

## Exact symmetries and the role of the pion cloud in deep-inelastic electron-nucleon scattering

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A careful analysis of exact symmetries, such as the charge conjugation symmetry and the electromagnetic and baryon current conservation, shows that exactly two nontrivial Feynman diagrams contribute to deep-inelastic inclusive pion electroproduction from the nucleon to  $O(g_{\pi NN}^2)$ . The same analysis reveals certain relationships between the two graphs. The two graphs are expressed as convolutions of the pion [ $g_{\pi}(y)$ ] and the nucleon [ $g_N(y)$ ] smearing functions and their respective deep-inelastic structure functions. The nucleon smearing functions are evaluated in three models of the nucleon off-shell dependence of the  $\pi NN$  vertex function and they turn out to have remarkably similar shapes. It is shown that this universality of  $g_N(y)$  persists in a wide class of models. Such universal  $g_N(y)$  peaks at  $y_0 = 1 - m_{\pi}/M_N = 0.85$  and allows a simple parton model interpretation. Furthermore, the normalized smearing functions approximately satisfy the Berger-Coester-Wiringa-Thomas ansatz  $g_{\pi}(y) = g_N(1-y)$  for two of the three models examined. Strong constraints on the nucleon off-shell dependence of the  $\pi NN$  vertex function [ $g_{\pi NN}(p_N^2)$ ] are obtained using the observed Gottfried sum rule violation as empirical input.

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In recent years we have seen repeated attempts [1–5] at establishing a relation between the pion cloud of the nucleon and its ocean quark content. At first [1,2] they were based on the  $SU(3)_f$ -breaking data in the nucleon ocean, whereas more recent work [3,5] relies on the Gottfried sum rule violation data [6]. These studies seem to vary in technical details and sometimes [4] even in the underlying philosophy: Some studies [1–3] use only one [“pion cloud”, Fig. 1(a)] Feynman diagram; others [4,5] use two [“pion cloud” + “nucleon recoil”, Figs. 1(a) and (b)]. This discrepancy seems to deserve attention especially in light of the fact that all these analyses are founded on the method developed by Sullivan [7] and Thomas [1]. Furthermore, there are various tacit and explicit assumptions in these analyses which remain completely unexplored (see below). It is the purpose of this paper to examine the theoretical foundations of the Sullivan-Thomas approach. This is accomplished by investigating the consequences of the exact symmetries such as  $C$ -conjugation, electromagnetic (EM), and baryon current conservation and approximate symmetries such as isospin  $SU(2)_f$ . We find on the basis of EM current conservation that there are exactly two graphs contributing to inclusive deep-inelastic scattering (DIS) pion electroproduction to  $O(g_{\pi NN}^2)$ . They can be expressed in terms of convolutions of the “smearing” and bare (valence) structure functions of the pion and the nucleon respectively.  $C$ -conjugation symmetry, together with the assumption of isospin symmetry, imposes a constraint on the two convolutions [see Eqs. (3) and (4)]. The baryon current of the valence quarks is found to be automatically conserved if the EM current conservation and the joint  $C$ -conjugation and  $SU(2)_f$  symmetry constraints are satisfied. Furthermore, we find that the shape of the nucleon smearing function  $g_N(y)$  is to a large extent model

independent and that the Berger-Coester-Wiringa-Thomas (BCWT) [5,8] conjecture  $g_{\pi}(1-y) = g_N(y)$  is approximately satisfied if all exact symmetry constraints are imposed. Gottfried sum rule violation data [6] are used to obtain a band of allowed values for the parameters describing the off-shell behavior of the  $\pi NN$  vertex function normalized to  $g_{\pi NN} = 13.6$ .

We start by examining the EM current conservation of inclusive ( $e, e'$ ) pion electroproduction to  $O(g_{\pi NN}^2)$  at arbitrary  $Q^2$  and  $\nu$ . We use two exact results: (i) The optical theorem relating the inclusive (total) electron scattering cross section and the imaginary part of the forward virtual photon Compton amplitude. (ii) The Ward-Takahashi (WT) identities for composite hadrons ( $\pi, N$ ) EM vertices with one (first kind) [9] and two external photons (second kind) [10].

First, we construct an EM current conserving (gauge invariant) virtual photon Compton scattering amplitude to  $O(g_{\pi NN}^2)$  with pions and nucleons with structure, i.e., with EM form factors. Here we use the methods developed by Gross and Riska [11]. The result is a set of

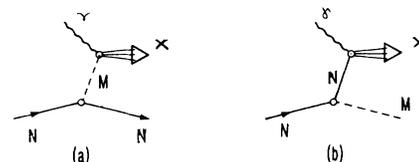


FIG. 1. The two Feynman diagrams appearing in the deep-inelastic inclusive pion electroproduction: (a) the “Sullivan process” diagram; (b) the “nucleon recoil” graph. The wavy line is the incoming virtual photon, the solid line the nucleon, and the dashed line the meson (pion).  $\times$  denotes arbitrary final states with baryon number 0 (a), or 1 (b).

