Rho exchange in charge-exchange reactions

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We have studied the role of rho meson exchange in the transition potential for the charge-exchange reactions $p(n,p)n$ and $p(p,n)\Delta^{++}$ at intermediate energies. The cross sections are calculated in the distorted-wave Born approximation. The calculated results are compared with the available experimental data. It is found that, for reproducing these data, while in the $p(n,p)n$ reaction, in addition to one-pion exchange, we need the rho exchange in the transition potential; in the delta excitation the rho exchange is not needed.

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I. INTRODUCTION

The nucleon-nucleon interaction has several parts to it. Out of them, the spin-isospin part of the interaction is the simplest to describe microscopically. In boson-exchange models, the Born term of the one-pion and -rho exchange potentials accounts for the major portion of this interaction [1]. The spin-isospin piece of the interaction is also of much interest because experimentally the spin-isospin correlations are dramatically manifested through the strong excitation of the isovector Gamow-Teller (GT) mode in the intermediate-energy (p,n) reaction [2]. Very recently, these studies have been extended further to the domain of the delta excitation, the first excited state of the nucleon, in nuclear collisions [3]. This excitation, as is now known, proceeds through the $NN\rightarrow N\Delta$ step, and because the delta has spin and isospin both equal to $\frac{3}{2}$, the interaction responsible for it is spin-isospin dependent.

The actual form of the one-pion and -rho exchange potentials in momentum space and the nonrelativistic limit are written as

$$V_\pi(\omega,\mathbf{q})=V_\pi(\omega,q)[(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+S_{12}(\hat{\mathbf{q}})](\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ , \qquad (1)$$

$$V_\rho(\omega,\mathbf{q})=V_\rho(\omega,q)[2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)-S_{12}(\hat{\mathbf{q}})](\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ , \qquad (2)$$

where

$$V_x(\omega,q)=-\frac{f_x^2}{3m_x^2}F_x^2(t)\frac{q^2}{q^2+m_x^2-\omega^2} \qquad (3)$$

and $S_{12}(\hat{\mathbf{q}})$ $[=3(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{q}})-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)]$ is the tensor operator. t $(=\omega^2-q^2)$ is the four-momentum transfer. f_x is the xNN coupling constant, where x denotes pion or rho. The form factor $F_x(t)$ in each interaction cuts off the interaction at short distances. Commonly, it is described by a monopole form

$$F_x(t)=\frac{\Lambda_x^2-m_x^2}{\Lambda_x^2-t}\ , \qquad (4)$$

where the length parameter Λ_x is a measure of its extension.

The above form of the spin-isospin interaction also holds if the nucleon-nucleon system is replaced by the $N\Delta$ system or $NN\rightarrow N\Delta$ transition. The coupling constants f_x and length parameters Λ_x are, of course, replaced by those corresponding to the $xN\Delta$ vertices. These quantities are denoted by f_x^* and Λ_x^*.

Structurally, as shown in Eqs. (1)–(3), the rho exchange makes the tensor part of the spin-isospin one-pion-exchange potential weaker and the central part stronger. The actual magnitude by which the ρ exchange changes the one-pion-exchange potential, of course, is determined by the values of the coupling constants f_ρ and f_ρ^* and the length parameters Λ_ρ and Λ_ρ^* at the ρNN and $\rho N\Delta$ vertices, respectively. The final form of the one-pion-plus-one-rho-exchange potential additionally depends upon the coupling constants f_π and f_π^* and the length parameters Λ_π and Λ_π^* for the pion. As a result of these parameters the values of f_π and f_π^* are well known from pion-nucleon scattering data and the decay width of the delta [4]. The values of the other parameters have large uncertainties.

The maximum uncertainty lies in the parameters of the rho coupling. This happens because the effects of the rho meson, which are varied, generally manifest in conjunction with other mesons. For example, along with ω exchange, ρ exchange helps in balancing some part of the attractive σ exchange at short distances, which, as we know, is important in producing the right binding energy in nuclei [5]. In nucleon-nucleon scattering, it has a large influence in determining the low partial-wave phase shifts [6].

It is only in the spin-isospin part of the nucleon-nucleon interaction, as mentioned earlier, that the rho exchange has a determining effect. Physical processes which directly depend upon the spin-isospin interaction, and hence on the one-pion- and -rho-exchange potentials, are charge-exchange processes, such as the Gamow-Teller part of the $p(n,p)n$ and $p(p,n)\Delta^{++}$ reactions. Because of the large mass difference between a nucleon and delta, together these reactions also span a large range of momentum transfer and, hence, the interaction in more detail. Experimentally [7,8], several measurements have been done by now on these reactions, and hence enough data at intermediate energies exist on them. In the present paper, we have theoretically studied these two reactions and have examined their sensitivity to the interaction parameters of the one-rho-exchange interaction and the length parameter associated with the one-pion-exchange potential. We have put some constraints on the values of these parameters by comparing the calculated results with the experiments. In the calculations the effect of other channels such as elastic scattering, etc., has been incorporated in the framework of the distorted-wave Born approximation. The justification for this approximation for the charge-exchange nucleon-nucleon reactions has been discussed in our earlier work [9].

Section II contains the general formalism. In Secs. III and IV, it is specialized to the cases of $p(n,p)n$ and $p(p,n)\Delta^{++}$ reactions, respectively, and subsequently used to analyze the experimental data. The conclusions are summarized in Sec. V.

