment for *each* mass-190 band displayed in Fig. 1 of Ref. [2] is explained by this assumption.

Note that the mass-190 data are available only for the low-frequency region ($\omega < 0.4$ MeV). The smooth increase with frequency of the measured values of $\mathcal{J}^{(2)}$ seems to suggest a crossing frequency for high-N orbitals greater than 0.4 MeV for most of the mass-190 superdeformed bands. Cranking calculations [8] indicate that the lowest crossing frequency for $N=7$ neutrons is $0.25 \sim 0.3$ MeV, resulting in a bump appearing in $\mathcal{J}^{(2)}$ for $\omega \simeq 0.3$ MeV that is too low in frequency to agree with data. Thus a higher crossing frequency also seems to be required empirically in the cranking calculation, though the reason is uncertain. If we accept this fact, the above hypothesis gives a natural explanation for why i is essentially zero for most superdeformed bands in this region.

Figure 2(c) displays the i versus ω plot for those bands that have been deemed exceptions and excluded from the "^{192}Hg family" in [2] because they do not follow the quantized alignment rule. According to our hypothesis there are no such exceptions. The difference between the bands in Fig. 2(b) and those in Fig. 2(c) is merely a question of crossing frequency: the former have crossing frequencies higher than 0.4 MeV and the latter happen to have lower crossing frequencies. From Fig. 2(b) one can already see that when the frequency exceeds 0.3 MeV, bands begin to exhibit differences. If there were no fission cutoff of the data we would have expected an alignment picture similar to Fig. 2(c) to occur in a higher frequency range, and we would perhaps not have called these bands identical bands. Likewise, if the data beyond 0.25 MeV were not measured for the bands in Fig. 2(c), we would probably have called these bands identical bands. The crossing frequency of high-N orbitals and the strength of band interactions depend sensitively on the details of aligned single-particle configurations. Therefore, it is not surprising that for some bands the high-N aligned orbitals happen to have lower crossing frequency and that for others they have higher crossing frequency.

C. A unified description of superdeformed bands

Thus, we present a unified picture: any two superdeformed bands in the mass-190 region will appear as SIB's if the frequency is below the high-N crossing frequency, and will begin to show differences beyond any one of the crossing frequencies. This picture is quantitatively consistent with all the mass-190 superdeformed data, as we will show in a separate publication [9,10].

This hypothesis also explains why there are so many identical bands found in the mass-190 region, but very few in the mass-150 region: data in this region are available only for higher frequencies where the effect of high-N crossings is seen explicitly. Thus, the properties of superdeformed bands in the mass-150 region depend strongly on the configurations of aligned high-N orbitals, which generally vary from band to band. Only when two bands have the same high-N aligned configurations can they appear as a pair of identical bands. So far only six pairs have been found that meet this condition in the mass-150 region: one pair of SIB's (^{152}Dy and ^{153}Dy*), and five pairs of DIB's: (^{152}Dy, ^{151}Tb*), (^{150}Tb, ^{149}Gd*), (^{148}Gd, ^{147}Gd), (^{147}Gd*, ^{146}Gd), and (^{151}Tb, ^{150}Gd*) (see Ref. [11]).

These six pairs of identical superdeformed bands have been explained without introducing the concept of quantized alignment [12,13]. In the first pair (^{152}Dy, ^{153}Dy*) the odd-neutron band is not a $K=\frac{1}{2}$ band; therefore, these are members of a SIB with $i \approx 0$ and no signature splitting, similar to the identical bands in the mass-190 region. In the other five pairs the nuclei in each pair differ by one proton hole, which is thought to be in a $[301]\frac{1}{2}$ orbital and to have pseudo-SU(3) symmetry [12], leading to a decoupling constant $a=1$ so that the resulting two bands have identical values for E_γ. Therefore these bands are DIB's, and because $a=1$ there is an alignment $i=\frac{1}{2}$ with respect to the reference nucleus, which is nothing but an even-odd spin difference ($i=\Delta I$).

It has been difficult to explain in a consistent manner why all the nuclei that are found to have quantized incremental alignment in [2] have $\Delta i=0$ or 1 for even-A nuclei, and $\Delta i=\pm\frac{1}{2}$ for odd-A nuclei, except for ^{151}Tb*, where Δi is zero. [The pairs (^{150}Tb, ^{149}Gd*), (^{148}Gd, ^{147}Gd), (^{147}Gd*, ^{146}Gd), and (^{151}Tb, ^{150}Gd*) were not considered in [2] since ^{152}Dy was taken as a reference for the mass-150 region.] From our point of view these observations are expected. For SIB's, $i \approx 0$; therefore, $\Delta i=-\Delta I$, which is the spin difference between states in two nuclei with the closest transition energies E_γ. It is then a trivial consequence that Δi must be 0 or ± 1 for even-A nuclei and $\pm\frac{1}{2}$ for odd-A nuclei. On the other hand, for the known DIB's, $a=1$ so that $\Delta i=0$ and $i=\Delta I$, because $i=\Delta I+\Delta i$. This is the case for ^{151}Tb*.

V. SUMMARY

In conclusion, our analysis suggests that there exists no model-independent evidence requiring the existence of quantized spin alignment. An alternative proposal that assumes essentially no alignment below the crossing frequency for high-N orbitals is equally plausible and explains present data in a more unified and economical fashion. Only for $K=\frac{1}{2}$ in odd-A nuclei with decoupling constant $a \neq 0$ will there be a constant alignment $i=a/2$, as has been observed in the mass-150 region for an $a=1$ case. Furthermore, we have argued that the quantized alignment hypothesis and the stable-core (zero alignment) hypothesis presented here are the *only* alternatives that can account for the incremental alignment that is observed for many (not all) mass-190 superdeformed bands.

However, in our opinion, whether the alignment is $i=0$ or some other value is not the central issue; this will be clarified if the spins of the states in question are measured. What we believe to be the key question for the many nearly identical superdeformed bands now observed is why superdeformed nuclei in a given region appear to have almost identical VMI cores, and that exhibit such stability from band to band and under local variations in particle number. In a separate publication [9] we describe how an extended version of the fermion dynamical symmetry model (FDSM) provides an explanation for this in terms of a supershell fermionic SU(3) symmetry [*not* a pseudo-SU(3) symmetry]. The preceding hypothesis that $i=0$ ($i=a/2$ for $K=\frac{1}{2}$) at low frequency finds a natural basis in such a model. Thus the explanation of incremental alignment suggested here on phenomenological grounds will be given a microscopic justification in [9]. A quantitative calculation of moments of inertia $\mathscr{J}^{(1)}$ and $\mathscr{J}^{(2)}$ based on this model that agrees very well with data will also be published elsewhere [10]. Finally, from Fig. 2(b) one may notice that for very low frequency (<0.15 MeV), i begins to deviate from zero. This is also expected in the model proposed in [9] as a consequence of a new alignment mechanism (*D-pair alignment*) that we believe to be directly related to the rapid loss of population from observed superdeformed bands at low frequency [13].

ACKNOWLEDGMENTS

We thank Jerry Garrett, Jim Griffin, Robert Janssens, Witek Nazarewicz, Mark Riley, and Ramon Wyss for useful discussions. Nuclear physics research at the University of Tennessee was supported by the U. S. Department of Energy through Contract No. DE-FG05-87ER40361. The Joint Institute for Heavy Ion Research has as member institutions the University of Tennessee, Vanderbilt University, and Oak Ridge National Laboratory. Nuclear physics research at Drexel University was supported by the National Science Foundation.

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