24 Feynman diagrams where the two external photons couple to the charged and neutral (due to nonzero form factors) lines in the  $\pi N$  self-energy diagram in *all* topologically distinct ways in addition to the pionic sea gull graphs. One thing worth noticing is the appearance of a new two-photon nucleon (“seagull”) vertex, due to the WT identity of the second kind:

$$q_1^\mu G_{\mu\nu}^N = \Gamma_\nu^N(-q_2, p_1 - q_2, p_1) - \Gamma_\nu^N(-q_2, p_2, p_2 + q_2) \quad (1)$$

relating the new structure-induced nucleon “seagull” vertex  $G_{\mu\nu}^N$  and the off-shell one-photon nucleon vertex  $\Gamma_\mu^N$ . It is the imaginary part of this graph in the forward direction which describes the DIS from the nucleon. Note that its existence is not precluded even if the left-hand side of Eq. (1) is zero; this only implies that this vertex is separately gauge invariant. Returning to the Compton scattering graphs, we choose the forward scattering kinematics and take the Bjorken limit ( $Q^2, \nu \rightarrow \infty; 0 \leq x = Q^2/2M\nu \leq 1$ ). All but four graphs containing seagull vertices vanish due to the rapid decay of elastic form factors in the  $Q^2 \rightarrow \infty$  limit. Two of these [Figs. 2(a) and 2(b)] serve to renormalize the bare nucleon [ $O(g_{\pi NN}^0)$ ] term, while the other two [Figs. 2(c) and 2(d)] are nontrivial contributions. To obtain their imaginary parts, we use Cutkosky rules [12] which immediately yield the two graphs in Fig. 1 previously discussed in the literature. This derivation is *not* just an academic exercise: it establishes that the four graphs in Fig. 2 are the complete gauge invariant set to  $O(g_{\pi NN}^2)$  in models with  $\pi$  and  $N$  degrees of freedom only [13]. The four graphs in Fig. 2 are separately gauge invariant in DIS, but that is a consequence of the forward scattering condition and the Bjorken limit and their contributions are expressed in terms of convolutions:

$$\begin{aligned} \delta F_{2,p}^\pi &= \int_x^1 dy g_\pi(y) \left[ 2F_{2,\pi^+}^{(b)} \left( \frac{x}{y} \right) + F_{2,\pi^0}^{(b)} \left( \frac{x}{y} \right) \right], \\ \delta F_{2,n}^\pi &= \int_x^1 dy g_\pi(y) \left[ 2F_{2,\pi^-}^{(b)} \left( \frac{x}{y} \right) + F_{2,\pi^0}^{(b)} \left( \frac{x}{y} \right) \right], \\ \delta F_{2,p}^N &= \int_x^1 dy g_N(y) \left[ 2F_{2,n}^{(b)} \left( \frac{x}{y} \right) + F_{2,p}^{(b)} \left( \frac{x}{y} \right) \right], \\ \delta F_{2,n}^N &= \int_x^1 dy g_N(y) \left[ 2F_{2,p}^{(b)} \left( \frac{x}{y} \right) + F_{2,n}^{(b)} \left( \frac{x}{y} \right) \right], \end{aligned} \quad (2)$$

of the “smearing functions”  $g_\pi(y)$  and  $g_N(y)$  and the *bare* (valence quark) pion and nucleon DIS structure functions  $F_{2,\pi,N}^{(b)}(x)$ . Here

$$\begin{aligned} g_\pi(y) &= \frac{y}{(4\pi)^2} \int_{t_{\min}^\pi}^\infty dt \frac{t |g_{\pi NN}(t)|^2}{(t + m_\pi^2)^2}, \quad t_{\min}^\pi = \frac{(yM_N)^2}{1-y}, \\ g_N(y) &= \frac{2m_\pi^2 y}{(4\pi)^2} \int_{t_{\min}^N}^\infty dt \frac{|\bar{g}_{\pi NN}(t)|^2}{(t + M_N^2)^2}, \\ t_{\min}^N &= y \left[ \frac{m_\pi^2}{1-y} - M_N^2 \right], \end{aligned}$$