We find that the cross sections for both charge-exchange reactions $p(n,p)n$ and $p(p,n)\Delta^{++}$ depend sensitively on the values of the interaction parameters. On comparing the calculated results with the measured ones, it is seen that data on the $p(n,p)n$ reaction can be reproduced only if the transition potential includes both the pion and rho exchange in it. The $p(p,n)\Delta^{++}$ reaction, on the other hand, requires a very weak coupling for the $\rho N\Delta$ vertex for the description of the experimental data. The coupling is found to be so weak that essentially the data can be reproduced by the one-pion-exchange potential only. For both reactions the values of the length parameters Λ_π $(=\Lambda_\pi^*)$ around 1.0–1.2 GeV/c and Λ_ρ $(=\Lambda_\rho^*)$ equal to 2 GeV/c reproduce the experimental data well. The values of the pion-coupling constants used are $f_\pi=1.008$ and $f_\pi^*=2.156$. f_ρ is taken equal to 7.815, and the value of f_ρ^* is found to have a value less than 3.

Comparing the above values of the interaction parameters with other work in the literature, the value of Λ_π in the analyses of NN scattering data [6], deuteron properties [10], electrodisintegration of the deuteron [11], the Goldberger-Triemen discrepancy [12], and dispersive theoretical approaches [13] also is around 1 GeV/c. For the value of the rho-meson coupling, there is much uncertainty; for example, the vector dominance model [14] gives $f_\rho=4.83$, while its determination from the nucleon form factor [15] and the nuclear phenomena [16,17] demands $f_\rho=7.815$. The value of f_ρ^* is known even with lesser certainty. The information which one can take with some confidence is the SU(2)×SU(2) quark-model relation, say,

$$f_\rho^*/f_\rho=f_\pi^*/f_\pi\equiv\alpha \; . \tag{5}$$

Quark models and various experiments, such as the $M1$ transition form factor in $eN\rightarrow e'\Delta$ [18], $NN\rightarrow NN$ potential, etc., give $1.7<\alpha<2$, except the Regge analysis [19] of $\pi N\rightarrow\pi\Delta$ at high energy, which gives $\alpha<1$.

II. FORMALISM

As given in our earlier work [9], including the effect of elastic and other channels, the transition matrix for the charge-exchange reactions in nucleon-nucleon scattering at intermediate energies can be given by

$$T_{\beta\alpha}(\mathbf{k}_f,\mathbf{k}_i)=(\chi_{\mathbf{k}_f}^{(-)},\langle\beta|v_{\beta\alpha}|\alpha\rangle,\chi_{\mathbf{k}_i}^{(+)}) \; , \tag{6}$$

where the χ's are the distorted waves. They are the solutions of potentials which describe the elastic scattering in respective channels. $v_{\beta\alpha}$ is the transition potential, which, as mentioned above, is a spin-isospin-dependent interaction. There are enough indications that this interaction, unlike the one which describes the elastic scattering, is dominantly given by the Born term of the potential generated by the exchange of isovector bosons, pions, and rhos [1]. $T_{\beta\alpha}$, as given by Eq. (6), has a useful structure that the effects due to the elastic and charge-exchange parts of the nucleon-nucleon interaction in it enter separately. This enables us to learn directly about $v_{\beta\alpha}$, once the χ's are known. Below the pion threshold, the scattering wave functions can be generated by the boson-exchange potentials. However, in the present work, since we only consider reactions above the pion threshold, the boson-exchange potentials are not of much use for describing the χ's. We have, therefore, used the eikonal approximation (which is valid at higher energies) and have written the χ's directly in terms of the elementary elastic-scattering amplitude (for details, see Ref. [9]) $f(k,q)$ as

$$\chi_{\mathbf{k}}(\mathbf{r})=e^{i\mathbf{k}\cdot\mathbf{r}}\left[1+\frac{i}{k}\int_0^\infty q\,dq\,J_0(qb)f(k,q)\right]^{1/2} , \tag{7}$$

where q is the momentum transfer and J_0 is the Bessel function. In the energy region of our interest, $f(k,q)$ can be reasonably parametrized as

$$f(k,q)=(k/4\pi)\sigma_T(i+\beta)\exp(-\alpha q^2/2) \; , \tag{8}$$

where σ_T is the total scattering cross section for the interacting particles, β is the ratio of the real to imaginary parts of the scattering amplitude, and α is the slope parameter. All these parameters are determined from the known scattering data. Therefore there is not much uncertainty in the description of the distorted waves χ's.

The description of χ by Eq. (7), of course, has an approximation. As described in our earlier work, we approximate the eikonal distorting function $D_k(r)$ as

$$D_{\mathbf{k}}^+(\mathbf{r})=\exp\left[-(i/\hbar v)\int_{-\infty}^{z}V(b,z')dz'\right] . \tag{9}$$

Combined with a similar approximation for the final state and writing in Eq. (6), this results in

$$\chi_{\mathbf{k}_f}^{(-)*}(r)\chi_{\mathbf{k}_i}^{(+)}(r)=e^{i\mathbf{q}\cdot\mathbf{r}}\exp[(i/2)\{\xi_i(b)+\xi_f(b)\}] \times\exp\left[-(i/\hbar v_p)\int_0^z V_p(b,z')dz' +(i/\hbar v_\Delta)\int_0^z V_\Delta(b,z')dz'\right] , \tag{10}$$

where $\xi(b)$ is the phase-shift function. The approximation in Eq. (7) thus amounts to neglecting in the above the quantity in the square brackets while evaluating $T_{\beta\alpha}$. This quantity represents the difference in the phase shifts suffered by the incident and outgoing particles in traveling a distance z from $z=0$. For the (p,n) reaction, this obviously vanishes. For the $p(p,n)\Delta^{++}$ reaction too, as estimated quantitatively in the Appendix, this approximation does not affect the final results beyond about 15%.

