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Chaos in nuclei with broken pairs

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A recent study of the onset of chaos in the low-lying collective states of nuclei, which uses the interacting boson model, is extended to higher spins and/or higher energies by including broken pairs. Spectral fluctuations are studied as a function of the quadrupole-quadrupole interaction between the fermion pair and the core and the pair-breaking interaction that mixes states with different number of fermions. The effects of the Coriolis force are discussed.

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I. INTRODUCTION

Random matrix theory (RMT) [1] was successful in explaining the fluctuation properties of the neutron resonances in the compound nucleus [2]. These resonances lie in the region of high level density, and the use of random ensembles of Hamiltonians was justified by the complexity of the compound nucleus. In recent years, however, it was conjectured that RMT describes quantal fluctuations of systems which are classically chaotic [3]. This conjecture was confirmed by numerous studies of systems in two degrees of freedom [4]. In particular, chaotic systems with time-reversal symmetry are associated with the Gaussian orthogonal ensemble (GOE) of random matrices. The interest in this phenomenon, known as quantum chaos, led to the investigation of the fluctuation properties of experimental low-lying levels in nuclei along and above the yrast line [5–8]. To obtain good statistics, it is necessary to include levels from various nuclei and different spins. The implicit assumption of such studies is that these levels have a similar statistical behavior.

However, the degree of chaoticity can depend on the Hamiltonian's parameters as well as on the spin, and so a careful study of the onset of chaos in nuclei requires the study of one Hamiltonian and one spin/parity class at a time. Since complete experimental data on one nucleus are unavailable, or that the number of known levels with a given spin is not large enough for a statistical analysis, it is useful to study in parallel realistic and tractable models of nuclei.

We have initiated [9–11] such an investigation for the low-lying collective states of nuclei by using the interacting boson model (IBM) [12]. By studying both quantal fluctuations of levels and $B(E2)$'s and the classical mean-field dynamics (in five degrees of freedom), we have determined the degree of chaoticity in the parameter space of the Hamiltonian and as a function of spin. Level fluctuations in the IBM were also studied near the SU(3) and O(6) limits in Ref. [13] and in the Casten triangle in Ref. [14]. However, the IBM, as a truncation of the shell model, is especially useful for the description of low-spin states ($J \lesssim 10\hbar$). To investigate high-spin and/or higher-energy states (above the yrast line), it is necessary to take explicitly into account noncollective fermion excitations.

The IBM can be extended to high-spin physics by including noncollective fermion states through the breaking of the correlated S and D pairs. High-spin states are described in terms of broken pairs. The model has been extended to one broken pair [15–17] (two-quasiparticle states) and two broken pairs [18–20] (four-quasiparticle states). Such models contain (in addition to the IBM parameters) two important new ingredients: the interaction of the unpaired fermions with the boson core and the pair-breaking interaction that mixes states with different number of fermions. In order to investigate how these two interactions affect the chaotic properties of the nucleus, we shall study a case where the core is regular [described by the SU(3) limit].

Another important question is the dependence of chaos on spin. The new physics here is that the Coriolis force, which increases with spin, will eventually cause the breaking of pairs and the alignment of the quasiparticles along the rotation axis. The interplay between the Coriolis and centrifugal forces and the pairing interaction may lead to an interesting dynamical behavior.

The outline of this paper is as follows. In Sec. II we review the model which includes broken pairs. In Sec. III we study the onset of chaos (for a regular bosonic core) as a function of the strengths of the dynamical and mixing interactions, while in Sec. IV we study the spin dependence of the degree of chaoticity. In both Secs. III and IV the model space includes only up to one broken pair.

II. MODEL

The model space [18] for an even-even nucleus with $2N$ valence nucleons is

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$$|N \text{ bosons}\rangle \oplus |(N-1) \text{ bosons} \otimes 1 \text{ broken pair}\rangle \ . \quad (1)$$

