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Linkage between β and γ vibrational excitations in deformed nuclei

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The consistent Q formalism of the interacting boson model (IBA) is systematically confronted with data on deformed vibrational states and found to be empirically supported. Exploiting this, we test a central IBA prediction, which is in stark contrast with traditional models, of an inherent linkage between the properties of the β and γ vibrations, and find this linkage to be confirmed by the data.

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In traditional collective models, β and γ vibrations are independent quadrupole excitations created by different operators ($Y_{2,0}$ and $Y_{2,\pm 2}$, respectively), with different matrix elements, energy systematics, and collectivity [1]. In view of this, it is one of the most surprising and intriguing features of the IBA (Ref. [2]) that the properties of the β vibration (for simplicity, we keep the notation "β" for the IBA, using it to denote the lowest $K=0^+$ excitation) and γ vibration are *intimately linked*: those of one are necessarily related to those of the other. This linkage has its origins in the structure of the SU(3) limit, is closely connected with the explicit recognition of the finite number of active nucleons, and persists in calculations for realistic deformed nuclei. It leads both to *collective* β-γ $E2$ transitions, which have been observed, and, as we shall see, it can also be seen rather directly in more readily available observables. Though this linkage is, in principle, known, it is little appreciated and has never been systematically studied, even though it is among the most notable differences between the IBA and geometrical models and even though the association of observables, such as β band energies, with others, such as the $E2$ decay properties of the γ band, is contrary to common perception.

A useful approach to many IBA calculations in the consistent Q formalism [3] (CQF) which prescribes that the quadrupole operator in the Hamiltonian and $E2$ operator be the same. A frequently used CQF Hamiltonian is

$$H=-\kappa Q\cdot Q\,, \tag{1}$$

where $Q=(s^\dagger d+d^\dagger s)+\chi_H(d^\dagger d)^{(2)}$. If the $E2$ operator is written

$$T(E2)=e_B[(s^\dagger d+d^\dagger s)+\chi_{E2}(d^\dagger d)^{(2)}]\,, \tag{2}$$

the CQF corresponds to the constraint

$$\chi_H=\chi_{E2}\ \text{(CQF)}\,. \tag{3}$$

Note that $\chi=-\sqrt{7}/2=-1.32$ gives the SU(3) limit while $\chi=0$ gives O(6). The CQF has the advantage that the SU(3) to O(6) transition region and, therefore, the structure of most well-deformed nuclei, is a function of only the *single* parameter χ. This not only simplifies practical calculations, but facilitates physical and geometrical interpretation. It is, for example, via the CQF, that the relation between the IBA and axial asymmetry [4], or finite boson number effects [5], are best studied.

Although CQF calculations for individual deformed nuclei work extremely well, there has never been a systematic study of whether the CQF is actually empirically *mandated,* that is, of what are the *maximal* deviations from consistent forms of the quadrupole operator allowed by the data.

It is the purpose of this Rapid Communication to discuss both subjects. We first test the CQF itself. Finding that the data strongly support the CQF, we exploit this formalism to explore the relationship between the β and γ vibrational modes.

The linkage of β and γ bands in the IBA arises in the structure of the SU(3) limit where these bands belong to the same excited $(\lambda,\mu)=(2N-4,2)$ representation. This leads (see Ref. [6]) to several well-known properties of SU(3): the β and γ bands are degenerate, that is, $E_\beta(J_\beta)=E_\gamma(J_\gamma)$ for $J_\beta=J_\gamma$; the $E2$ selection rule $\Delta(\lambda,\mu)=0$ implies that $B(E2{:}\beta\rightarrow g)$ and $B(E2{:}\gamma\rightarrow g)$ vanish; and $B(E2{:}\beta\leftrightarrow\gamma)$ values are not only nonzero but, in fact, collective. Moreover, for finite N_B, there is β-γ band mixing.

This linkage persists when SU(3) is broken (i.e., for $\chi\neq-1.32$). The principal off-diagonal matrix elements that break SU(3) have $\Delta K=0$. Hence the β and g bands mix and repel: $E_\beta(J)/E_\gamma(J)>1$. This β-g $\Delta K=0$ mixing is transferred to the γ band via β-γ band mixing and is an important source for γ-g band mixing, due to which IBA $\gamma\rightarrow g$ $E2$ branching ratios deviate from the Alaga values (and agree with the data).