The description of $T_{\beta\alpha}$ by Eq. (6) also involves another assumption that the transition $\alpha\rightarrow\beta$ mainly takes place from the elastic channel. The effect of other channels on the elastic channel is, however, contained through Eq. (7). For the $pp\rightarrow n\Delta^{++}$ reaction, this assumption is quite valid at intermediate energies. For the $np\rightarrow pn$ channel at the same energies, there may be some contribution from channels such as

$$p+n\rightarrow\begin{cases}n+\Delta^{+}\rightarrow n+p,\\ \Delta^{0}+p\rightarrow n+p\ .\end{cases} \tag{11}$$

However, being second-order processes, their contribution might not be large.

III. $p(n,p)n$ REACTION

For the $p(n,p)n$ reaction, the differential cross section in terms of the $T_{\beta\alpha}$ is given by

$$\frac{d\sigma}{d\Omega}=\left[\frac{m^2}{2\pi\varepsilon}\right]^2\frac{k_f}{k_i}\langle|T_{pn}|^2\rangle \tag{12}$$

or

$$k_i^2\frac{d\sigma}{dt}=\left[\frac{m^2}{2\varepsilon}\right]^2\frac{1}{\pi}\langle|T_{pn}|^2\rangle\ , \tag{13}$$

where ε $(=\varepsilon_p+\varepsilon_n)$ is the total energy in the c.m. system and t $[=(k_i-k_f)^2]$ is the four-momentum transfer squared. In this case, because the energy transfer is zero, the three- and four-momentum transfers are equal. The angular brackets around $|T_{pn}|^2$ denote the sum and average over the spins in the initial and final states, respectively.

The experimentally measured cross section for the $p(n,p)n$ reaction has contributions from the spin-flip channel (GT) as well as the non-spin-flip channel (F). Therefore the T_{pn} in Eqs. (12) and (13) consists of a sum of the spin-isospin-dependent term $t_{\sigma\tau}$ and the only isospin dependent term t_τ, i.e.,

$$T_{pn}(\mathbf{k}_f,\mathbf{k}_i)=t^{C}_{\sigma\tau}(q)\langle(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\rangle+t^{\rm NC}_{\sigma\tau}(q)\langle S_{12}(\hat{\mathbf{q}})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\rangle+t_\tau(q)\langle(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\rangle\ , \tag{14}$$

where bracketed operators denote their expectation values over the spin-isospin wave function of the pn system. From Eq. (12), after carrying out the spin-isospin operation, we can write

$$\langle|T_{pn}|^2\rangle=12|t^{C}_{\sigma\tau}(q)|^2+(96\pi/5)\sum_M|t^{{\rm NC},M}_{\sigma\tau}(q)|^2+4|t_\tau(q)|^2\ , \tag{15}$$

where

$$t^{C}_{\sigma\tau}(q)=\langle\chi^{(-)}_{\mathbf{k}_f}(\mathbf{r})|V^{C}_{\sigma\tau}(r)|\chi^{(+)}_{\mathbf{k}_i}(\mathbf{r})\rangle \tag{16}$$

and

$$t^{{\rm NC},M}_{\sigma\tau}(\mathbf{q})=\langle\chi^{(-)}_{\mathbf{k}_f}(\mathbf{r})|V^{\rm NC}_{\sigma\tau}(r)Y_{2M}(\mathbf{r})|\chi^{(+)}_{k_i}(\mathbf{r})\rangle\ . \tag{17}$$

The superscripts C and NC denote the central and noncentral pieces, respectively. We observe in Eq. (15) that the contributions of these two parts to the spin-isospin t matrix are incoherent. They also occur incoherently in the isospin term t_τ.

For the one-pion-plus-one-rho exchange potential, using Eqs. (1)–(3), V^C and $V^{\rm NC}$ are given by

$$V^{C}_{\sigma\tau}(t)=\tfrac{1}{3}[V_\pi(t)+2V_\rho(t)]\ , \tag{18}$$

$$V^{\rm NC}_{\sigma\tau}(t)=\tfrac{1}{3}[V_\pi(t)-V_\rho(t)]\ . \tag{19}$$

The configuration-space representation of these potentials, required in Eqs. (16) and (17), are obtained by the usual Fourier transform.

For the proton emerging in the forward direction (i.e., $q=0$), the dominant contribution to $|T_{pn}|^2$ in Eq. (15) comes from the central terms only. The noncentral part contributes only up to a few percent. Therefore, for zero-degree emission, to a good approximation, we can write

$$\langle|T_{pn}|^2\rangle(0^\circ)\approx12|t^{C}_{\sigma\tau}(q)|^2+4|t_\tau(q)|^2\ . \tag{20}$$

Since in the present work our aim is to examine the sensitivity of the $p(n,p)n$ cross section to the pion- and rho-exchange potentials through $t_{\sigma\tau}$, for t_τ we have used the phenomenologically determined t matrix of Petrovitch and Love [20]. Microscopically, unlike the $\sigma\tau$ part of the interaction, the V_τ, in boson-exchange models, gets constructed from second- and higher-order Born terms only.

Experimental measurements on the $p(n,p)n$ reaction exist for the 0° emission angle from several groups in the 200–800 MeV incident neutron-energy range [7]. These cross sections, when multiplied by the squared beam momentum, have a constant value around 150 mb. Using Eqs. (13) and (20) for the cross section and Eq. (7) for the distorted waves, we have calculated $p^2(d\sigma/dt)(0^\circ)$. First, we calculate them for the one-pion-exchange potential only. The coupling constant f_π is taken equal to 1.008. This value describes well the pion-nucleon scattering data and is the generally accepted value [4]. The cutoff mass parameter Λ_π for pions, on the other hand, is a much less determined quantity. Analyses of the data on different experimental processes suggest that the value of Λ_π could vary to correspond to a very soft form factor (500 MeV/c) to a point form factor. We have therefore done calculations for several values of Λ_π. In Fig. 1 we show the calculated 0° cross section at 200–700 MeV beam energy for $\Lambda_\pi=0.65$, 1.0, and ∞ GeV/c. The parameters of the elementary elastic-scattering amplitude $f(k,q)$ in Eq. (8) are taken from Ref. [21]. For comparison, the available data from various experiments on the $p(n,p)n$ reaction are also shown in Fig. 1. From these results we