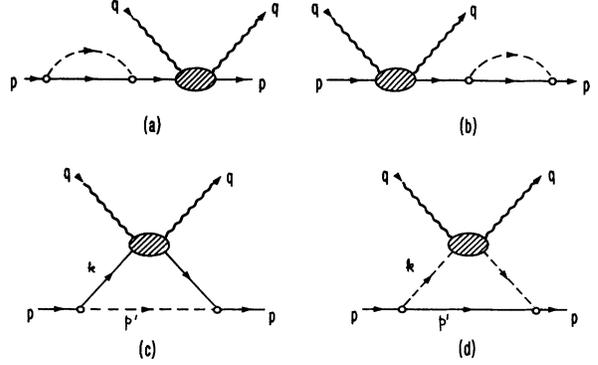


FIG. 2. The four Feynman diagrams describing the forward virtual Compton scattering from the nucleon in the deep-inelastic limit, to  $O(g_{\pi NN}^2)$ . The first two graphs (a), (b) ensure (finite) renormalization of the Born, i.e.,  $O(g_{\pi NN}^0)$ , graph. The second two graphs (c), (d) are nontrivial contributions whose imaginary parts correspond to graphs in Figs. 1(b) and 1(a), respectively.

where  $m_\pi = 140$  MeV,  $M_N = 940$  MeV, and

$$\begin{aligned} g_{\pi NN}(t = -p_\pi^2) &= g_{\pi NN}(p_\pi^2, M_N^2), \\ \bar{g}_{\pi NN}(t = -p_N^2) &= g_{\pi NN}(m_\pi^2, p_N^2) \end{aligned}$$

are the  $\pi$  and  $N$  off-shell dependences of the  $\pi NN$  vertex function, respectively. Here we *assumed* that the off-shell structure functions  $F_{2,\pi,N}(x, p_{\pi,N}^2)$  equal the on-shell ones. The *dressed* nucleon structure functions  $F_{2,N}^{(d)}$  equal

$$F_{2,N}^{(d)} = ZF_2^{(b)} + \delta F_2^\pi + \delta F_2^N,$$

where  $Z$  is the (finite) bare nucleon probability which is less than 1 due to the finite renormalization induced by the  $\pi N$  self-energy insertions in Feynman diagrams in Figs. 2(a) and 2(b). Note immediately that  $\delta F_2^\pi$  does not contribute entirely to the ocean quark distribution because the charged pion carries one *valence* dressed nucleon quark. Consequently,  $\delta F_2^N$  is not entirely valence quark distribution either: one of the quarks belongs into the ocean.

The charge conjugation symmetry requires that the zeroth moment of the ocean quark distribution equal the zeroth moment of the ocean antiquark distribution [14]. This requirement places a constraint on the zeroth moments of the  $\pi$  and  $N$  convolutions. To see this, remember that the  $\bar{d}$  quark in  $\pi^+$  is accompanied by a  $d$  quark in the neutron. Integrated values of these two contributions have to be equal,

$$\begin{aligned} \int_0^1 dx \int_x^1 \frac{dy}{y} g_\pi(y) \bar{d}_{\pi^+}^{(v)} \left( \frac{x}{y} \right) \\ = \int_0^1 dx \int_x^1 \frac{dy}{y} g_N(y) d_n^{(v)} \left( \frac{x}{y} \right) \end{aligned} \quad (3)$$

and similarly for the  $\bar{u}$  quark in  $\pi^-$  and a  $u$  quark in the proton,

$$\int_0^1 dx \int_x^1 \frac{dy}{y} g_\pi(y) \bar{u}_{\pi^-}^{(v)} \left( \frac{x}{y} \right) = \int_0^1 dx \int_x^1 \frac{dy}{y} g_N(y) u_p^{(v)} \left( \frac{x}{y} \right). \quad (4)$$

Now, if we assume the isospin invariance of the  $\pi$  and  $N$  structure functions and make the further simplifying assumption that the pion and the bare nucleon consist of valence quarks only, we can rewrite this constraint in terms of  $\pi$  and  $N$  convolutions:

$$\int_0^1 \frac{dx}{x} \delta F_{2,p}^\pi = \frac{5}{6} \int_0^1 \frac{dx}{x} [2\delta F_{2,p}^N - \delta F_{2,n}^N], \quad (5)$$

$$\int_0^1 \frac{dx}{x} \delta F_{2,n}^\pi = \frac{5}{9} \int_0^1 \frac{dx}{x} [2\delta F_{2,n}^N - \delta F_{2,p}^N].$$

Equations (5) lead to a constraint on the integral of the smearing functions,

$$n_\pi = \int_0^1 g_\pi(y) dy = n_N = \int_0^1 g_N(y) dy = n. \quad (6)$$

This constraint turns out to play an important role in the test of baryon current conservation. Note that the BCWT conjecture  $g_\pi(y) = g_N(1-y)$  also fulfills Eq. (6).

