Possibility of observing massive neutrino admixtures in nuclear orbital electron capture rates

F. X. Hartmann
Chemistry Department, Brookhaven National Laboratory, Upton, New York 11973
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The unusual case in nuclear orbital electron capture whereby either a neutrino in a weak interaction eigenstate or in a mass eigenstate is emitted by the same nucleus is examined. The case of the low-energy Q value nuclear electron capture decays is considered where, for at least one electronic shell, the emission of a weak eigenstate neutrino having an admixture of a second mass eigenstate neutrino (with mass in the keV/c^2 range) is energetically prohibited. This feature, in particular, opens the possibility for a new approach to searching for massive neutrino admixtures using relative electron capture rates. Five candidates are specifically identified, of which one, ^{157}Tb, is conceivably of immediate interest to the 17 keV/c^2 neutrino mass question. Finally experimental prospects and limitations are described.

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Evidence for the emission of a massive 17 keV/c^2 neutrino in beta decay [1] and independent theoretical arguments [2] that the neutrino weak-interaction eigenstates are not mass eigenstates prompts interest in the possibility for observing such massive neutrino admixtures, if any, in nuclear orbital electron capture decay rates. In particular, certain nuclei having electron capture decay energies suitably positioned energetically with respect to electron binding energies will produce neutrinos which have differing expectation values for the mass of the emitted neutrino. This feature would then be reflected as deviations of electron capture rates from those expected based on the standard electroweak model. We examine that possibility here—one case holds promise for further experimental study.

Ultralow energy nuclear beta decays—electron capture (ec) or β^- decay—are important to particle physics in providing sensitive tests for placing limits [2] on the electron neutrino or antineutrino rest masses. These masses are of general fundamental interest; if nonzero, they strongly influence the need for modifications to the standard model and provide insight into the grand unification mass scale. Additionally, astrophysical questions concerning neutrino oscillations and the mass in the universe also prompt continuing interest in pursuit of neutrino mass studies in ultralow-energy nuclear physics.

Current limits on neutrino masses in low-energy nuclear physics are based on the influence of mass effects in phase-space factors of reactions having very low weak-interaction decay energies, Q. The neutrino momentum q which enters into the rate expressions for such decays is dependent on the neutrino total energy E and the neutrino rest mass m_ν:

$$q=(1/c)(E^2-m_\nu^2c^4)^{1/2}\,, \tag{1}$$

with c the speed of light. In the β^--decay studies of ^{3}H, the rate of detection of electrons having energy E_{e^-} near Q_{β^-} is sensitive to neutrino masses, when

$$m_\nu c^2 \approx E = Q_{\beta^-} - E_{e^-}\,. \tag{2}$$

In order to study electron neutrino (as opposed to antineutrino) masses, one correspondingly desires

$$m_\nu c^2 \approx Q_{\rm ec} - E_x\,, \tag{3}$$

where E_x is the binding energy of the captured electron in the daughter nucleus corrected for the interactive energy shifts.

In electron capture, as first discussed by De Rujula [3], the neutrino mass can affect the shape of the spectrum of x rays that are emitted along with the neutrino in the process called inner bremsstrahlung electron capture (IBEC). The neutrino mass can also affect the decay rate, λ_x, for capture from a given atomic subshell x, through the phase-space factors $w_x q_x$, with q as defined above and w_x the neutrino total energy $Q_{\rm ec}-E_x$, a situation pursued in capture rate studies [4]. However, if weak eigenstate neutrinos are not mass eigenstates, it is a unique feature of electron capture that by judicious choice of nucleus, the binding energies E_x can be exploited so as to project out specific neutrino mass eigenstates and, as seen below, provide constraints on any such massive admixtures.

