Nonlocal field theory model for nuclear matter

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Nuclear matter is investigated in the relativistic Hartree approximation to a nonlocal σ-ω model containing short distance vertex form factors to simulate an underlying QCD substructure. At the Hartree level only the nucleon momentum dependence of the distributed vertex enters and the resulting finite nonlocal field theory model is solved in Euclidean metric with simple Gaussian forms for the so-called sideways form-factors. To reproduce saturated nuclear matter the nonlocal model selects form-factor ranges at the nucleon mass scale.

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The Walecka model or quantum hadrodynamics (QHD) [1] provides an effective and practical relativistic model for nuclear matter and nuclei in terms of point nucleon and meson fields. At the mean field level, a simple relativistic mechanism is provided for nuclear matter saturation and a successful and efficient phenomenology for ground and excited states of finite nuclei has been developed. QHD is renormalizable, and quantum loop effects can be addressed explicitly. However, point coupling causes quantum loops at momentum scales as high as 5 GeV [2] to strongly influence the final results at the Fermi momentum scale. The same mechanism can produce a tachyon pole in the scalar meson propagator and appears to be a general problem of models lacking asymptotic freedom in the coupling of fermions to scalars [3]. Quantum effects at the two-loop level [4] tend to destroy reasonable phenomenological interplay between short range repulsion and long range attraction in nuclear matter that occurs naturally at the mean field level.

It is not physically satisfying to have hadronic quantum loop effects strongly dependent upon short distance scales where the physically relevant degrees of freedom are not exclusively hadronic. Some progress has been made towards identification of effective models of interacting composite hadrons by integration over the fields of simplified models of QCD [5]. The resulting structure is complex. One feature is clear however: the three-point vertex is distributed due to the underlying substructure. When quark fields are integrated out of QCD models to produce composite $\bar{q}q$ mesons [5, 6], the three-point quark-meson vertex at the on-mass-shell point for the meson is the $\bar{q}q$ meson internal form factor. This is a sideways form factor for quark-meson coupling as it falls off with increasing quark momentum. With a quark model of a nucleon, this effect necessarily produces a falloff with the nucleon momentum [7]. The need for a sideways form factor in QHD has recently been argued from the basis of consistency with large-N_c QCD [8]. Rather than attempt to base our considerations on a particular quark model, we explore how QHD operates for nuclear matter when modified by simple phenomenological sideways form factors. This investigation is complementary to a recent work [9] which explores the effect of a conventional form-factor suppression of large meson momentum; and also complementary to work in progress [10] to generate that mechanism from meson dressing within QHD [11].

A distributed three-point coupling of an ω meson field to nucleon fields can be written in momentum space as

$$S_{\rm int} = \int d^4k' d^4k\, \bar{\psi}(k')\Gamma_\mu\left(\frac{k'+k}{2};k'-k\right) \times\omega_\mu(k'-k)\psi(k)\,. \quad (1)$$

We use a Euclidean metric throughout such that $a\cdot b = a_\mu b_\mu$ and $\{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu}$. For the special case of the Hartree approximation in a uniform system, the self-consistent meson fields contain only a zero-momentum component $\omega_\mu(q) = \delta^4(q)\omega_\mu$ and we have $S_{\rm int} = \int d^4k\,\bar{\psi}(k)\Gamma_\mu(k)\omega_\mu\psi(k)$.

We define a nonlocal QHD model (NL-QHD) for a uniform system by the action

$$S = \int d^4k\,\bar{\psi}(k)G^{-1}(k-g_v\omega)\psi(k) + \frac{1}{2}\int d^4x(m_s^2\phi^2 + m_v^2\omega^2)\,, \quad (2)$$