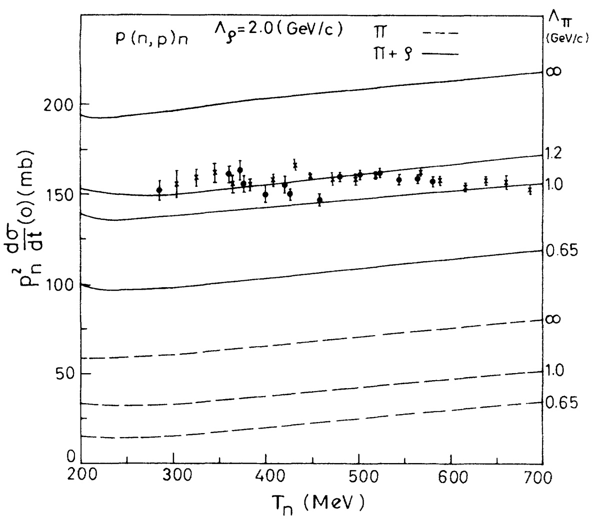

FIG. 1. Dependence of calculated $p^2(d\sigma/dt)(0°)$ on the incident neutron energy (T_n) for the $p(n,p)n$ reaction. The dashed and solid curves are, respectively, for the one-pion and one-pion-plus-one-rho-exchange transition potentials for various values of Λ_π and $\Lambda_\rho=2$ GeV/c. The experimental points are taken from Ref. [3].

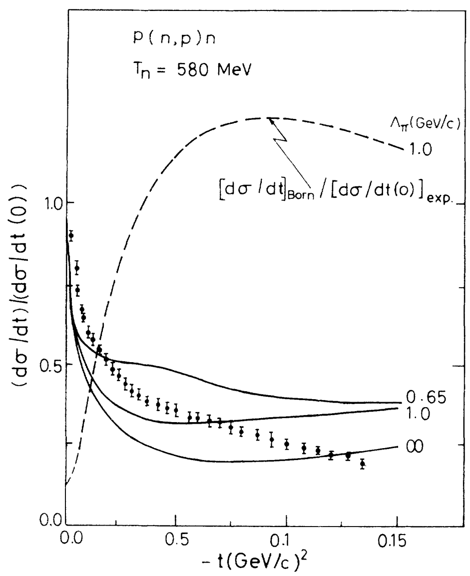

FIG. 2. Normalized four-momentum-transfer distribution for the $p(n,p)n$ reaction at 580 MeV incident neutron energy. The solid curves show the calculated results using the distorted-wave Born approximation and one-pion-exchange transition potential with various values of Λ_π. The Born-approximation results are given by the dashed curve, which is normalized to the experimental 0° cross section [≈ 105 mb/(GeV/c)^2]. The experimental points are taken from Ref. [3].

observe that, while the calculated results for all values of Λ_π reproduce the beam-energy dependence of the measured cross section, they all fall far below their magnitudes. The maximum calculated cross section is around 50 mb, while the measured one is near to 150 mb.

Experimental data on differential cross sections also exist for the $p(n,p)n$ reaction at some energies as a function of the momentum transfer. Using Eq. (15), as an illustrative example, we have calculated these cross sections at 580 MeV beam energy for the one-pion-exchange potential. They are shown (normalized to the 0° cross section) in Fig. 2 for $\Lambda_\pi=0.65$, 1.0, and ∞ GeV/c. The $|t_\tau|^2$ values, required here, are taken from Ref. [20]. For comparison, the experimental data are also shown. We find that in no region of the momentum transfer t and for no value of Λ_π do the calculated results agree with the measured values. In the low momentum-transfer region, the calculated results underestimate, and in the large momentum-transfer region, they overestimate the cross section.

Without the inclusion of the "distortion" [through Eq. (7)], the disagreement of the above calculated cross sections with the measured ones would have been even worse. The effect of distortion is large. For example, the finite value of the 0° cross section and the reproduction of its measured beam-energy dependence is due to the effect of distortion only. Without it, the Born estimate V_π, as given by Eqs. (1) and (3), gives a vanishing cross section at $q=0$. In the differential cross section, as shown in Fig. 2, the Born-approximation results (normalized to the 0° experimental cross section) are in complete discord with the measured cross sections. The distortion reduces this discord considerably.

Thus we find that with the one-pion-exchange potential prescription for the spin-isospin potential it is not possible to account for the measured cross sections on the $p(n,p)n$ reaction. However, if we examine the nature of this disagreement *vis-á-vis* the contribution of rho exchange to $V_{\sigma\tau}$ [Eqs. (18) and (19)], it seems that the inclusion of rho exchange would go in the right direction to produce better agreement between the calculated and measured cross sections. As required for the better agreement, the ρ exchange strengthens the central part and weakens the tensor part of $V_{\sigma\tau}$. To get quantitatively the contribution of the rho-exchange potential, we require the values of the coupling constant f_ρ and the length parameter Λ_ρ. These parameters, as mentioned earlier, are known with much less certainty. We take the strong ρ-meson coupling values, i.e., $f_\rho^2=61.07$ and $\Lambda_\rho=2$ GeV/c, which seem to describe many of the nuclear physics phenomena without much difficulty [6]. In Fig. 1 we show the zero-degree cross section in the beam energy range 200–700 MeV for the one-pion-plus-one-rho-exchange potential for $\Lambda_\pi=0.65$, 1.0, 1.2, and ∞ GeV/c. As we see, the measured cross sections are well reproduced in both magnitude and shape for $\Lambda_\pi=1.0$–1.2 GeV/c. This combination of the pion- and rho-exchange potentials is also known to be required for describing many other nuclear physics phenomena.