For the case of massive neutrino admixtures, then, the electron neutrino wave function is given by $|\nu_e\rangle = U_{ej}|\nu_j\rangle$ for the j mass eigenstates. In the lowest order, only neutrinos of mass m_j satisfying

$$m_j c^2 < Q_{\rm ec} - E_x \tag{4}$$

are emitted; these are denoted by m_k. The leptonic piece $L_\mu^+(x)$ of the beta-decay transition matrix element in the usual theory [5] gives

$$\langle \nu_e | L_\mu^+(x) | \Psi_e \rangle = \sum_k \langle \nu_k | U_{ke}^* | \nu_e \rangle \langle \nu_e | L_\mu^+(x) | \Psi_e \rangle\,. \tag{5}$$

Consequently, the overall capture rate from shell x is

$$\lambda_x = \frac{G_{\beta^2}}{4\pi^2} C_x n_x B_x \beta_x^2 \sum_k |U_{ek}|^2 q_{x,k} w_{x,k} = \sum_k \lambda_{x,k}\,, \tag{6}$$

where the subscripts x,k refer to an neutrino of mass m_k emitted following capture from the shell or subshell x. Also note (for use later) the traditional notation for the forms of the other factors in the electron capture decay rate: G_β is the weak-interaction coupling constant, C_x is the squared nuclear matrix element, n_x is the electron occupation probability, B_x is the overlap and exchange correction, and β_x is the electron wave-function amplitude over the nuclear volume.

Since relative capture rates can potentially eliminate C_x, given knowledge of β_x, B_x, and Q_{ec} one can, in principle, constrain m_j and U_{ej}. In practice, though, it is fruitful to consider a special case, as now described. For the current case of immediate interest, we take $k=1,2$, $m_1=0$ keV/c^2, and $m_2 \approx$ keV/c^2. For the two-neutrino case $U_{e1}=\cos\theta$, $U_{e2}=\sin\theta$, where θ is the mixing angle. In this case, the ratio of any two capture rates (annotated for the illustrative case of K and L capture, but applicable to any two suitably positioned shell binding energies or subshell energies with summations as appropriate) is conveniently written for the moment as

$$\frac{\lambda_L}{\lambda_K}=\frac{\beta_L^2 B_L}{\beta_K^2 B_K}\frac{(Q_{ec}-E_L)^2}{(Q_{ec}-E_K)^2}\frac{\cos^2\theta+\sin^2\theta[1-m_2^2c^4/(Q_{ec}-E_L)^2]^{1/2}}{\cos^2\theta+\sin^2\theta[1-m_2^2c^4/(Q_{ec}-E_K)^2]^{1/2}} . \tag{7}$$

A cursory inspection of Eq. (7), as written, reveals little opportunity for a sensitivity to a 17 keV/c^2 mass. However, for $m_2c^2+E_K > Q_{ec}$, K capture is not allowed, and one thus obtains the following important result:

$$\frac{\lambda_L}{\lambda_K}=\frac{\beta_L^2 B_L}{\beta_K^2 B_K}\frac{(Q_{ec}-E_L)^2}{(Q_{ec}-E_K)^2}\left[1+\frac{\sin^2\theta}{\cos^2\theta}\left(1-\frac{m_2^2c^4}{(Q_{ec}-E_L)^2}\right)^{1/2}\right] . \tag{8}$$

Equation (8) is written so as to highlight the effects of the mass m_2 and its admixture $\sin^2\theta$. Unlike the case of neutrino mass research in electron capture where an enhanced effect arises for $Q_{ec}-E_x \approx m_\nu c^2$ in the electron capture ratio and, subsequently, in the inner bremsstrahlung shape, here the persistent presence of the small "admixture ratio" $\sin^2\theta/\cos^2\theta$ suggests that, in an opposite sense, the best approaches would keep $Q_{ec}-E_L > m_2c^2$ [contrary to Eq. (3)] so that the effects of that admixture ratio are enhanced. Moreover, the best case has Q_{ec} less than the K-shell binding energy by more than 17 keV but larger than E_L by as much as possible. In this more useful limit then, the ratio λ_L/λ_K deviates from the single massless neutrino limit by the percent admixture of a second massive neutrino whose mass lies between $Q_{ec}-E_K$ and $Q_{ec}-E_L$.