The model is based on the IBM-1; the boson space consists of s and d bosons, and no distinction between protons and neutrons is made. A boson can break to form a noncollective fermion pair that is represented by a two-quasiparticle state. The model Hamiltonian is

$$H = H_B + H_F + V_{BF} + V_{\text{mixing}} \ . \quad (2)$$

The first term is the IBM-1 boson Hamiltonian; the second term is the fermion Hamiltonian. V_{BF} is the boson-fermion interaction of the interacting boson-fermion model [21]. V_{mixing} is the pair-breaking interaction that mixes states with a different number of quasiparticles. In the present paper, we consider the example of an SU(3) boson core which corresponds to the case of a well-deformed nucleus. In order to simplify the presentation and to consider only the most important effects, we restrict the various terms in our model Hamiltonian (2) to the following: The fermionic part is

$$H_F = \sum_\alpha E_\alpha a_\alpha^\dagger \tilde{a}_\alpha \ , \quad (3)$$

where E_α are the quasiparticle energies. The boson core is described by the interacting-boson-model Hamiltonian (in the self-consistent Q model [22])

$$H_B = E_0 + c_0 \hat{n}_d + c_2 Q^\chi \cdot Q^\chi + c_1 \mathbf{L}^2 \ . \quad (4)$$

Here $\hat{n}_d = d^\dagger \cdot \tilde{d}$ is the d-boson number operator, $\mathbf{L}$ is angular momentum operator, and Q^χ is the quadrupole operator,

$$Q^\chi = (d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d})^{(2)} + \chi (d^\dagger \times \tilde{d})^{(2)} \ , \quad (5)$$

which depends on the parameter χ. In the following we shall restrict the fermion space to a unique-parity high-j orbital. The basic structure of the spectrum is then determined by the dynamical boson-fermion interaction

$$V_{BF} \equiv H_{\text{dyn}} = \Gamma_0 \sum_j (u_j^2 - v_j^2) \langle j \| Y_2 \| j \rangle (a_j^\dagger \times \tilde{a}_j)^{(2)} \cdot Q^\chi \ , \quad (6)$$

and the pair-breaking interaction

$$V_{\text{mixing}} = -u_2 \sum_j 2 u_j v_j \langle j \| Y_2 \| j \rangle (a_j^\dagger \times a_j^\dagger)^{(2)} \cdot \tilde{d} + \text{H.c.} \quad (7)$$

In the present investigation, we have neglected the effects of the exchange and monopole boson-fermion interaction [18] and the monopole mixing interaction [19]. The interaction between the unpaired fermions is dominated by the long-range dynamical force (6). Therefore we have not included the residual two-body fermion interaction (for example, a delta force) in our model Hamiltonian.

Our purpose is to study the chaoticity of the nuclear spectrum as a function of the dynamical coupling Γ_0, the mixing coupling u_2, and the total spin J. We thus choose a bosonic core which displays regular behavior: $\chi = -\sqrt{7}/2$: $c_0 = 0$ in the SU(3) limit (describing a rotational nucleus). The other parameters are $c_2 = -0.04$ MeV and $c_1 = -0.005$ MeV, and the number of bosons in $N = 9$.

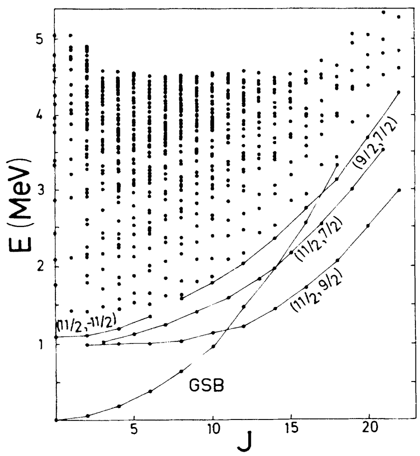

FIG. 1. Excitation energy versus spin diagram obtained by diagonalizing the Hamiltonian in (2)–(7) with up to two quasiparticles in the $h_{11/2}$ orbit. The boson core is an SU(3) prolate with $N = 9$ bosons, $c_2 = -0.04$ MeV, and $c_1 = -0.005$ MeV. The quasiparticle in the $h_{11/2}$ orbit has an energy of 0.85 MeV and an occupation probability of $v^2 = 0.2$. The dynamical interaction has $\Gamma_0 = 0.6$ MeV, and the mixing strength is $u_2 = 0.275$ MeV. (α_1, α_2) describes bands with a dominant configuration of two quasiparticles with spin projections α_1 and α_2 along the rotation axis. Solid dots denote energy levels.

