RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 45, NUMBER 6 JUNE 1992

Signature splitting in octupole bands of deformed nuclei

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(Received 9 December 1991)

Energy staggering in octupole bands in deformed nuclei is studied. The data show dramatic variations, even reversals, and a singularity in the staggering, across the rare-earth region. A simple model analysis, which is almost parameter independent, is able to reproduce the phenomenology, including the discontinuity, and is amenable to a simple physical interpretation.

PACS number(s): 21.10.Re, 21.60.Ev

Signature splitting, or energy staggering, in rotational bands is an actively studied [1] phenomenon, especially at high spins and is yielding important insights into nuclear structure. Unfortunately, little attention has been paid heretofore to energy staggering at low spin despite the fact that it is often in this spin region that the interpretation is clearer since there are fewer states and it is often possible to know all states (in certain spin and energy regimes) empirically. In particular, the vast amount of data on octupole states in well-deformed nuclei has largely escaped attention from this point of view.

It is the purpose of this Rapid Communication to study these data, focusing on the deformed rare-earth nuclei and on the lowest $K=1^-$ bands where the data display the most interesting phenomenology. We will show that, despite drastic variations (even reversals) in empirical staggering phenomenon, a simple model analysis, which is virtually parameter independent, yields excellent agreement with the data and is amenable to a transparent physical interpretation. A secondary result will be the realization that, albeit for different reasons in regions of different structure, the *same* staggering pattern dominates octupole states in *nearly all* nuclei.

To proceed, we exploit the simplicities of the interacting boson model (IBA) [2], and take

$$H=H_{sd}+H_{df}$$
$$=\kappa Q_d\cdot Q_d+\kappa' L_d\cdot L_d+\varepsilon_f n_f+A_1(L_d\cdot L_f)$$
$$+A_2(Q_d\cdot Q_f)+A_3{:}E^\dagger_{df}\cdot E_{df}{:}\,. \qquad (1)$$

This is taken from Ref. [3] (the now-standard approach) where the notation is discussed. The f-boson number, n_f, is either 0 (positive parity) or 1 (negative parity). Though this Hamiltonian appears complicated, the ordering of the K bands and the staggering results are *virtually independent* of nearly all parameters, except A_2 and A_3 (the coefficients of the "quadrupole" and "exchange" terms) *through* the ratio $F\equiv A_3/A_2$. For example, though χ (appearing in Q_d) is critical for the structure of the positive-parity intrinsic (e.g., β,γ) excitations, the lowest octupole states, being coupled primarily to the ground-state band, are nearly independent of χ. Also, ε_f merely sets the overall excitation energy of the negative parity states, not their internal energy relations. We therefore adopt a single parameter set for the present calculations: $\chi=-1.32$ [SU(3)], $\kappa=-0.015$ MeV, $\kappa'=0.0005$ MeV, $\varepsilon_f=1.88$ MeV, $A_1=0$, $A_2=-0.03$ MeV, and the boson number $N_B=14$. We will present calculations as a function of F (by varying A_3).

Figure 1 shows the calculated intrinsic energies for the $K=0^-$, 1^-, 2^-, and 3^- bands against F. Interpreting F as simulating the position of the Fermi surface, this figure, implicit in Ref. [3], displays the systematic change in the order of these bands as the shell fills, which is a well-known feature of these nuclei both empirically and microscopically [4].