The interesting feature in Eq. (8) is, once again, that K capture makes only $|\nu_1\rangle$. Finally, it is useful to note now that if the Q_{ec} value is high (which it typically is) such that $|\nu_e\rangle$ is emitted following both K and L capture, then the factor on the right-hand side in Eq. (7) becomes

$$\frac{\cos^2\theta+\sin^2\theta[1-m_2^2c^4/(Q_{ec}-E_L)^2]^{1/2}}{\cos^2\theta+\sin^2\theta[1-m_2^2c^4/(Q_{ec}-E_K)^2]^{1/2}} \xrightarrow{Q_{ec}>E_K} \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta+\sin^2\theta}=1 \tag{9}$$

for Q_{ec} (typically) a few times E_K. In this case, the test for a second massive neutrino can include the measurement of a capture ratio of a second isotope having a larger Q_{ec} value, to eliminate the greater uncertainties introduced by the overlap and exchange factors. In this case, Q_{ec} and E_x must still be known.

Of the twelve known nuclei [6] having very low values of Q_{ec} which are comparable to electron binding energies, only five decay by electron capture. Of these five, summarized in Table I, only one is of immediate and direct interest to establishing the upper limit on the electron (or ν_1, as appropriate) neutrino mass: ^{163}Ho [7]. Of the remaining four nuclei, only one is well suited to addressing the 17-keV neutrino mass question: ^{157}Tb.

The $\frac{3}{2}^+$ ground state of the nucleus ^{157}Tb decays by electron capture with a half-life of 150 yr to the $\frac{3}{2}^-$ ground state of ^{157}Gd (99.66%) and the 54.5-keV $\frac{5}{2}^-$ excited state in ^{157}Gd (0.34%) [8], as depicted in Fig. 1. Because the K-shell binding energy in the daughter ^{157}Gd nucleus is 50.239 keV, K capture cannot energetically produce any 17-keV neutrinos, whereas L capture ($L=LI+LII$) can. Consequently, since $m_2c^2 < Q_{ec}-E_L$, the λ_L/λ_K capture ratio (or λ_{LI}/λ_K, etc.) is approximately proportional to the theoretical λ_L/λ_K ratio by the percent admixture of the 17-keV neutrino, or, specifically, any neutrino with a mass greater than 12.1 keV.

The best currently obtainable value for $Q_{ec}=62.292 \pm 0.574$ keV is from Ref. [9]. This value is consistent with differences based on atomic mass predictions of Wapstra, Audi, and Hoekstra in Ref. [10] which give 62 ± 4 keV. It is of course critical that the Q_{ec} values

TABLE I. Candidates for a second massive neutrino admixture.

Nuclide	Q_{ec} [a] (keV)	Half-life [b] (yr)	E_x (shell) [b] (keV)	E_x' (shell) [b] (keV)	m_{ν_2} range (keV/c^2)
^{157}Tb	62.3	150	50.239 (K)	8.376 (L)	12.1–53.9
^{163}Ho	2.58	4570	2.050 (M)	0.419 (N)	0.53–2.16
^{179}Ta	115	1.7	65.351 (K)	11.272 (L)	50–104
^{193}Pt	61	50	13.880 (L)	3.298 (M)	47–58
^{205}Pb	60	1.4×10^7	15.347 (L)	3.740 (M)	45–56

[a]Reference [6] except ^{157}Tb, Refs. [9,10]; ^{163}Ho, Ref. [7].
[b]Reference [8] except ^{163}Ho, Ref. [7].