The baryon current conservation in deep-inelastic lepton-nucleon scattering is reflected in the valence baryon number sum rule [15], which, put simply, says that the zeroth moment (i.e., the integral over  $dx$ ) of the valence quark distribution function equals the sum of the squared charges of the valence quarks. To check the valence baryon number sum rule we first note that the Sullivan-Thomas procedure creates ocean quarks in mod-

els which started without them. Hence we must identify and subtract out this ocean term in evaluating the valence baryon number sum rule. When we do so the valence baryon number sum rule is automatically conserved, provided we assume isospin invariance of the valence quark distributions in the pion and nucleon and the validity of Eq. (6). Then integration of  $(1/x)F_2(x)$  leads to the normalization conditions for the  $\pi, N$  smearing functions:

$$n = \int_0^1 g_\pi(y) dy = \int_0^1 g_N(y) dy = \frac{1-Z}{3}. \quad (7)$$

At this point we are ready to discuss the Gottfried sum rule [16], which is just the difference between the *total* (not only valence) baryon number sum rules of the proton and neutron. Our procedure has ensured that the valence quarks reproduce the expected value of  $1/3$ . But we also see that the pion cloud contributes in an  $SU(2)_f$ -violating way, thus providing a source of deviation from the canonical (valence) value  $1/3$ . Ignoring the  $\Delta$  degrees of freedom, we readily obtain  $n = 0.07 \pm 0.03$  from the Gottfried sum rule violation data [6].

Now that we have established the consequences of the exact symmetries we proceed to evaluate the smearing functions  $g_\pi(y)$  and  $g_N(y)$ . The pion smearing function  $g_\pi(y)$  is well studied [1] and believed to be known (at least up to overall normalization). The nucleon smearing function  $g_N(y)$ , on the other hand, is completely unknown. This is due to our ignorance of the basic ingredient entering it: the dependence of the  $\pi NN$  coupling constant  $g_{\pi NN}$  on the "off-shell mass"  $p_N^2 \neq M_N^2$  of the nucleon:  $\bar{g}_{\pi NN}(t) = g_{\pi NN}(p_N^2)$ . We use three choices for  $\bar{g}_{\pi NN}(t)$ .

(i) The suggestion by Hwang *et al.* [4] of a *monopole*

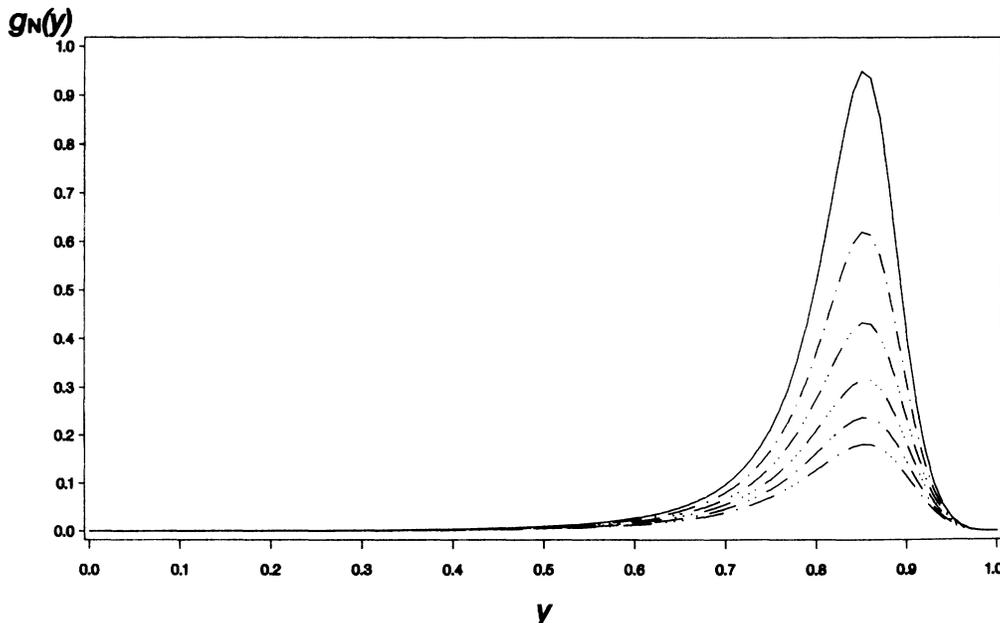


FIG. 3. The "nucleon smearing function"  $g_N(y)$  as calculated with a monopole nucleon off-shell dependence of the  $\pi NN$  vertex function (see text). Solid ( $\Lambda_{1N} = 815$  MeV), dash-dotted ( $\Lambda_{1N} = 820$  MeV), dash-double dotted ( $\Lambda_{1N} = 825$  MeV), dash-triple dotted ( $\Lambda_{1N} = 830$  MeV), dot-dash-double dotted ( $\Lambda_{1N} = 835$  MeV), dot-dash-triple dotted ( $\Lambda_{1N} = 840$  MeV).