An interesting issue, beyond the scope of the present paper, is how the IBA can yield such a systematics. The or-

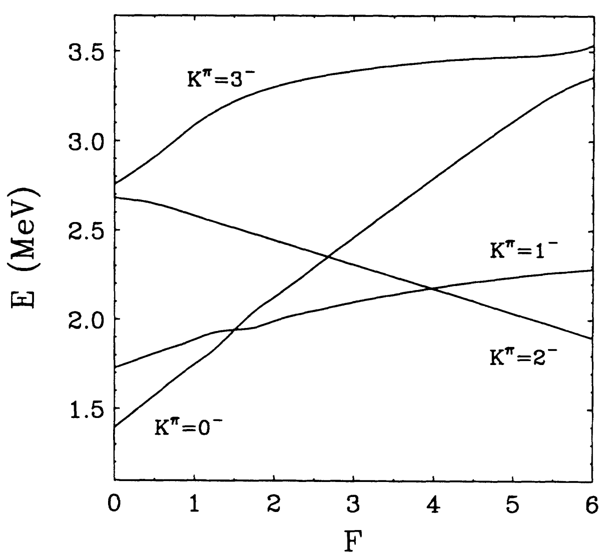

FIG. 1. Intrinsic energies of octupole excitations calculated as a function of F with the Hamiltonian of Eq. (1) and the parameters given in the text. The main features, especially the locations of the crossing points, are virtually parameter independent.

dering of the K bands can be understood [4] microscopically in terms of the interplay of quadrupole and octupole degrees of freedom via the availability of particular combinations of Nilsson orbits with appropriate asymptotic quantum numbers, and with $\Delta K=0$, 1, 2, or 3 near the Fermi surface. However, the IBA is unaware of such details and yet produces the same pattern, suggesting that its origins may be inherent in very general features of shell structure deserving of more explicit elucidation.

To study the energy staggering, we use an index, S_J, defined in Ref. [5],

$$S_J=\frac{[E(J)-E(J-1)]-[E(J-1)-E(J-2)]}{E_{2_1^+}}. \quad (2)$$

If there is sufficient staggering in rotational energies, S_J will alternate between positive and negative values with J. For γ bands in O(6) nuclei, where levels appear in couplets as 2_γ^+, $(3_\gamma^+,4_\gamma^+)$, $(5_\gamma^+,6_\gamma^+),\ldots$, S_J is positive for *odd J* and negative for even J. We have termed this pattern "quadrupole" staggering [5]. Negative-parity bands in such nuclei show [6] the opposite pattern, called "octupole" staggering, with S_J large and positive for *even J*. We now study the patterns arising in deformed nuclei.

Figure 2 shows examples of S_J for $K=1^-$ bands for two F values and empirical results [7] for a couple of typical nuclei. These are not fits nor are they intended for detailed comparison. Recall that all parameters except F were kept constant (including $N_B=14$), and none were optimized for these nuclei. The point is merely to show that strong staggering appears empirically and theoretically, that the model can produce patterns close to those seen empirically, and, perhaps most importantly, once a pattern (empirical or calculated) commences at low J, it continues with no phase changes. Thus we can simplify the analysis by considering a single S_J, namely, S_4 for which abundant data exist.

Figure 3 (top panel) shows the calculated values of S_4 against F. The most striking feature is the dramatic discontinuity near $F=1.5$, where S_4 suddenly switches sign. Thereafter S_4 gradually decreases towards 0.33, the rotor value. Since S_4 is positive for negative-parity states in O(6) (octupole staggering) [6], as well as for U(5) and intermediate situations, the negative values of S_4 for small F in deformed nuclei represent virtually the only cases where octupole excitations in the IBA display quadrupole staggering. Clearly, it is of considerable interest to see if such cases exist empirically and if the data reflect the singular behavior in Fig. 3 (top panel).

In order to do so we must devise a simple empirical quantity that mirrors the ratio, F. By interpreting F as mimicking the Fermi surface, an appropriate empirical analog becomes the average *fractional shell filling*

$$F_{\rm sh}=\frac{1}{2}\left(\frac{p}{32}+\frac{n}{44}\right), \quad (3)$$

where $p(n)$ is the number of valence proton (neutron) particles or holes counted to the nearest magic number, and 32 and 44 are the major shell degeneracies. Experimental S_4 values up to neutron midshell at $N=104$ are plotted against $F_{\rm sh}$ in the lower part of Fig. 3. The data are all taken from the recent thorough compilation by Sood, Headly, and Sheline [7]. We want to emphasize two related points, namely, first, the remarkable agreement between theory and experiment and, second, the fact that the physical *origin* of the staggering pattern is the