where the inverse nucleon propagator is

$$G^{-1}(k) = i\gamma\cdot kF_v(k^2) + M - g_s(\phi - v)F_s(k^2)\,, \quad (3)$$

and $v =<\phi>$ is the vacuum expectation of the scalar field. This ensures that the parameter M is the physical mass of the free nucleon. The functions $F(k^2)$ are the phenomenological form factors chosen with the normalization $F_v(k^2=-M^2) = F_s(k^2=-M^2) = 1$. For simplicity, we choose the form $F(k^2) = \exp[-\beta^2(1+k^2/M^2)]$, thereby introducing parameters β_s and β_v. In (3) we have chosen to couple the vector field via minimal substitution. This choice leads to the correct Einstein relation between the energy and momentum for a nucleon in uniform scalar and vector fields. The implied irreducible three-point vertices at tree level are identified from $\Gamma^s(k) = \delta G^{-1}/\delta\phi$ and $\Gamma^v_\mu(k) = \delta G^{-1}/\delta\omega_\mu$, both evaluated at the vacuum configuration $\phi = v$ and $\omega_\mu = 0$. Thus $\Gamma^s(k) = -g_s F_s(k^2)$ and $\Gamma^v_\mu(k) = -g_v \frac{\partial}{\partial k_\mu} i\gamma \cdot k F_v(k^2)$. Since the latter obeys the Ward identity, our choice for vector field coupling would yield the correct limit in the massless field case. Our chosen action reduces to the QHD action in the point coupling limit $F_s = F_v = 1$ for a uniform system if a fourth-order polynomial $V(\phi)$ is added for renormalization purposes. We treat the finite nonlocal model without such a term.

A nucleon Fermi sea can be treated through a chemical potential constraint which here corresponds to the shift $k_4 \to k_4 + i\mu$, and the chemical potential μ will become the Fermi energy. A systematic procedure for developing approximate solutions can be defined through a loop expansion of the generating functional $Z(\mu) = N \int D\bar\psi\, D\psi\, D\phi\, D\omega \exp(-S)$. The functional path integral is a sensible approach to quantization of the nonlocal action S because it coincides with the general structure of effective hadronic field models produced by (path) integration over point QCD fields [5]. After integration over the nucleon fields, we have $Z(\mu) = N' \int D\phi\, D\omega \exp(-\mathcal{S})$ where the meson action that includes all nucleon loop effects is

$$\mathcal{S}[\mu,\phi,\omega] = -\left[\mathrm{Tr}\ln G^{-1}(\mu)G(0)\right] - \mathrm{Tr}\ln G^{-1}(0) + \frac{1}{2}\int d^4x(m_s^2\phi^2 + m_v^2\omega^2)\,. \quad (4)$$

The trace Tr includes a discrete trace over isospin and relativistic spin indices as well as a 4-space integral.

The energy density $\mathcal{E}$ for a static uniform system can be obtained directly by forming the effective action Γ defined from $\ln Z$ by the Legendre transformation in standard fashion and using $\Gamma = \int d^4x\,\mathcal{E}$. The loop expansion [12] for Γ when truncated at the lowest or classical term defines the Hartree approximation which is $\Gamma[\rho_B,\phi,\omega] = \mathcal{S}[\mu,\phi,\omega] - \mu(\partial\mathcal{S}/\partial\mu)$, where $\partial\mathcal{S}/\partial\mu = -\int d^4x\,\rho_B$ and ρ_B is the baryon density. The equations of motion are $\delta\Gamma/\delta\phi = \delta\Gamma/\delta\omega_\mu = 0$ and physical nuclear matter resides at the value of ρ_B which minimizes the energy density. It is convenient to write $\mathcal{E} = U_F + U_v$, where U_F is the Fermi-sea component due to the first term of (4) and U_v is the vacuum component due to the ($\mu = 0$) remainder.

We first identify the field configurations that minimize $U_v[\phi,\omega]$. It is easily verified that $\omega_\mu = 0$ is the vacuum value because ω can be removed from the vacuum Tr ln term by a shift of the integration momentum. Only $\tilde U_v = U_v[\phi,\omega] - U_v[v,0]$ is relevant and the result is

$$\tilde U_v[\phi,\omega] = -\frac{1}{2}\mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\ln\left[\frac{k^2F_v^2(k^2) + \{M - g_s(\phi - v)F_s(k^2)\}^2}{k^2F_v^2(k^2) + M^2}\right] + \frac{1}{2}\left[m_s^2(\phi^2 - v^2) + m_v^2\omega^2\right]\,. \quad (5)$$

The vacuum expectation value of ϕ at the Hartree level is defined by $\partial\tilde U_v/\partial\phi = 0$ at $\phi = v$, and this yields

$$v = -\frac{Mg_s}{m_s^2}\mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\frac{F_s(k^2)}{k^2F_v^2(k^2) + M^2}\,. \quad (6)$$