The effect of ρ exchange can also be calculated on the differential cross section. However, to get correct results up to larger scattering angles, one should describe the distortion more accurately. The eikonal approximation, used here, should give reasonably correct results only near forward angles. In any case, to see the trend of the changes in the differential cross section due to the ρ exchange, in Fig. 3 we show the calculated results for the

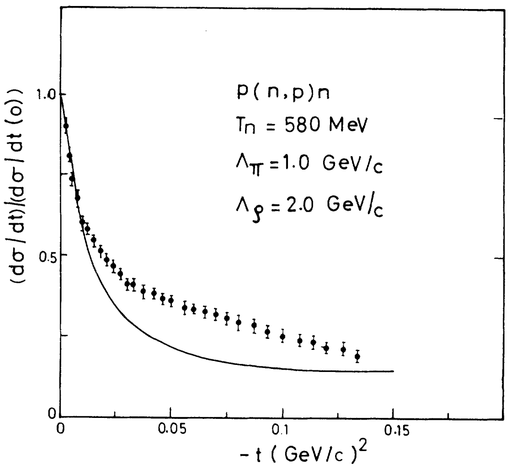

FIG. 3. Same as Fig. 2. The solid curve shows the calculated result using the distorted-wave Born approximation and one-pion-plus-one-rho-exchange transition potential with $\Lambda_\pi=1.0$ GeV/c and $\Lambda_\rho=2.0$ GeV/c.

pion-plus-rho-exchange transition potential. The results are shown for $\Lambda_\pi=1.0$ GeV/c. The parameters for the rho coupling are taken as in the zero-degree cross sections. The measured cross sections are also shown for comparison. As we see, the inclusion of the ρ exchange reduces the disagreement between the calculated and measured cross sections considerably. The cross sections near the forward direction are completely reproduced. The extent of disagreement at larger angles, though not removed completely, also gets reduced.

IV. $p(p,n)\Delta^{++}$ REACTION

The $p(p,n)\Delta^{++}$ reaction differs from the $p(n,p)n$ reaction in several respects. First, it can only be initiated by the spin-isospin-dependent interaction. Terms analogous to the Fermi transition do not contribute. Second, the momentum transfer in the delta production reaction is much larger (> 300 MeV/c). Therefore the $p(p,n)\Delta^{++}$ reaction samples the large momentum part of the spin-isospin potential. In addition, the boson-coupling parameter at one vertex corresponds to the $p\to\Delta^{++}$ transition.

The differential cross section for the $p(p,n)\Delta^{++}$ reaction, like Eq. (13), is written as

$$k_i^2\frac{d\sigma}{dt}=(m_\Delta m^3/4\pi\varepsilon^2)\langle|T_{\Delta^{++}p}(\mathbf{k}_f,\mathbf{k}_i)|^2\rangle \ . \quad (21)$$

$\langle|T_{\Delta^{++}p}|^2\rangle$, as shown in our earlier work [9], is given by

$$\langle|T_{\Delta^{++}p}|^2\rangle=4|t_{ST}^C|^2+(32\pi/5)\sum_M|t_{ST}^{\mathrm{NC},M}|^2 \ , \quad (22)$$

where

$$|t_{ST}^C|^2=|t_{ST}^C(\mathbf{k}_f,\mathbf{k}_i)-t_{ST}^C(\mathbf{k}_f,-\mathbf{k}_i)|^2 \quad (23)$$

and

$$|t_{ST}^{\mathrm{NC},M}|^2=|t_{ST}^{\mathrm{NC},M}(\mathbf{k}_f,\mathbf{k}_i)-t_{ST}^{\mathrm{NC},M}(\mathbf{k}_f,-\mathbf{k}_i)|^2 + \tfrac{3}{2}\,\mathrm{Re}[t_{ST}^{\mathrm{NC},M}(\mathbf{k}_f,\mathbf{k}_i)t_{ST}^{\mathrm{NC},M}(\mathbf{k}_f,-\mathbf{k}_i)] \ . \quad (24)$$

Here S and T represent the transition spin and isospin matrices, respectively, for $p\to\Delta^{++}$. The expressions for t^C and t^{NC} are same as those given in Eqs. (16) and (17) except that the interactions V^C and V^{NC}, now, are for $pp\to n\Delta^{++}$. Because of the large momentum transfer, unlike the $p(n,p)n$ reaction, the dominant contribution to $\langle|T_{\Delta^{++}p}|^2\rangle$ now comes from the noncentral term.

For the one-pion-plus-one-rho-exchange potential, here, too, V_{ST}^C and V_{ST}^{NC} are given by Eqs. (18) and (19), with $V(\omega,q)$, in the one-pion- or one-rho-exchange model, given by

$$V_x(t)=\frac{f_x}{m_x}F_x(t)\frac{f_x^*}{m_x}F_x^*(t)\frac{q^2}{t-m_x^2} \ . \quad (25)$$

Here f_x^* is the coupling constant for the $xp\Delta^{++}$ vertex. For the form factor F, again the monopole form is used.

In the above expressions, we have, of course, assumed that the delta has a fixed mass. In the actual case, however, this is not true. The delta has a mass distribution. Incorporating this, the missing-mass spectrum in the $p(p,n)\Delta^{++}$ reaction can be written as

$$\frac{d\sigma}{d\mu}=\int_{-1}^{+1}(\mu^2m^3k_f/\pi\varepsilon^2k_i)\rho(\mu^2)\langle|T_{\Delta^{++}p}|^2\rangle d\cos\theta \ , \quad (26)$$

where μ is the mass of the delta and $\rho(\mu^2)$ is its distribution function. The form of $\rho(\mu^2)$ is fixed by the analysis of the data on the $\pi^+p\to\pi^+p$ scattering [22].