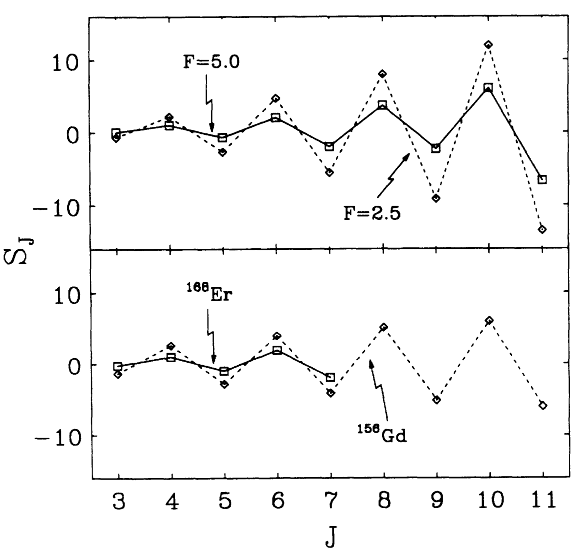

FIG. 2. S_J values. Top panel: Calculated values for two F values. Bottom panel: Experimental results for two typical nuclei, ^{156}Gd and ^{168}Er. The positive values for even J define "octupole" staggering. The data are from Ref. [7].

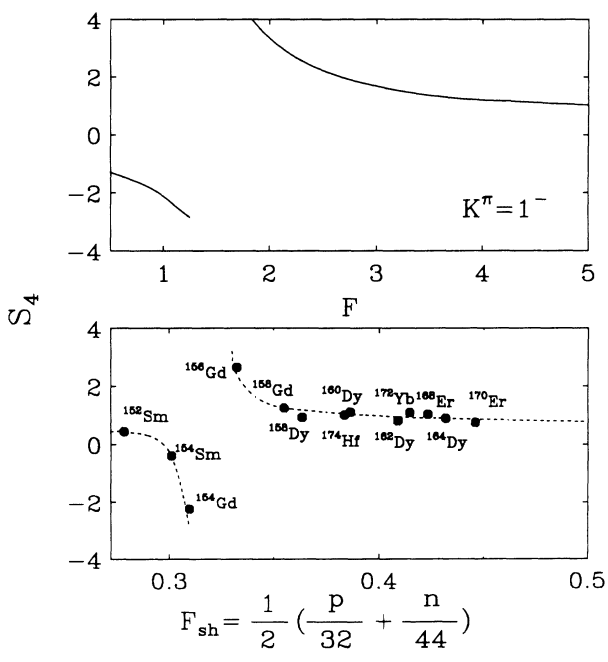

FIG. 3. S_4 values for rare-earth nuclei. Top panel: Theoretical values plotted against F. Note that for F very close to 3.9, where the $K=1^-$ and 2^- bands cross, S_4 is somewhat ambiguous due to the difficulty of associating states with particular bands. Bottom panel: Experimental values plotted against $F_{\rm sh}$ (see text).

same in each.

Addressing the first point, the principal conclusion of this Rapid Communication is the striking agreement between the schematic model and the data. Both show a singularity where S_4 suddenly switches sign, as well as very similar behavior before and after the singularity. The agreement in Fig. 3, of course, also further supports the interpretation of F as related to the Fermi surface, although we note that the association of F with a specific F_{sh} is only approximate in these schematic calculations, and detailed fits of band ordering and separations will require tuning of F, especially beyond Gd. As these nuclei are already in an asymptotic S_4 region, however, this will have little effect on the predicted staggering.