The vacuum quantities in (5) and (6) are finite due to the suppression of high momenta by the form factors. The vacuum effective potential $\tilde U_v$ is a nonlinear function of the shifted field $\hat\phi = \phi - v$. The first term in (5) is the baryon loop term, which we denote by $L(\hat\phi)$. With the form-factor range parameters β_s =0.8 and β_v= 0.2 that we shall use for nuclear matter, the calculated vacuum effective potential for ω_μ =0 is shown in Fig. 1 by the solid line. This is the combined result of $L(\hat\phi)$ and the scalar mass term which are also displayed. An absolute minimum will always exist in this finite model, because in the asymptotic regions where $|\hat\phi| \longrightarrow \infty$ the logarithmic decrease of $L(\hat\phi)$ cannot overcome the quadratic growth of the scalar mass term. The loop term here contains contributions that are quadratic and higher in $\hat\phi$. The quadratic term must be used to renormalize the scalar mass. The result can be expressed as

$$\tilde U_v[\hat\phi,\omega] = \frac{1}{2}\left(\bar m_s^2\hat\phi^2 + m_v^2\omega^2\right) + L^{(3)}(\hat\phi)\,, \quad (7)$$

where the physical scalar mass is $\bar m_s^2 = m_s^2 + L''(\hat\phi = 0)$, and $L^{(3)}(\hat\phi)$ is the loop function without linear and quadratic terms in $\hat\phi$. We have chosen the bare mass m_s so that $\bar m_s$= 550 MeV.

In contrast, the usual procedure followed in the point coupling QHD [1] leads to the replacement of $L^{(3)}(\hat\phi)$ in (7) by $L^{(5)}_{PT}(\hat\phi)$ where the latter is the point coupling

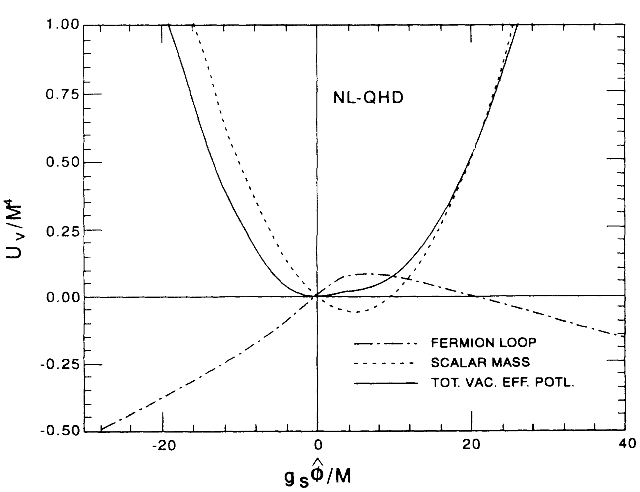

FIG. 1. The total vacuum effective potential U_v and its two separate contributions plotted as a function of the scalar field with dimensionless units for both axes.

limit of the loop term of (5) with the (divergent) first four powers of $\hat{\phi}$ renormalized to zero. Then v=0 and $\bar{m}_s$ is also the bare mass. The resulting $\tilde{U}_v(\hat{\phi})$ has no absolute minimum. This can be seen from the closed form expression [4] for $L^{(5)}_{PT}(\hat{\phi})$ which is dominated at large $|\hat{\phi}|$ by $L^{(5)}_{PT}(\hat{\phi}) \approx -\frac{1}{4\pi^2}M^{*4}\ln|M^*/M|$, where $M^* = M - g_s\hat{\phi}$. There is a local minimum (by definition) at $\hat{\phi}$=0 and the effect of the nucleon Fermi sea will produce a local minimum in the total energy at a physically sensible $\hat{\phi} > 0$. Strictly speaking though, it would also be possible to arbitrarily lower the energy by allowing consideration of very large (and unphysical) values of $|\hat{\phi}|$. Such phenomena have been pointed out previously [13].