The experimental data on the $p(p,n)\Delta^{++}$ reaction are available from threshold to 5.5 GeV/c beam momentum [8]. For the purpose of the present paper, we consider the missing-mass spectrum at 2.23 and 4.0 GeV/c beam momenta and the differential cross section $d\sigma/dt$ at 2.23 GeV/c incident momentum. First, we calculate these cross sections for the one-pion-exchange potential. The value of f_π^* is taken equal to 2.156. This value along with $f_\pi=1.008$ describes the pion-nucleon potential and the delta width well. For the value of Λ_π^*, we have assumed $\Lambda_\pi^*=\Lambda_\pi$. In quark models this assumption may be justified, as the spatial behavior of quark wave functions in the nucleon and delta might not be different from each other.

For the above calculations, another set of new quantities which we need pertain to distortion in the $n\Delta^{++}$ channel. Experimentally, practically no direct information exists on it. In view of this, we have taken guidance from the fact that the delta is an intrinsically excited state of a nucleon and thus carries about 300 MeV more energy than the nucleon at the same kinetic energy. Therefore it is plausible that the scattering parameters for the $n\Delta^{++}$ system at kinetic energy T may be similar to those for protons at $T+300$ MeV kinetic energy. Hence, as a reasonable guess, we have taken the distortion parameters for the final state in the $p(p,n)\Delta^{++}$ reaction corresponding to the nucleon-nucleon system at this enhanced energy.

In Fig. 4 we show the calculated differential cross section $d\sigma/dt$ for several values of Λ_π. Comparing them with the experimentally measured cross sections [8], we find that the latter can be reproduced very well for

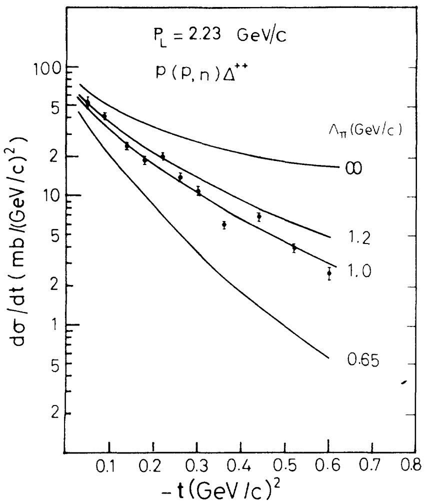

FIG. 4. Four-momentum-transfer distribution for the $p(p,n)\Delta^{++}$ reaction at 2.23 GeV/c incident-beam momentum. The solid curves show the calculated results using the one-pion-exchange transition potential with various values of Λ_π. The experimental points are taken from Ref. [4].

$\Lambda_\pi = 1.0-1.2$ GeV/c. This is in contrast with the $p(n,p)n$ reaction where the one-pion-exchange potential could not reproduce the experimental data for any value of Λ_π. The value of Λ_π required here is, however, the same as that needed to reproduce the $p(n,p)n$ data in the pion-plus-rho-exchange model.

In Figs. 5 and 6, we show the calculated missing-mass spectra $d\sigma/d\mu$ for the beam energies 2.23 and 4.0 GeV/c. The results are shown for the best-fit value of Λ_π, which is equal to 1.0 GeV/c. Again, the agreement with the experimental data is very good.

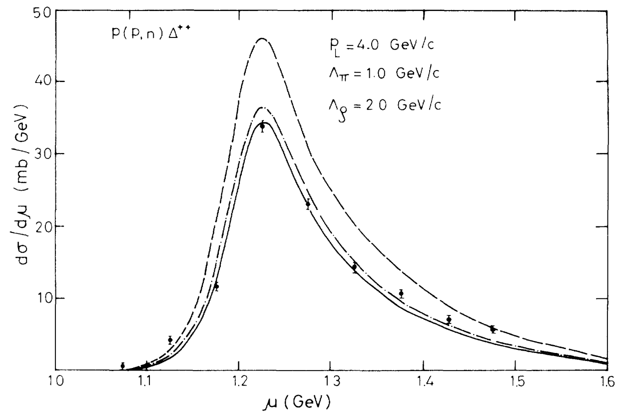

FIG. 6. Same as Fig. 5 for 4.0 GeV/c beam momentum. Experimental points are from Ref. [4].

We also calculate the effect of one-rho exchange on the $p(p,n)\Delta^{++}$ reaction. For this, as in the one-pion exchange, we need additionally the value of the coupling constant f_ρ^* and the length parameter Λ_ρ^*. For the Λ_ρ^* we still feel that it is reasonable to assume $\Lambda_\rho^* = \Lambda_\rho$. The value of f_ρ^*, as mentioned in the Introduction, is unknown to a great extent. For the present purposes, we take $f_\rho^* = 14.4$ ($\alpha = 1.8$ [Eq. (5)]), which, to a great extent, is favored by the nucleon-nucleon potential [6]. Calculated results with the ρ exchange potential plus the one-pion-exchange potential for $d\sigma/dt$ are shown in Fig. 7. The value of Λ_π is taken equal to 1.0 GeV/c and varied between 1.5 and 2.5 GeV/c. On comparing these results with those due to the one-pion exchange only (in Fig. 4), we find that the effect of the rho exchange is large. This is understandable as the $p(p,n)\Delta^{++}$ reaction