To proceed, we define two observables, associated with these modes,

$$R_{\beta\gamma}=E_{0_\beta^+}/(E_{2_\gamma^+}-E_{2_g^+})\,, \tag{4}$$

where the energy difference in the denominator of $R_{\beta\gamma}$ cancels any influence of $L\cdot L$ terms in H (which cannot change energy differences of states of the same spin), and

$$R_{62}=\frac{B(E2{:}6_\gamma^+\rightarrow 4_g^+)}{B(E2{:}6_\gamma^+\rightarrow 6_g^+)}\Bigg/\frac{B(E2{:}2_\gamma^+\rightarrow 0_g^+)}{B(E2{:}2_\gamma^+\rightarrow 2_g^+)}\,. \tag{5}$$

[We obtain R_{62} from empirical values [5] of the band mixing parameter Z_γ which is closely related to R_{62} but averages γ-g band mixing over several transitions (spins) and is nearly immune to effects of $M1$ components in the

$J_\gamma \rightarrow J_g$ transitions. Comparison with measured R_{62} values discloses no problems with this approach.]

Figure 1 (top) shows $R_{\beta\gamma}$ as a function of χ_H. There is a nearly linear increase from unity to ~ 1.8 for χ_H values typical ($\chi_H \sim -0.5$) of deformed nuclei. In contrast R_{62} depends on *both* χ_H and χ_{E2}. Figure 1 (bottom) shows a contour plot of R_{62} against χ_H and χ_{E2}. Any empirical R_{62} value has a family of solutions: to pick a specific one, a further constraint is needed. To do this, we fix χ_H from $R_{\beta\gamma}$, as in Fig. 1, and determine χ_{E2} to reproduce each R_{62} (always for the appropriate N_B). [Two of the nuclei studied, ^{156}Gd and ^{158}Dy, cannot be described simply as lying between SU(3) and O(6) but exhibit remnants of U(5). Previous IBA calculations [7] incorporated small εn_d terms in Eq. (1) which we used in extracting χ_H and χ_{E2}.] The test of the CQF is to compare these extracted values of χ_H and χ_{E2}.

The results in Fig. 2 show a close correlation between χ_H and χ_{E2}: with few exceptions χ_H and χ_E nearly scale with each other. Although the exact CQF relationship, $\chi_H = \chi_{E2}$, is not fulfilled, optimum χ_{E2} values tend to be only slightly greater than χ_H and usually well within experimental uncertainties. This establishes that the CQF is not merely a convenient tool to simplify calculations and to expose the geometrical content of the IBA, but, at least within the context of simple IBA Hamiltonians, that it is an approximation mandated by the data.

Besides supporting the CQF as a reasonable approach to IBA calculations, Fig. 2 implicitly contains important information on the linkage of β and γ band properties in the IBA. To see this, we exploit the simplifying properties of the CQF to invert the procedure just discussed. We adopt the CQF, setting $\chi_{E2} = \chi_H \equiv \chi$, and determine χ solely from values of R_{62} extracted from $E2$ decay data alone. Then, using these χ values, we calculate the ratio $R_{\beta\gamma}$ and compare with experiment. The results are shown in Fig. 3. If the relationship between the β band energy and the $E2$ decay properties of the γ band were precisely as given by the CQF, all the data points would lie on the diagonal in Fig. 3. Remarkably, there is a close correlation: except for ^{164}Dy (and, to a lesser extent, ^{182}W), the calculated points lie within $\sim 20\%$ of experiment. This relationship holds even though χ spans a wide range from $\chi \simeq -0.3 \rightarrow -1.0$. [We note in passing that the anomalous case,

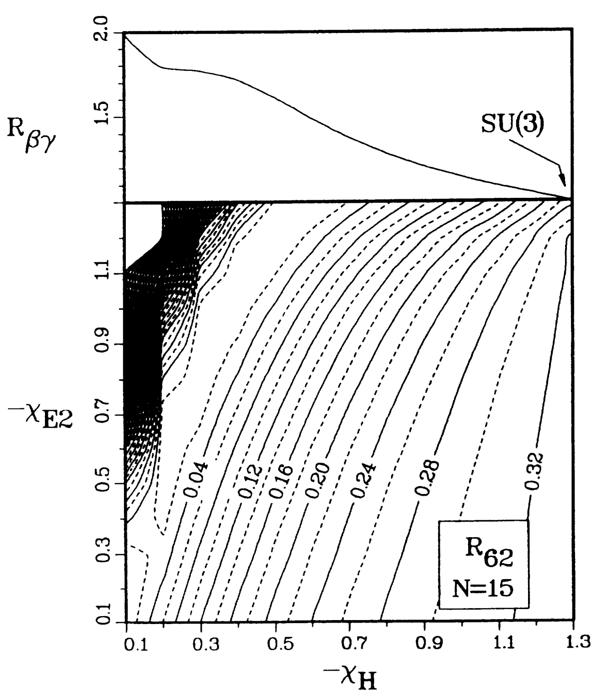

FIG. 1. Top: $R_{\beta\gamma}$ vs χ_H. Bottom: Constant R_{62} contours against χ_H and χ_{E2}.