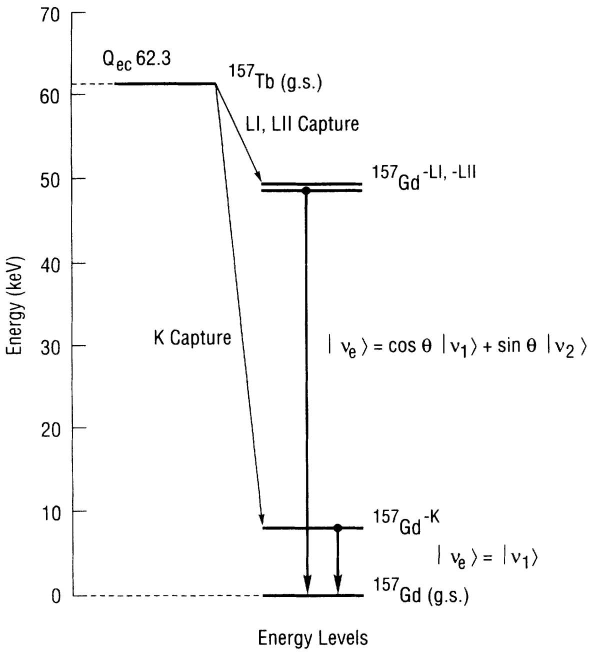

FIG. 1. Electron capture decay scheme for ^{157}Tb. The case of ^{157}Tb illustrates the emission of two different massive neutrinos. The Q_{ec} value is too close to the K-shell binding energy to permit emission of any massive neutrino in excess of 12.1 keV/c^2.

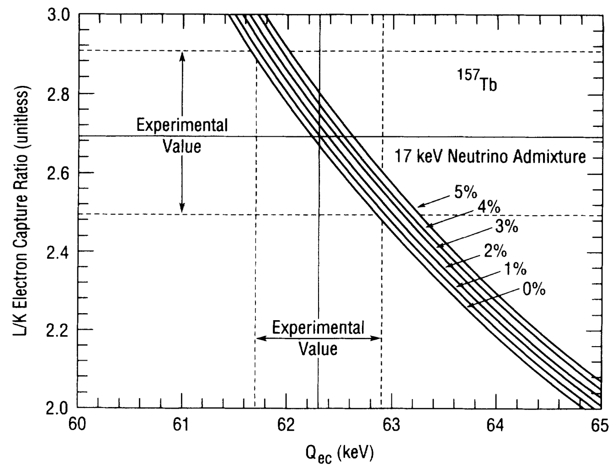

FIG. 2. The constraints on ν_2 in ^{157}Tb. The dependence of the percent admixture of a 17-keV neutrino ($m_2 > 12.1$ keV/c^2) is depicted as a function of two potential experimental measurements: The L-to-K electron capture ratio and the ^{157}Tb-^{157}Gd mass difference. The current measurement of 2.69 for the capture ratio illustrates the state of current experimental knowledge (1% with a 15% upper limit). We find ^{157}Tb to be the most favorable case available to constrain a 17-keV real neutrino in weak-interaction electron capture rates.

must remain determined experimentally in a manner which is independent from the λ_L/λ_K ratio. Currently, that capture ratio is 2.69 ± 0.20 [11] (or 2.65 ± 0.20 [12]). The current experimental λ_L/λ_K ratio, the Q_{ec} and the dependence on various massive neutrino admixtures are plotted in Fig. 2. (A 1% admixture with an upper limit of 15% as indicated is clearly based on existing data and can be improved with follow-on measurement.) The relevant atomic physics parameters for the ^{157}Tb system are summarized in Table II. Using these values and $Q_{ec} = 62.3$ keV, $\lambda_L/\lambda_K = 2.67$ for no 17-keV neutrino, whereas $\lambda_L/\lambda_K = 2.75$ for a 3% keV neutrino (at the higher range of any such admixture at which the experiment becomes useful). Furthermore, this ratio is proportionally less for smaller neutrino admixtures as indicated in the figure. The atomic physics parameters are also subject to error and an approach to basing those parameters on a second isotope is discussed further below.