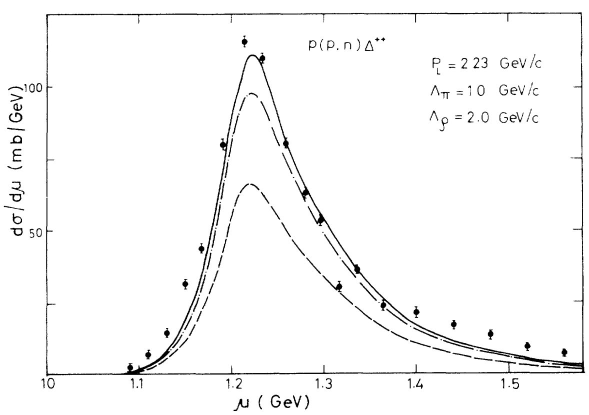

FIG. 5. Missing-mass spectrum for the $p(p,n)\Delta^{++}$ reaction at 2.23 GeV/c incident-beam momentum. The solid curve shows the calculated result using the one-pion-exchange transition potential with $\Lambda_\pi = 1.0$ GeV/c. The dashed curve shows the calculated result using the one-pion-plus-one-rho-exchange transition potential. Λ_ρ is 2.0 GeV/c. The dot-dashed curve shows the calculated results using the one-pion-plus-one-rho-exchange transition potential when the coupling strength f_ρ^* is reduced to 20% of its earlier value. The experimental points are taken from Ref. [4].

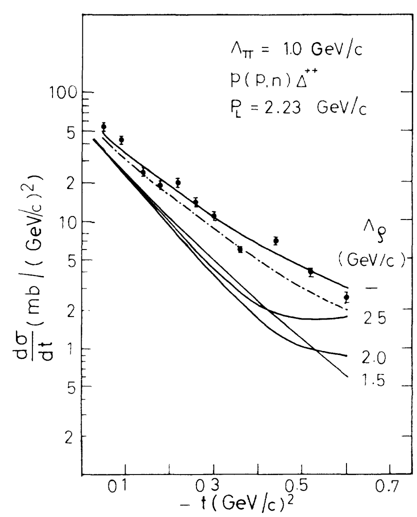

FIG. 7. Four-momentum distribution for the $p(p,n)\Delta^{++}$ reaction at 2.23 GeV/c incident-beam momentum. The solid curves show the calculated results using one-pion-plus-one-rho-exchange transition potential with $\Lambda_\pi = 1.0$ GeV/c and various values of Λ_ρ. The calculated results when the coupling strength f_ρ^* is reduced to 20% of its earlier value is shown by the dot-dashed curve. The experimental points are from Ref. [4].

involves higher momentum transfer and the heavier bosons (m_ρ=770 MeV) contribute more in this domain. However, on comparison with experiments, we find that, for all values of Λ_ρ, the addition of rho exchange causes a big disagreement between the calculated and measured cross sections. This is surprising. Again, because of the large momentum transfer, one expects that the $p(p,n)\Delta^{++}$ reaction should have preferred the pion-plus-rho-exchange potential.

The calculated results for the missing-mass spectrum $d\sigma/d\mu$ for the one-pion-plus-one-rho-exchange potential are shown in Figs. 5 and 6. Here we have shown the results for Λ_ρ=2.0 GeV/c only. The trend of the results for other values of Λ_ρ is similar to that seen earlier for $d\sigma/dt$. Here we observe that the introduction of the rho exchange has a large effect, but again it goes in a direction which introduces a large disagreement with the experimental data.

In order to see that, up to a certain value of f_ρ^*, the one-pion-plus-one-rho-exchange potential could be consistent with the data, in Figs. 5–7 we have also shown results with the coupling of the rho meson for $p\rightarrow\Delta^{++}$ reduced to 20% of its value used earlier. It appears that with this much reduction the calculated results, to a certain extent, can be made consistent with the experimental data within experimental errors. This may be interpreted as saying that the maximum value of f_ρ^* consistent with the $p(p,n)\Delta^{++}$ is around $0.2\times 14.4\approx 3$.

V. CONCLUSIONS AND DISCUSSION

Thus, from the results of this paper, while we conclude that, in general, the one-pion-plus-one-rho-exchange potential is a valid description of the spin-isospin potential in the $p(n,p)n$ and $p(p,n)\Delta^{++}$ reactions, the role of rho exchange in the two reactions varies. In the nucleon-nucleon sector, the rho-exchange potential, with f_ρ=7.814, is found absolutely necessary to account for the data on the $p(n,p)n$ reaction. For the $pp\rightarrow n\Delta^{++}$ transition, on the other hand, it appears that such a potential is not required at all. This seems very surprising. However, one should remember that the regions of the energy (ω) and the momentum (q) which appear in the rho-meson propagator in two cases are very different. In the $p(n,p)n$ reaction, both these quantities are very near to zero, and in the $p(p,n)\Delta^{++}$ reaction, both of them are quite large. Therefore the coupling of the rho meson to the nucleon (or maybe quarks in it) occurs at very different values of the energy and momentum in the two reactions. In addition, on the quark level, the intrinsic energy of quarks in the delta is higher than that in the nucleon. From the studies of Brown [23], in the nucleon sector, we already know that the interaction of nucleons in the rho channel is very complicated. The strength associated with it depends upon the available energy. At higher energies the rho coupling for nucleon-nucleon potential seems to become weaker. What happens to this coupling in a transition from a nucleon to a real delta is even more difficult to answer. The present analysis suggests that this coupling probably is much weaker than usually assumed. The data on the $p(p,n)\Delta^{++}$ reaction seem to accept a value for f_ρ^* of at the most around 3. The normally accepted value is around 14.4. A similar conclusion has also been reached by Dmitriev, Sushkov, and Gaarde [24] about rho exchange in the $p(p,n)\Delta^{++}$ reaction.

Finally, before we conclude the paper, we wish to make some comments on certain approximations used in this work. For the distorted waves χ in Eq. (6), we have used the eikonal approximation. Normally, one should like to solve the relevant wave equation for it exactly. In a recent paper by Kelkar and Jain [25], an estimate of error due to this approximation for beam energies of 400 MeV–3 GeV has been made. They have calculated the (p,Δ^{++}) cross section in nuclei using the exact and eikonal approximated χ. They find that the eikonal approximation, at the most, can underestimate the final cross section by 15%.