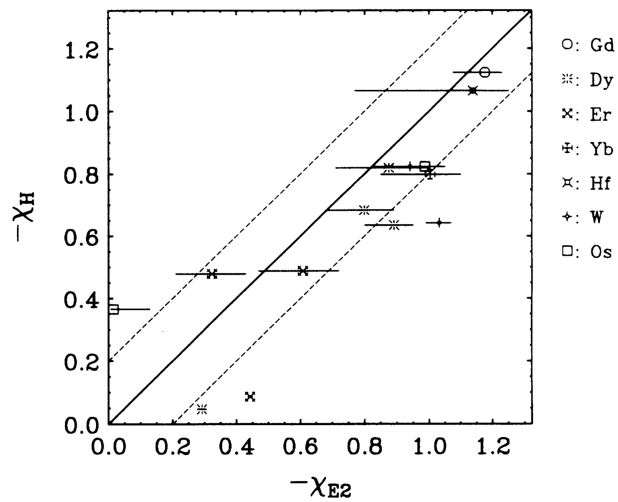

FIG. 2. Comparison of χ_H and χ_{E2} values extracted from the data for $R_{\beta\gamma}$ and R_{62} for rare-earth deformed nuclei. The error bars reflect experimental uncertainties in R_{62}. The solid diagonal line corresponds to $\chi_H = \chi_{E2}$, the dashed lines to $\chi_H = \chi_{E2} \pm 0.2$.

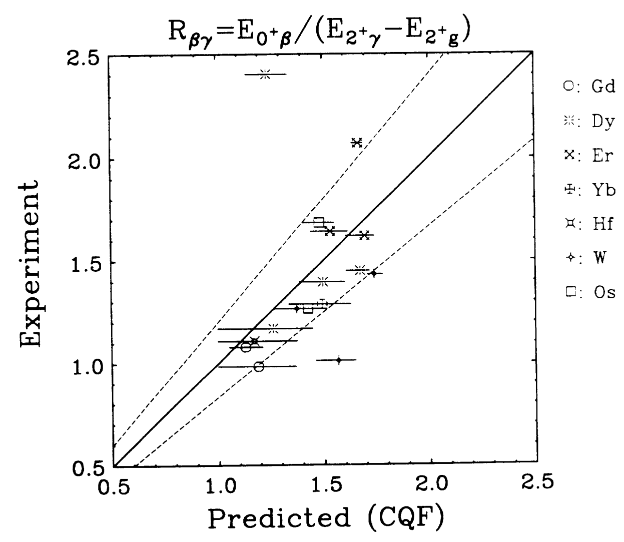

FIG. 3. Comparison of empirical and calculated values of $R_{\beta\gamma}$. The error bars reflect uncertainties in experimental R_{62} values used. The solid diagonal line corresponds to $R_{\beta\gamma}$ (expt.) $= R_{\beta\gamma}$ (IBA), the dashed lines to $\pm 20\%$ deviation from agreement.