The case of ^{157}Tb is the best nucleus found for searching for neutrino admixtures through the approach discussed here. The electron capture rates then best constrain the percent admixture of the second massive neutrino, at the detriment of being less sensitive to the magnitude of the mass itself. On the other hand, having a fixed and finite range about the 17 keV/c^2 number to which the ^{157}Tb experiment is sensitive could, alternatively, be viewed as advantageous since values up to 22 keV are suggested in experiments which study the inner bremsstrahlung electron capture line shape [13]. It should, however, be emphasized that the interpretation of the IBEC shapes—first done in detail in regards to neutrino mass studies—is difficult and subject to theoretical uncertainties as well [7].

Practically, ^{157}Tb can be prepared by neutron irradiation of ^{156}Dy followed by radiochemical and isotopic separation. Atoms of ^{157}Dy thus produced subsequently decay by an 8-h half-life to the desired ^{157}Tb nucleus. Electron capture to the first excited nuclear state in ^{157}Gd does not interfere since capture to this state is allowed for $Q_{ec} = 7.8$ keV; thus only M and higher shells contribute ($E_M = 1.888$ keV). In order to count, directly, the K and L capture events, the ^{157}Tb could, for example, be made into a volatile metal β diketonate and counted internally within an appropriately designed elevated temperature proportional counting detector, as developed for the ^{136}Ho neutrino mass experiment [14]. Since the decay energies here are larger than the 400 eV of the N capture in ^{163}Ho, it is also possible to consider depositing the required 10^{10} atoms on a thin film which is then internally counted. The key is to distinguish K and L capture events. Especially useful is the fact that ^{158}Tb (150 yr half-life) and (to a lesser extent) ^{155}Tb (5.3 days half-life) are electron capturing nuclei with Q_{ec} values sufficiently high to result in decays to excited gamma-ray emitting states (at 1.02–1.04 MeV for ^{158}Tb and 489 keV for ^{155}Tb). This aspect is of further important use in exploiting measure-

TABLE II. Atomic parameters for ^{157}Tb.

Shell	E (in $Z=64$) (keV)	β[a] (no units)	B_K/B_L[a] (no units)
K	50.239	1.1964	1.035
LI	8.376	0.435 39	
LII	7.931	0.091 339	

[a]Reference [5].

ments of relative capture rates in coincidence with the emitted gamma ray so as to allow for direct experimental checks on the common atomic physics factors, in Eq. (8), which would otherwise be subject to further questioning if left to the current state of theoretical knowledge alone. In fact, in eliminating atomic physics factors one ultimately needs only to measure ratios of K/K' and L/L' of the two isotopes in the same detector geometry. There remains the practicality of building a large volume, high-resolution heated proportional detector for internal counting of a gas or positioning of a thin source and the difficulty in pushing the limits (to 1%) of measuring capture ratios. An approach using internally doped cryogenic bolometers is also an alternate route to consider. Regardlessly, a ^{157}Tb experiment only requires the measurement of routine parameters of electron capture decay; it is a fortunate aspect that its overall properties make it an ideal case.

The primary uncertainty in the measurement of the 17-keV neutrino admixture arises due to the error in the capture ratio. This admixture is essentially directly proportional to that ratio. One would expect such a 1% measurement to be readily achievable, through careful technique, with further refinement feasible.

In summary, with the report of evidence for a second massive neutrino in beta decay, the possibility for observing such a massive neutrino admixture in electron capture decay rates was examined as well. Such possible but difficult experiments would provide additional approaches towards studying potential neutrino mass effects. The most favorable aspect to pursue in this regard is not in the "traditional" factors treating phase-space effects (as could have been expected based on the ^{163}Ho experiment) but in the factor treating the nuclear transition matrix element. That factor leads to a modification of the electron capture ratio by the percent admixture of the second massive neutrino. Finally, of the known low-energy weak-interaction electron capturing nuclei, one is relevant to the 17-keV neutrino mass question—and its properties are, fortunately, favorable for experimentation.

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