For the fermion orbit, we take the $h_{11/2}$ orbital with occupation $v^2 = 0.2$ and $E_{11/2} = 0.85$ MeV. The energy spectrum shown in Fig. 1 is calculated for $\Gamma_0 = 0.6$ MeV and $u_2 = 0.275$ MeV. Note that we have particle-type fermions coupled to a prolate core for which the dynamical boson-fermion interaction is repulsive for the lowest two-quasiparticle (2qp) states, and the resulting 2qp spectrum is of decoupled type. GSB is the ground-state band which is predominantly composed from N-boson states, with small admixtures of 2qp states. The lowest 2qp states are grouped into rotational bands characterized by the signature: $r = +1$ (-1) for states with even (odd) spins. To the lowest 2qp bands we assign the "algebraic projection" quantum numbers (α_1, α_2) [18]. The strong Coriolis force aligns the unpaired fermions along the axis of rotation, and the projections (α_1, α_2) of their angular momenta (j_1, j_2) on the rotation axis are approximately good quantum numbers. In particular, the most aligned 2qp band $(\frac{11}{2}, \frac{9}{2})$ crosses the ground-state band and becomes yrast around $J = 12\hbar$. At this crossing the Coriolis force becomes strong enough to break a pair, and there is a gain in energy by maximally aligning the two fermions along the rotation axis. For nonyrast states the Coriolis mixing is much stronger and the classification of 2qp states into rotational bands becomes more difficult.

At energies of several MeV, the density of states became quite large and we have a sufficient number of levels (at each spin) for a statistical analysis.

III. LEVEL FLUCTUATIONS

We have diagonalized the Hamiltonian in Eqs. (2)–(7) and analyzed the fluctuations of its spectrum $\{E_i\}$. For that purpose we first separate the smooth (nonuniversal) part of the spectrum. This is accomplished by constructing a staircase function $N(E)$ and calculating an average part N_{av} by fitting a smooth function [11] to $N(E)$. We then obtain the fluctuating part from $N(E) = N_{av}(E) + N_{fluct}(E)$, and the unfolded levels are defined by $\tilde{E}_i = N_{av}(E_i)$. The analysis is done separately for each spin-parity (J^π) class of levels.

Two statistical measures are used for $\{\tilde{E}_i\}$, the nearest-neighbor level spacing distribution $P(S)$ and the Δ_3 statistics of Dyson and Metha. $P(S)$ is fitted to a Brody distribution [23] parametrized by ω:

$$P_\omega(S) = AS^\omega \exp(-\alpha S^{1+\omega}) , \tag{8}$$

where $\alpha = \Gamma[(2+\omega)/(1+\omega)]^{1/2}$ and $A = (1+\omega)\alpha$ are chosen so as to ensure the normalization of P and $\langle S \rangle = 1$. The distribution (8) interpolates between the Poisson distribution ($\omega = 0$), which characterizes a regular system, and the Wigner distribution ($\omega = 1$), which characterizes a chaotic system.

The Δ_3 statistics

$$\Delta_3(\alpha, L) = \min_{A,B} \frac{1}{L} \int_\alpha^{\alpha+L} [N(\tilde{E}) - (A\tilde{E} + B)]^2 d\tilde{E} \tag{9}$$

is a measure of the deviation of the unfolded staircase function from a straight line. To get a smoother $\bar{\Delta}_3(L)$, we average $\Delta_3(\alpha, L)$ over n_α intervals $(\alpha, \alpha+L)$, which overlap by $L/2$ successively:

$$\bar{\Delta}_3(L) = \frac{1}{n_\alpha} \sum_\alpha \Delta_3(\alpha, L) . \tag{10}$$

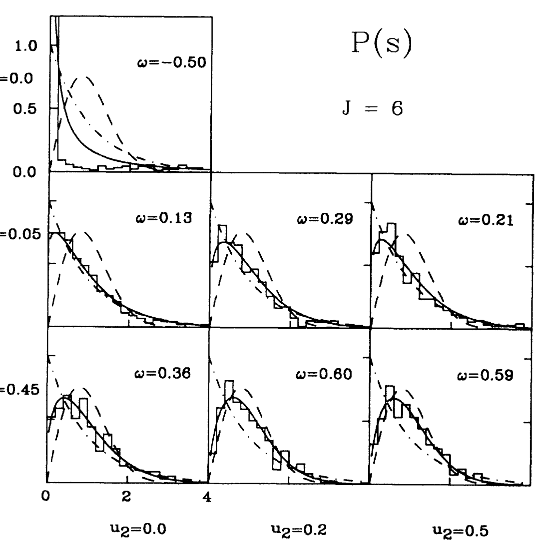

FIG. 2. Nearest-neighbor level spacing histogram $P(S)$ for the Hamiltonian in (2)–(7) and for several values of the dynamical interaction strength Γ_0 and mixing strength u_2. All other parameters are as in Fig. 1. The solid line is the fit to the distribution $P_\omega(S)$ [Eq. (8)] with the quoted ω; the dashed line is the Wigner distribution (GOE), and the dot-dashed line is the Poisson distribution. Note the saturation in ω as u_2 increases for a given Γ_0. For a fixed u_2, ω increases (i.e., more chaos) with Γ_0.