The $\tan\theta$-like shape of the discontinuity at $F=1.5$ is a characteristic signature of two-state mixing where a crossing of the unperturbed states occurs at the singular point. The origin of the predicted staggering is then immediately clear from Fig. 1. Since the $K=0^-$ band has only alternate spins $1^-, 3^-, 5^-, \ldots$, it repels only those levels of the $K=1^-$ band. When the $K=0^-$ band is below the $K=1^-$ band, the $1^-, 3^-, 5^-, \ldots$, levels of the latter are forced upwards, towards the $2^-, 4^-, 6^-, \ldots$, giving couplets arranged as $(1^-,2^-)$, $(3^-,4^-)$, $(5^-,6^-)$, ...; that is, quadrupole staggering. When the $K=0^-$ band passes above the $K=1^-$ band, the opposite staggering appears. Other details of the S_4 can be easily understood, in this context, in terms of the energy difference $\Delta E_K = E_{1^-}(K=0^-) - E_{1^-}(K=1^-)$. For $F \sim 0$, ΔE_K is large and negative; hence S_4 is negative but small (small mixing). As $|\Delta E_K|$ decreases with increasing F, the mixing increases and $|S_4|$ grows larger. At the crossing point, S_4 becomes positive and large since $\Delta E_K \sim 0$. Thereafter, S_4 smoothly decreases towards 0.33 as ΔE_K grows more positive.

The *observed* staggering can also be attributed to mixing. Figure 4 shows that the order of the two bands indeed switches between ^{154}Gd and ^{156}Gd and their larger $|S_4|$ values are associated with smaller ΔE_K.

Finally, we comment on the near universality of octupole staggering observed for negative-parity excitations. In O(6) and U(5) (and intermediate) regions, octupole excitations are weakly coupled to a vibrational-like core. In first order, the 3^- level is an octupole ($l=3$) excitation coupled to the ground state, the 4^- and 5^- levels both require coupling of that mode to the 2_1^+ level, and hence have nearly the same energy, and so on. It is then natural

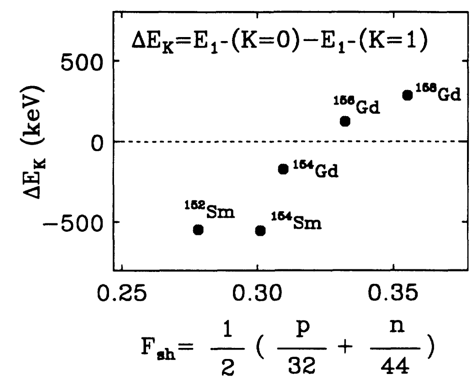

FIG. 4. The empirical energy difference ΔE_K (see text) near the discontinuity in S_4. The data are from Ref. [7].

that the staggering in negative-parity bands in such nuclei is shifted in phase relative to that in the γ band, a weakly coupled $l=2$ excitation. In deformed nuclei the staggering in $K=1^-$ bands is also nearly always of octupole type (S_4 positive and greater than 0.33 for J even), but its origin is different: it arises from $\Delta K=1$ mixing with the $K=0^-$ band where the latter is nearly always *above* the $K=1^-$ band.

To summarize, the *sdf* IBA in deformed nuclei predicts a characteristic staggering pattern for $K=1^-$ octupole excitations which is nearly parameter independent. This pattern features a marked discontinuity (at $F \sim 1.5$), arising from the characteristic ordering of negative-parity K bands, as a result of simple two-state mixing of the $K=1^-$ and 0^- bands. The data exhibit a similar pattern, including the discontinuity, and again this has a mixing origin. Lastly, an upshot of these results is that the lowest negative-parity states almost universally display the same (octupole) staggering (excepting the very few nuclei before the discontinuity in deformed regions), even though this staggering has very different physical origins in regions of different structure.

We thank B. R. Barrett, D. D. Warner, and P. von Brentano for enlightening discussions. This work was supported under Contracts No. DE-AC02-76CH000016 and No. DE-FG02-88ER40417 with the U.S. DOE, No. 62-91-37 with the IAP (Bucharest), and by the BMFT.

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