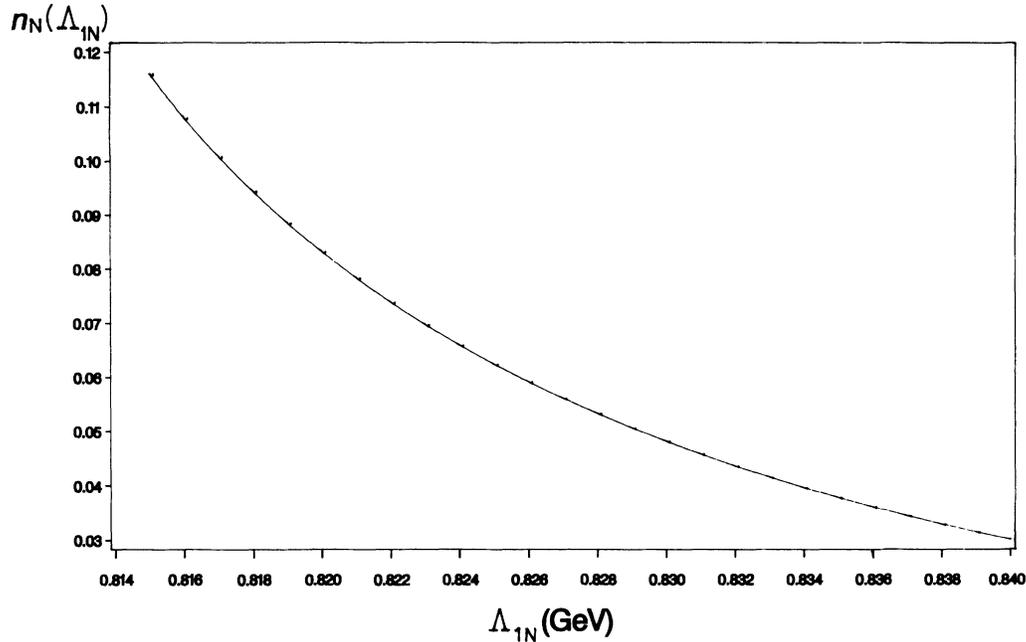


FIG. 4. The integral over the nucleon smearing function  $g_N(y)$  from  $y=0$  to  $y=1$ ,  $n_N$  as a function of the nucleon monopole cutoff  $\Lambda_{1N}$ .

dependence with an as yet unknown cutoff parameter  $\Lambda_{1N}$ :

$$\tilde{g}_{\pi NN}(t) = 13.6 \left[ \frac{\Lambda_{1N}^2 - M_N^2}{\Lambda_{1N}^2 + t} \right]. \quad (8)$$

The monopole  $g_N(y)$  is shown in Fig. 3 for six values of the cutoff  $\Lambda_{1N}$ . All the curves are peaked at

$y=y_0=0.85$ , but the height of their maxima rapidly increases as  $\Lambda_{1N} \rightarrow M_N - m_\pi = 0.8$  GeV at which point  $g_N(y)$  develops a nonintegrable singularity at  $y=y_0$ . This apparent disaster is readily understood if we note that the pole from  $\tilde{g}_{\pi NN}(t)$  enters the integration region of the integral in the definition of  $g_N(y)$  [see below Eq. (2)] for  $\Lambda_{1N} \leq M_N - m_\pi$ , and the integral diverges at  $y=y_0$ . Thus we have obtained a lower limit on  $\Lambda_{1N} > M_N - m_\pi$