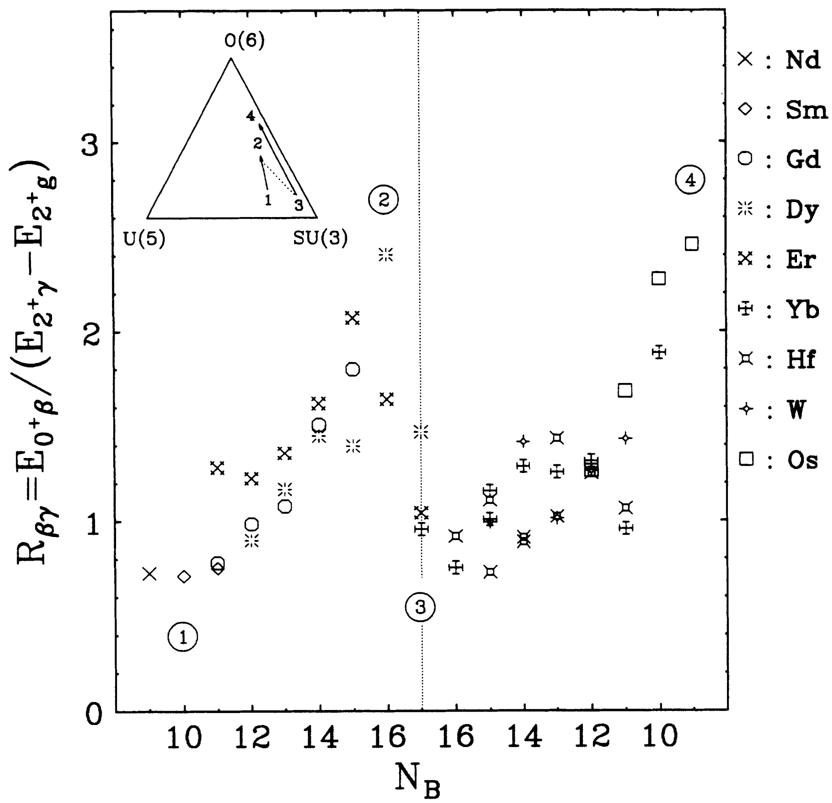

FIG. 4. $R_{\beta\gamma}$ vs N_B for rare-earth nuclei. Elements up to the onset of SU(3) after Er are plotted in the first half of the shell, those from Yb onwards, spanning the SU(3)$\rightarrow$O(6) transition, in the second. The inset shows a schematic illustration of the "path" taken by the nuclei from Gd to Os in the IBA symmetry triangle.

^{164}Dy, is unique in another way: summed empirical $B(M1)$ values for the isovector 1^+ scissors mode behave [8] smoothly in this region with, again, the exception of ^{164}Dy.] We stress that the β-γ relationship characterizing the IBA is not dependent on using the CQF. The same linkage occurs with the general IBA Hamiltonian, for example, using the multipole form or incorporating a term in εn_d: the CQF merely simplifies the analysis, reducing it to a one-parameter problem.

This link between β and γ vibrations should somehow be directly visible in the data itself. Though never previously noticed, this is in fact the case as shown in Fig. 4: the empirical $R_{\beta\gamma}$ values are not randomly scattered but show clear monotonic trends with N_B in each half shell. Interestingly, $R_{\beta\gamma}$ increases with N_B in the first half and decreases in the second half. The sawtooth pattern at midshell occurs at just the point, near $N=102$ in the Yb region, where the SU(3) limit ($R_{\beta\gamma}=1$) may be most closely approached [9]. $R_{\beta\gamma}=1$ also in the other nuclei often associated with near SU(3) character [2], near ^{156}Gd. This plot suggests a complex structural "trajectory" across the rare-earth nuclei with islands of near SU(3) at the beginning of the deformed region, and at midneutron shell, and with trends from SU(3) towards O(6) both between these islands and beyond midshell when the Os-Pt nuclei are approached (see the inset in Fig. 4). This path, and the basic β-γ linkage revealed in Figs. 3 and 4, presents a challenge to microscopic understanding.

0^+ intrinsic excitations have always been enigmatic to nuclear structure theories, both macroscopic and microscopic. Phenomenologically, their properties seem erratic; microscopic predictions, especially of $E2$ properties, are seldom highly successful. The present results, however, may shed some light on their structure. Historically, emphasis has been on $B(E2{:}K=0_2^+\rightarrow K=0_g^+)$ transitions as the supposed signature of collectivity. However, the IBA suggests a different view: here, the lowest $K=0^+$ excitation has collective transitions to the γ band instead. Such transitions have been observed. Along with the predicted and empirical correlation of β and γ band energies noted here this suggests a collective mode but one without collective transitions to the ground-state band: the fluctuating strengths of the latter may now be seen, not as a basic shortcoming of theories, or an indicator of noncollective character, but simply as reflecting the not uncommon behavior of forbidden transitions. Briefly, the emphasis in the past (on $\beta\rightarrow g$ transitions) has been on the wrong signature—on the noise rather than the signal.

To conclude, we have shown that the CQF of the IBA is an excellent approximation in deformed rare-earth nuclei: deviations from $\chi_H=\chi_{E2}$ are nearly always <0.2. Exploiting this, we have seen a relation between the β and γ vibrational modes close to that inherently predicted by the IBA. The concept of linkage between β and γ vibrational properties deserves serious microscopic study and may lead to a substantial revision in our understanding of these modes.

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