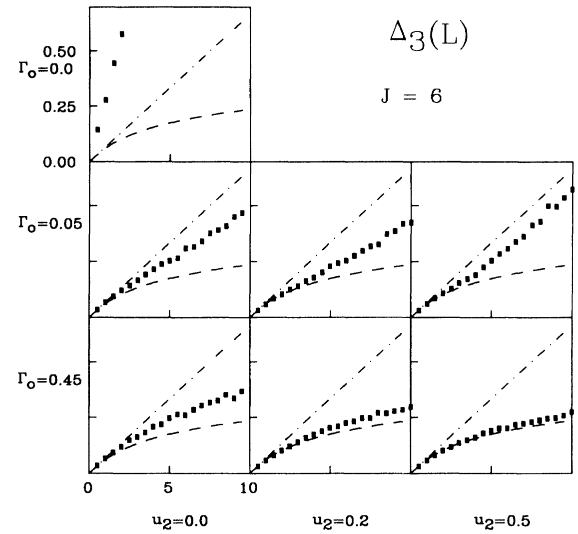

FIG. 3. As in Fig. 2, but for the Δ_3 statistics (solid squares). The dashed line is the GOE Δ_3 statistics, and the dot-dashed line is Δ_3 for the Poisson statistics.

The Poisson statistics gives

$$\bar{\Delta}_3(L) = L/15 , \tag{11}$$

while, in the GOE (chaotic) limit,

$$\bar{\Delta}_3(L) \underset{L \gg 1}{\approx} \tfrac{1}{2}\ln L - 0.007 . \tag{12}$$

Figures 2 and 3 describe the distribution $P(S)$ and Δ_3 statistics, respectively, for the Hamiltonian (2)–(7) with

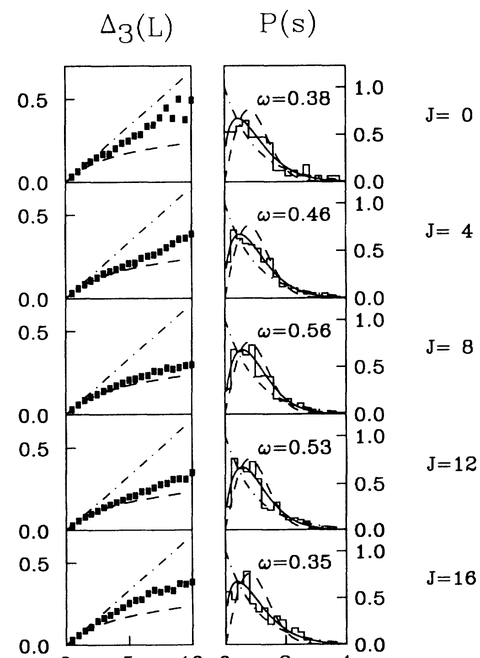

FIG. 4. Level spacing distribution $P(S)$ (right column) and Δ_3 statistics (left column) for the Hamiltonian in (2)–(7) and for several even spins from Fig. 1. The dashed lines correspond to the GOE statistics, while the dot-dashed lines correspond to the Poisson statistics.

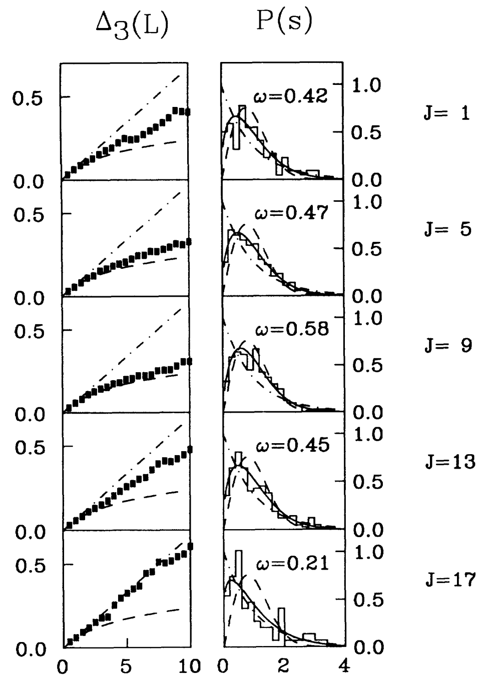

FIG. 5. As in Fig. 4, but for odd spins.