The Fermi-sea contribution to the energy density is identified from the first term of (4) and is $\int d^4x\, U_F = -(1 - \mu\frac{\partial}{\partial\mu})\mathrm{Tr}\ln[G^{-1}(\mu)G(0)]$. Spectral resolution of the nucleon propagator together with contour integration produces a summation of the Fermi-sea energies. Details for this procedure can be found in Ref. [7] for the related case of quarks and mesons. The eigenvalue problem to be solved is

$$\left[i\gamma\cdot pF_v(p^2) + M - g_s\hat{\phi}F_s(p^2)\right]u(p) = 0\,, \tag{8}$$

where $p = k - g_v\omega$ and k is the nucleon momentum. This is equivalent to a determination of $p^2 = -M^{*2}$, where $M^*(\hat{\phi})$ is a nonlinear function satisfying the transcendental equation

$$\frac{M^*}{M}F_v(-M^{*2}) = 1 - \frac{g_s\hat{\phi}}{M}F_s(-M^{*2})\,. \tag{9}$$

It is necessary to be able to continue the form factors into a limited region of the timelike (non-Euclidean) domain to find these physical solutions. We accept the consequences of the chosen Gaussian form factors. The energy of a Fermi-sea nucleon in the uniform medium characterized by $\omega = (i\omega_0, \boldsymbol{\omega})$ and $\hat{\phi}$ is determined from the eigenvalue and the identification $k_4 = iE(\mathbf{k})$ and thus $E(\mathbf{k}) = g_v\omega_0 + \sqrt{\mathbf{p}^2 + M^{*2}}$. The condition (9) for the effective mass M^* is identical to that of the point-coupling case when the form factors are unity. The total energy density is $\mathcal{E} = U_F + \tilde{U}_v$. It is easily verified that the ω field that minimizes $\mathcal{E}$ is $\omega_0 = g_v\rho_B/m_v^2$ and $\boldsymbol{\omega} = 0$ where ρ_B is the baryon density $2k_F^3/3\pi^2$.

The total energy density in relativistic Hartree approximation (RHA) is then given by

$$\mathcal{E}(\hat{\phi}, k_F) = \frac{1}{2}\bar{m}_s^2\hat{\phi}^2 + \frac{g_v^2}{2m_v^2}\rho_B^2 + 4\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\sqrt{\mathbf{k}^2 + M^*(\hat{\phi})^2} + L^{(3)}(\hat{\phi})\,. \tag{10}$$

The mean-field theory (MFT) result is obtained by omitting the last term which is the vacuum baryon loop. Equation (10) has the same form as the point-coupling RHA result except that $M^*(\hat{\phi})$ is a different function (having the same limit in free space) and the baryon loop $L^{(3)}(\hat{\phi})$ has cubic and quartic terms in $\hat{\phi}$. The nuclear matter solution is obtained by minimization of $\mathcal{E}$ with respect to $\hat{\phi}$. With the range parameters β_s=0.8 and β_v=0.2, the RHA saturation curve is shown in Fig. 2 as the solid line. We label this calculation as NL-QHD (RHA). We have chosen m_v=783 MeV and the coupling constants g_s and g_v have been adjusted to obtain the saturation point at k_F=1.42 fm^{-1} and 15.75 MeV binding energy per nucleon. The obtained values of the coupling constants expressed as $C_v^2 = g_v^2M^2/m_v^2$ and $C_s^2 = g_s^2M^2/\bar{m}_s^2$ are listed in Table I along with the calculated incompressibility $K_v^{-1} = k^2d^2\mathcal{E}/dk^2$ at $k = k_F$.

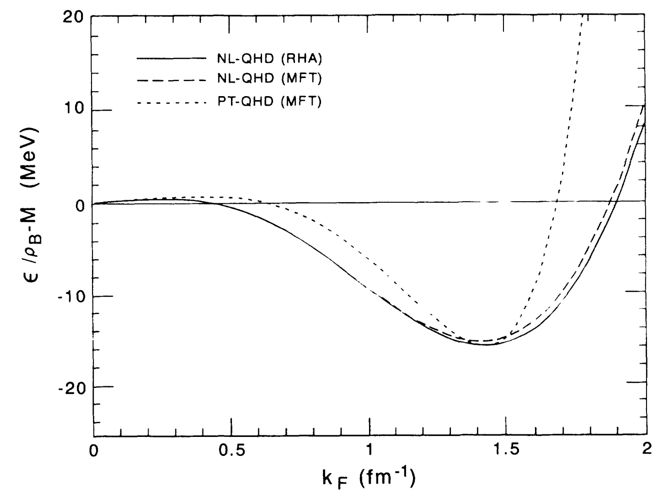

FIG. 2. The binding energy curve for nuclear matter. The solid line is the fitted result of the present nonlocal model. The long dashed line is the nonlocal result obtained by omitting the baryon loop but retaining the RHA parameters. The short dashed line is the fitted result of point-coupling QHD in MFT.