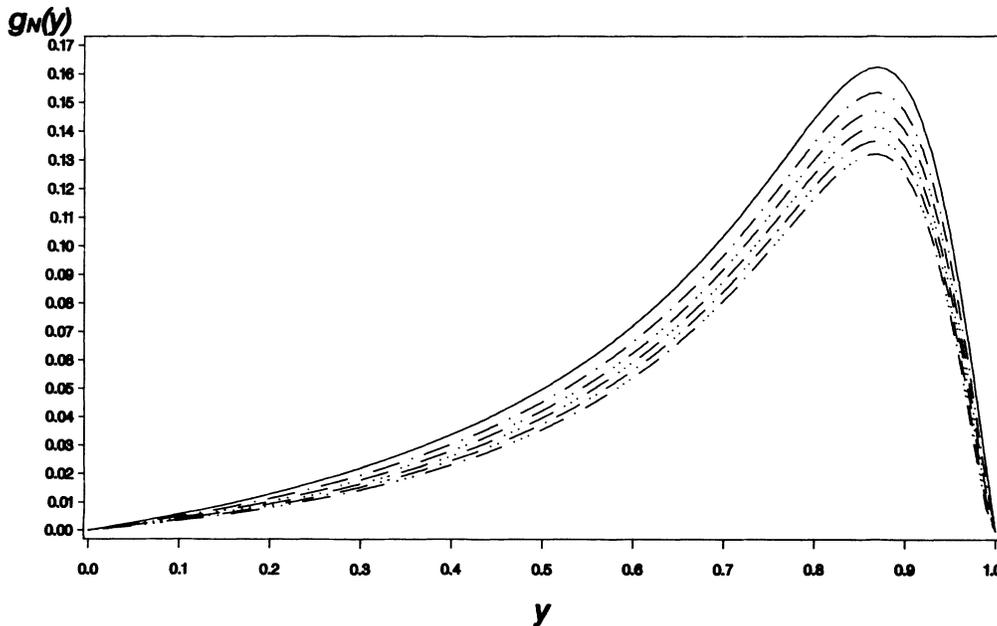


FIG. 5. Same as Fig. 3, but with exponential nucleon off-shell dependence. Solid ( $\alpha_N=0.00$ ), dash-dotted ( $\alpha_N=0.02$ ), dash-double dotted ( $\alpha_N=0.04$ ), dash-triple dotted ( $\alpha_N=0.06$ ), dot-dash-double dotted ( $\alpha_N=0.08$ ), dot-dash-triple dotted ( $\alpha_N=0.10$ ).

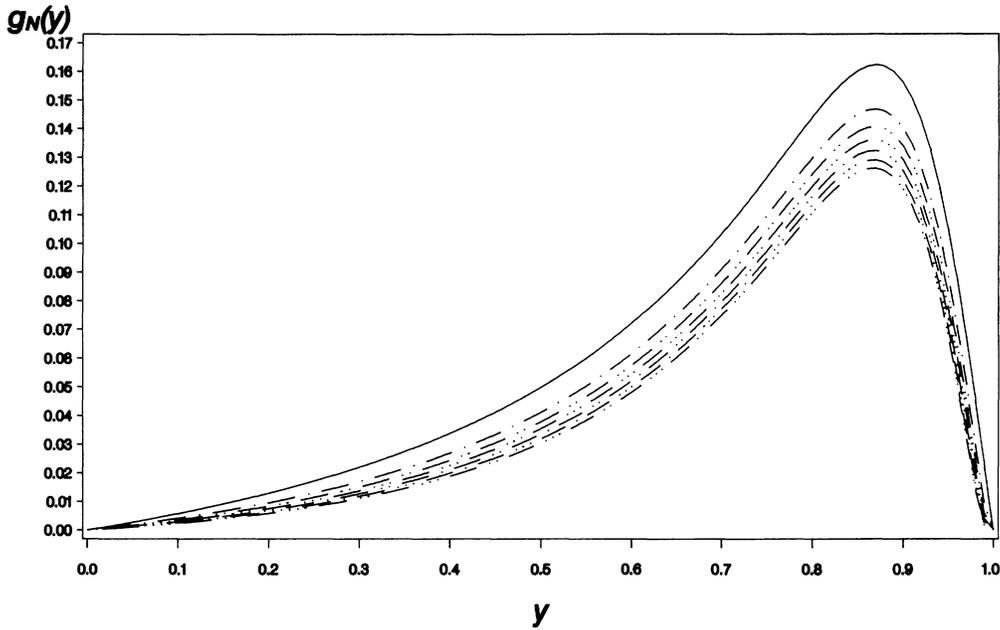


FIG. 6. Same as Figs. 3 and 5, but with Gaussian nucleon off-shell dependence Eq. (10). Solid ( $\beta=0.00$ ), dash-dotted ( $\beta=0.02$ ), dash-double dotted ( $\beta=0.04$ ), dash-triple dotted ( $\beta=0.06$ ), dot-dash-double dotted ( $\beta=0.08$ ), dot-dash-triple dotted ( $\beta=0.10$ ), double-dot-dash-triple dotted ( $\beta=0.12$ ).

without any empirical input. We further restrict the allowed values of  $\Lambda_{1N}$  by using the experimental values of the “pion number”  $n=0.07\pm 0.03$  (see Fig. 4). We find that  $\Lambda_{1N}$  lies in a very narrow range of values:  $\Lambda_{1N}=823\pm_6^{11}$  MeV.