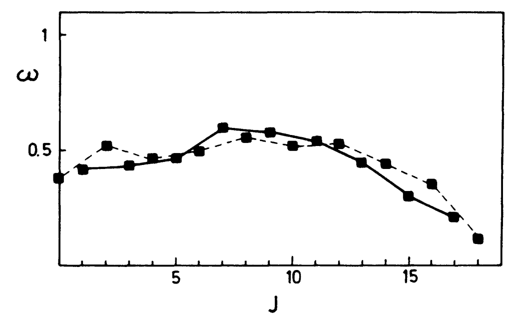

FIG. 6. Parameter ω of the level spacing distribution [see Eq. (8)] versus spin for the Hamiltonian in (2)–(7) with $\Gamma_0=0.6$ MeV and $u_2=0.275$ MeV (Fig. 1). Note the sharp decrease in ω (i.e., the spectrum becomes more regular) for spins $J \gtrsim (10\text{–}12)\hbar$. $J=12\hbar$ is the spin for which the lowest two-quasiparticle band crosses the ground-state band.

an SU(3) boson core and the $h_{11/2}$ fermion orbital (Fig. 1) for several values of Γ_0 (0, 0.05, and 0.45 MeV) and u_2 (0, 0.2, and 0.5 MeV). The levels analyzed are the $J=6^+$ levels. Only 0qp and 2qp states are considered. The case of no interaction between the quasiparticles and core ($\Gamma_0=0$) is "overintegrable" ($\omega<0$) because of large number of exact degeneracies. A more generic situation is thus a small Γ_0 (such as $\Gamma_0=0.05$, $u_2=0$) for which the statistics is very close to the Poisson limit ($\omega=0.13$).

For a fixed dynamical coupling ($\Gamma_0 \neq 0$), we see that the statistics becomes less regular when the strength of the mixing interaction u_2 increases until a certain saturation is reached ($\omega \approx 0.2-0.3$ for $\Gamma_0=0.05$ and $\omega \approx 0.6$ for $\Gamma_0=0.45$). For a fixed value of the mixing coupling u_2, the degree of chaos increases when the quadrupole-quadrupole coupling Γ_0 increases. The Δ_3 statistics (Fig. 3) is consistent with the level spacing distribution (Fig. 2).

IV. CORIOLIS EFFECTS

The Coriolis and centrifugal forces increase with spin. To see how they affect the fluctuation properties of the levels, we have analyzed various spins between 0 and $20\hbar$ for the illustrative spectrum shown in Fig. 1. The level spacing distribution $P(S)$ and Δ_3 statistics are shown in Fig. 4 (even spins) and Fig. 5 (odd spins). When the spin increases, there is initially a steady increase in chaoticity (ω increases) and a maximum of the onset of chaos is reached around $J \approx 10\hbar$, not far from the crossing of the two-quasiparticle band with the ground-state band ($J=12\hbar$). In this region the Coriolis interaction is comparable to the pairing interaction, and this produces maximal chaoticity. For spins above $J \approx 12\hbar$, we see a rapid decrease in chaoticity with increasing spin. This may be attributed to a regularity induced by the decoupling of the quasiparticles from the core and their spin alignment along the rotation axis.

Figure 6 shows the dependence of ω on the spin for the even spins (solid line) and odd spins (dashed line).

V. CONCLUSIONS

The study of the onset of chaos in realistic models of nuclei is extended to higher spins and energies by including broken pairs in the interacting boson model. In this paper spectral fluctuations were analyzed. It is found that both the quadrupole-quadrupole interaction (between the quasiparticles and core) and the mixing interaction can cause a partial onset of chaos in an otherwise regular core. For a given interaction, the degree of chaoticity seems to be maximal for spins in the vicinity of the crossing between the ground-state band and the lowest two-quasiparticle band. In this region Coriolis interaction is comparable to the pairing interaction.

Work in progress includes the study of the distribution of electromagnetic transition intensities as an additional probe of the onset of quantal chaos in nuclei with broken pairs. We are also studying the effects of including more than one broken pair in the model space.

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