With the chosen form-factor ranges, the incompressibility is reduced by a factor of 2 compared to the point-coupling model. It is thus much easier to reconcile these results with the value derived from the breathing mode excitation of nuclei, which is typically 200 MeV. The vacuum component (one baryon loop) of the energy density in this model is very small and attractive. This is illustrated in Fig. 2 by the long dashed curve labeled NL-QHD (MFT) obtained by removing the last term of (10) but retaining the same parameters as the RHA case. When refitted to nuclear matter, the obtained parameters are shown in Table I. For comparision, we show in Fig. 2 as the short dashed curve, the point-coupling MFT result fitted to the empirical saturation point. The obtained parameters are also shown in the table. The characteristic changes in the incompressibility and M^* in the nonlocal cases are due principally to the nonlinear dependence of M^* upon $\hat{\phi}$.

This NL-QHD model apparently has two additional parameters (β_s and β_v) compared to the point-coupling model. One way to view their role is through the production of nonzero strength for the cubic and quartic terms of the loop function $L^{(3)}(\hat{\phi})$. (For the present solution, the cubic and quartic terms contribute only 0.4 MeV and -0.04 MeV respectively to the binding energy

TABLE I. Parameters employed and quantities obtained from the nuclear matter calculation described in the text.

	PT-QHD (MFT) (fit)	NL-QHD (MFT)	NL-QHD (RHA) (fit)
β_s	0	0.8	0.8
β_v	0	0.2	0.2
C_s^2	267.57	182.35	183.72
C_v^2	196.31	53.44	54.19
M^*/M	0.56	0.84	0.83
K_v^{-1}(MeV)	545	206.5	206.3

per nucleon.) These strengths are actually parameters also in the point-coupling case because there is no need to renormalize their infinite values to zero. However when the regulation mechanism is the presence of intrinsic form factors in the hadronic action, there are additional self-consistency requirements that limit the parameter space of possible solutions. It is not possible to obtain solutions unless the range parameters are such that the internal constraints of the NL-QHD model are satisfied. A major constraint is that the vacuum effective potential (5) should have an absolute minimum at $\phi = v$. The condition (6), together with renormalization of $\bar{m}_s$ to 550 MeV, guarantees only a local minimum and we must reject range parameters that produce a lower minimum elsewhere. This can happen if $|v|$ is too large in (5), and (6) then requires that the scalar coupling constant be bounded from above. Without a sufficiently soft scalar form factor (large β_s) the upper limit on g_s will not provide enough attraction to properly bind nuclear matter. If the scalar form factor is too soft, the nucleon loop term becomes negligible, but (9) produces $M^* \to M$ with the result that the scalar field would decouple from the Fermi-sea nucleons and no nuclear matter solution would be possible. The NL-QHD model selects a limited range of form factors. We find that solutions are only possible if $\beta_s > \beta_v$ and $0.72< \beta_s <1.2$. A range of solutions has been studied with $\beta_v <0.4$ and $\beta_s < 1.0$. Each one produces a reduced incompressibility, small vacuum contribution, and increased effective mass of the same order as the displayed solution. The range parameters in Table I produce high momentum behavior $\exp(-k^2/\Lambda^2)$, where $\Lambda_v \approx 5$ M and $\Lambda_s \approx 5M/4$, i.e., $\Lambda_v \approx 4.72$ GeV and $\Lambda_s \approx 1.18$ GeV. Thus the expectation that the sideways form factors should suppress momenta above about the nucleon mass scale is fulfilled by the solutions allowed.

In summary, the addition of sideways form factors to the QHD model at the RHA level produces a nonlocal field theory model which is still capable of reproducing nuclear matter saturation despite the additional nonlinearities. The internal constraints of the model automatically select momentum suppression above about the nucleon mass scale and the resulting vacuum contribution is quite small. The residual effects of the nonlocal coupling that show up at both the RHA and MFT levels are an increase in M^* from $0.56M$ to $0.8M$ and about a factor of 2 decrease in the incompressibility compared to the point-coupling model. A nonlocal field theory model developed further along these lines to include more general form factors may be more easily reconciled with approaches based on boson-exchange models that ignore the quantum vacuum.

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