(ii) The exponential parametrization of the nucleon

off-shell dependence:

$$\tilde{g}_{\pi NN}(t) = 13.6 \exp[-\alpha(t/M_N^2 + 1)]. \quad (9)$$

The nucleon smearing function  $g_N(y)$  peaks again at  $y=y_0$  (Fig. 5) independently of the value of the parame-

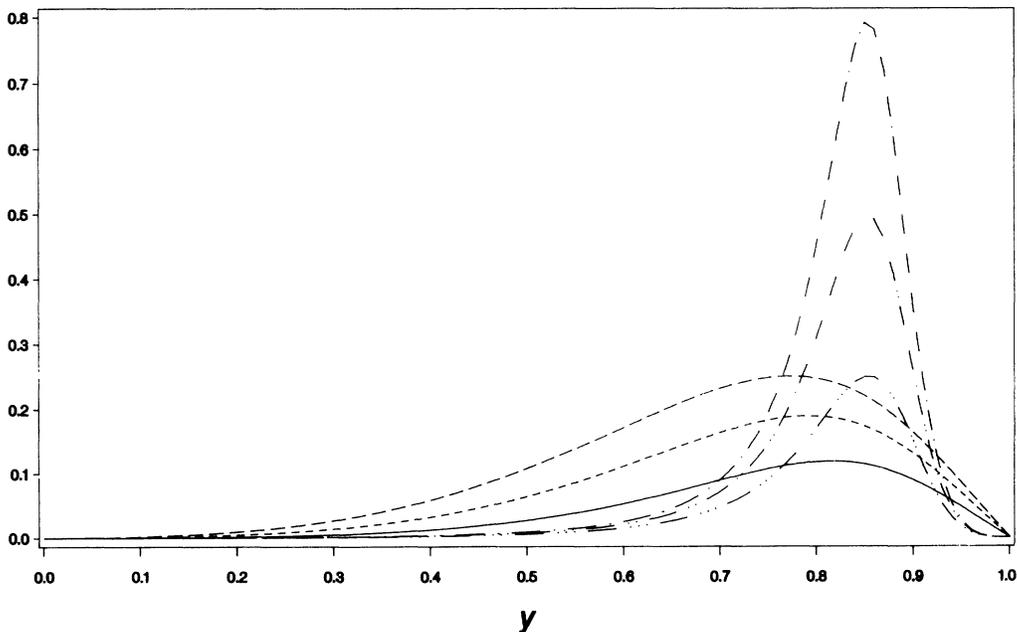


FIG. 7. Check of the BCWT ansatz for the monopole forms of  $g_{\pi NN}(t)$  and  $\tilde{g}_{\pi NN}(t)$ :  $g_N(y)$  and  $g_\pi(1-y)$ . The curves are normalized to the Gottfried sum rule violation values  $n=0.07\pm 0.03$ . For  $g_\pi(1-y)$ : Solid ( $\Lambda_{1\pi}=445$  MeV), short dashes ( $\Lambda_{1\pi}=575$  MeV), long dashes ( $\Lambda_{1\pi}=688$  MeV). For  $g_N(y)$ : dashes-triple dots ( $\Lambda_{1N}=834$  MeV), dashes-double dots ( $\Lambda_{1N}=823$  MeV), dashes-dots ( $\Lambda_{1N}=817$  MeV).  $g_{\pi NN}(t)$  is given by Eq. (8), but with  $\Lambda_{1\pi}$  instead of  $\Lambda_{1N}$ .