For writing the one-boson-exchange potential [Eqs. (1)–(4)], we have used the nonrelativistic approach. For small energy transfer, it is, of course, well justified. For large energy transfer, which occurs in our calculations, the uncertainty in our final results needs to be estimated. In a recent paper by Jain, Kelkar, and Londergan [26], this estimate, in fact, has been made for beam energies of 400 MeV–3 GeV. Their calculations give the ratio of the one-pion-exchange potential for $pp\rightarrow n\Delta^{++}$ using the relativistic Lagrangian and its nonrelativistic reduction in the four-momentum transfer (t) range 0.1–0.6 (GeV/c)^2. It is found that, up to 1 GeV, this ratio is near to unity in the whole t range. Around 3 GeV, however, it starts deviating from unity for t below 0.15 (GeV/c)^2 or so.

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APPENDIX

Here we give a quantitative estimate of the error introduced in the results presented in this paper due to the approximation [given in Eq. (7)] to the eikonal wave function. The cross section for the $p(p,n)\Delta^{++}$ reaction, due to large momentum transfer, is mainly determined by the

TABLE I. t_{SI}^{NC} for the two values of a [$=(V_\Delta^0/\sigma_\Delta)/(V_p^0/\sigma_p)$] in $p(p,n)\Delta^{++}$ reaction.

	Re $t_{ST}^{NC,M=0}(q,\omega)$ (MeV fm^3)		
		a=0.5	
$-t$ [(GeV/c)^2]	a=1	r_0=2.0 fm	r_0=1.5 fm
0.027	−153.58	−153.36	−153.11
0.1	−38.22	−38.03	−37.82
0.2	−9.65	−9.51	−9.34
0.3	−1.69	−1.58	−1.45
0.4	0.91	0.98	1.08
0.5	1.59	1.63	1.71
0.6	1.46	1.50	1.55
0.7	1.02	1.04	1.07

noncentral part $t_{ST}^{\rm NC}$ of the t matrix. We therefore estimate the error in this term.

$t_{ST}^{\rm NC}$, as written in Eq. (14), is

$$t_{ST}^{{\rm NC},\,M}(\mathbf{q})=\int d\mathbf{r}\,\chi_{\mathbf{k}_\Delta}^{(-)*}(\mathbf{r})V_{\sigma\tau}^{\rm NC}(r)Y_{2M}(\hat{\mathbf{r}})\chi_{\mathbf{k}_p}^{(+)}(\mathbf{r})\ . \quad \text{(A1)}$$

The product of the distorted waves χ is given by

$$\chi_{\mathbf{k}_\Delta}^{(-)*}(\mathbf{r})\chi_{\mathbf{k}_p}^{(+)}(\mathbf{r})=e^{i\mathbf{q}\cdot\mathbf{r}}\exp\left[-(i/\hbar)\left\{(1/v_p)\int_{-\infty}^{z}V_p(\mathbf{b},z')dz'+(1/v_\Delta)\int_{z}^{\infty}V_\Delta(\mathbf{b},z')dz'\right\}\right]\ , \quad \text{(A2)}$$

where $\mathbf{q}=\mathbf{k}_p-\mathbf{k}_\Delta$. In terms of the phase-shift function $\xi(b)$, this product can be further written as

$$\chi_{\mathbf{k}_\Delta}^{(-)*}(\mathbf{r})\chi_{\mathbf{k}_p}^{(+)}(\mathbf{r})=\exp[i\{\mathbf{q}\cdot\mathbf{r}+\tfrac{1}{2}[\xi_p(\mathbf{b})+\xi_\Delta(\mathbf{b})]\}]e^{iX(\mathbf{b},z)}\ , \quad \text{(A3)}$$

where

$$X(b,z)=(1/\hbar v_\Delta)\int_0^z V_\Delta(b,z')dz' -(1/\hbar v_p)\int_0^z V_p(b,z')dz'\ . \quad \text{(A4)}$$

In our calculations it is this factor, $X(b,z)$, that we have neglected. To estimate the effect due to this, for simplicity, we take a Gaussian form of the distorting potential in $X(b,z)$ and write

$$X(b,z)=-\frac{V_p^0 r_0}{\hbar v_p}e^{-(b/r_0)^2}\,\mathrm{erf}(z/r_0)\{1-a\}\ , \quad \text{(A5)}$$

where

$$a=\frac{V_\Delta^0/v_\Delta}{V_p^0/v_p}\ , \quad \text{(A6)}$$

and the V^0's are the strengths of the distorting potentials. r_0 is the range of the potential.

In Eq. (A6), a, in fact, is the ratio of the mean free paths for the delta and proton. In case they are equal, X vanishes. For calculating the effect, if they are not equal, we approximate V_p^0, in Eq. (A5), using the optical theorem and the Born approximation for the scattering amplitude as

$$V_p^0\approx -i\frac{\hbar\sigma_p^T v_p}{2(\pi r_0^2)^{3/2}}\ , \quad \text{(A7)}$$

where σ_p^T is the total cross section for pp scattering. The term in the denominator is the volume integral of the Gaussian form. We calculate $t_{ST}^{\rm NC}$ for σ_p^T equal to 40 mb and r_0 equal to 1.5 and 2 fm. The effect of our approximation, as we see in Eq. (A5), is determined by the value of a. We take it equal to $\frac{1}{2}$. For comparison, the results are also given for $a=1$, which corresponds to the results given in the text. The results for various values of the momentum transfer are given in Table I. As we see, the difference among various results is not much. Because of its dominating contribution, in Table I we have shown results for $M=0$ and Re($t_{ST}^{\rm NC}$) only.

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