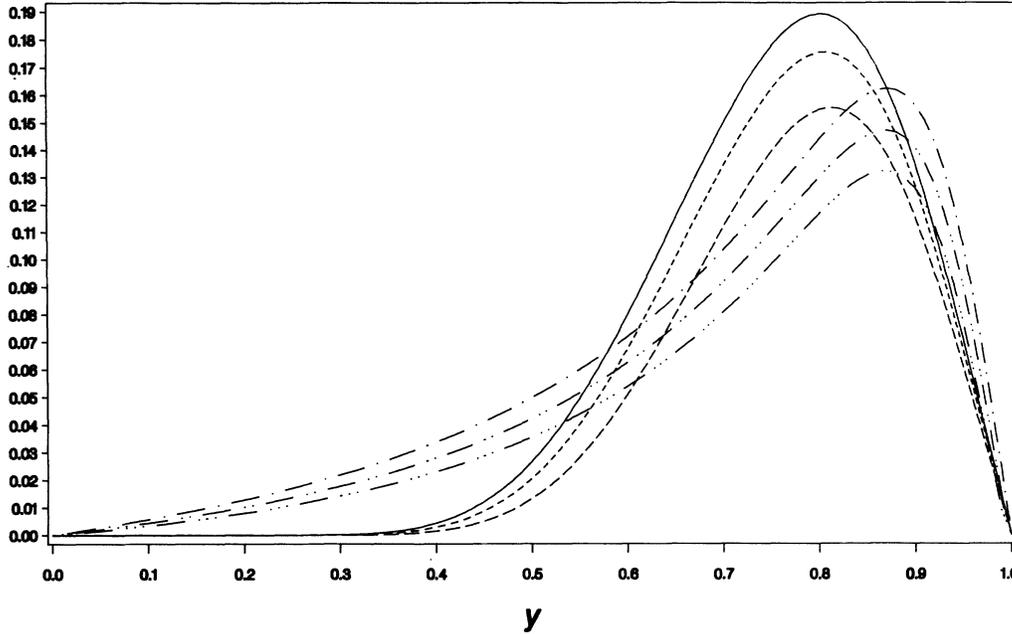


FIG. 8. Check of the BCWT ansatz for the exponential forms of  $g_{\pi NN}(t)$  and  $\tilde{g}_{\pi NN}(t)$ :  $g_N(y)$  and  $g_\pi(1-y)$ . The curves are normalized to the following values of  $n$ : 0.062, 0.054, 0.047. For  $g_\pi(1-y)$ : long dashes ( $\alpha_\pi=0.054$ ), short dashes ( $\alpha_\pi=0.047$ ), solid ( $\alpha_\pi=0.043$ ). For  $g_N(y)$ : dashes-triple dots ( $\alpha_N=0.10$ ), dashes-double dots ( $\alpha_N=0.04$ ), dashes-dots ( $\alpha_N=0.00$ ).  $g_{\pi NN}(t)$  is given by Eq. (9), but with  $\alpha_\pi$  instead of  $\alpha_N$ .

ter  $\alpha_N$ . Contrary to the monopole case, the “pion number”  $n$  can never exceed the value of 0.062, which lies within the experimental bounds  $n=0.07\pm 0.03$ , and is achieved for  $\alpha_N=0$ . This feature is readily understood if one remembers the definition of  $g_N(y)$ : the integral over  $dt$  is finite even with  $\tilde{g}_{\pi NN}(t)=13.6$ , i.e.,  $\alpha_N=0$ , and thus provides the maximal possible value of  $n$ . It is interesting, though, that simple improvement on the experimental error bars could eliminate this model.

(iii) The Gaussian nucleon off-shell dependence:

$$\tilde{g}_{\pi NN}(t) = 13.6 \exp[-\beta(t/M_N^2 + 1)^2], \quad (10)$$

which is motivated by Speth and Tegen’s [18] quark model predictions. Unfortunately, this calculation is nonrelativistic and hence does not allow a unique expression as a function of  $t$ . Once again (Fig. 6),  $g_N(y)$  peaks at  $y=y_0$  and, again, there is a limiting value for  $n(\beta)$  at  $\beta=0$ , for the same reasons as in point (ii) above.

We have witnessed a curious model independence of the position of the peak of  $g_N(y)$ , and to a lesser extent model independence of the shape of  $g_N(y)$ . The former has a straightforward explanation when one remembers the definition of  $g_N(y)$  and the fact that all three  $\tilde{g}_{\pi NN}(t)$  considered are monotonically decreasing for  $t > -(M_N - m_\pi)^2$ . Now note that the maximum of the integral appearing in  $g_N(y)$  is completely determined by the minimum, as a function of  $y$ , of  $t_{\min}^N(y)$ . The extremal condition  $dt_{\min}^N(y)/dy=0$  readily leads to  $y_{\max}=y_0=1-m_\pi/M_N$ : Thus, we have established the invariance of

the position of the peak of  $g_N(y)$ , for all monotonically decreasing  $\tilde{g}_{\pi NN}(t)$ , for  $t > -(M_N - m_\pi)^2$ .

This result is particularly appealing because it allows a simple parton model interpretation:  $g_\pi(y)$  and  $g_N(y)$  would be the probabilities of finding a “ $\pi$  parton” or an “ $N$  parton” (bare nucleon), respectively, with momentum fraction  $y$  in the “dressed nucleon,” if the intrinsic DIS structure functions of these “partons”  $F_{\pi,N}(x)$  were pointlike, i.e., Dirac  $\delta$  functions. Then it is intuitively clear that a “ $\pi$  parton” with mass  $m_\pi$  is most likely to carry the  $(m_\pi/M_N)$  fraction of the total momentum, i.e., that  $g_\pi(y)$  ought to peak at  $y=m_\pi/M_N=0.15$ , and that the bare nucleon is most likely to carry the complement to unity of the pion’s momentum fraction. This argument is the motivation for the BCWT ansatz  $g_N(y)=g_\pi(1-y)$ , but unfortunately all studies conducted so far [1–5] have shown that the position of the maximum of  $g_\pi(y)$  is a sensitive function of the  $\pi NN$  form factor  $g_{\pi NN}(t)$ . We check the validity of this ansatz for our models [cases (ii) and (iii) are treated as one due to their great similarity] and find that in case (i) only the gross features coincide (Fig. 7), while in case (ii) (Fig. 8) the agreement is much better, although not perfect. We find it very surprising that this basically kinematic constraint is so restrictive on the form of the  $\pi NN$  vertex function and intend to investigate this relation further.

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