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Fluctuations and intermittency in multifragmentation processes

H. W. Barz,[1] J. P. Bondorf,[2] R. Donangelo,[3] I. N. Mishustin,[4] H. Schulz,[5] and K. Sneppen[2]
[1]*Institut für Kern- und Hadronenphysik am Forschungszentrum Rossendorf, Postfach 19, O-8051 Dresden, Germany*
[2]*The Niels Bohr Institute, 2100 Copenhagen, Blegdamsvej 17, Denmark*
[3]*Instituto de Física, Universidade Federal do Rio de Janeiro, 21945 Rio de Janeiro, Brazil*
[4]*V. I. Kurchatov Institute of Nuclear Research, Moscow, U.S.S.R.*
[5]*National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824*

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Nuclear multifragmentation processes may show features such as intermittency that signal scale-invariant properties of fluctuations in the fragment distributions. We test this behavior using several models and show that intermittency may be caused by a combination of finite-size effects and the mixing of fragmentation events corresponding to different initial conditions.

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The fluctuations that cause the breakup of a highly excited nuclear system into many fragments have been studied very intensely in the last few years. It was recently pointed out that the fragment size distribution shows signs of intermittency [1]. If this were truly the case, it would indicate that the fragmentation process exhibits scale-invariance properties similar to those found, e.g., in turbulent flows.

Intermittency can be inferred from the data through consideration of the factorial moments (see Ref. [2] and references therein)

$$F_k(\Delta)=\frac{\left\langle \sum_{i=1}^{Z_0/\Delta} N_i(N_i-1)\cdots(N_i-k+1)\right\rangle}{\sum_{i=1}^{Z_0/\Delta}\langle N_i\rangle^k}. \tag{1}$$

Here Z_0 is the total charge of the disassembling nuclear system which is divided in bins of length Δ, and N_i represents the number of fragments with charges in the interval $[(i-1)\Delta+1, i\Delta]$, $i=1,2,\ldots,Z_0/\Delta$. The ensemble averaging $\langle\cdots\rangle$ in Eq. (1) is performed over all the fragmentation events considered. One expects for an intermittency behavior that the moments F_k follow a power law as a function of the resolution Δ of the form $F_k\sim\Delta^{-f_k}$, i.e., the moments show self-similarity for the different resolutions $F_k(a\Delta)=a^{-f_k}F_k(\Delta)$.[1] When this analysis is applied to the reaction of gold on emulsion at 1 GeV per nucleon incident energy [6] one observes [1] that (i) $\ln[F_k(\Delta)]>0$ for all k,Δ; (ii) $\ln[F_k(\Delta)]$ increases roughly linearly with $-\ln(\Delta)$. These findings were interpreted as a signal for intermittency in the nuclear fragmentation process. It was also shown in Ref. [1] that the percolation approach to multifragmentation gives a similar intermittency behavior as the data.

[1]For a discussion of the intermittency pattern in connection with fractal geometry and chaos we refer to Refs. [3–5] and references therein.

Despite the fact that the data are exclusive in the sense that they were analyzed event by event, one has to be aware that they do include fragmentation events corresponding to many different impact parameters. As a consequence, they are associated to a wide range of excitation energies of the fragmenting transient system, and so give rise to large fluctuations of the initial conditions for the nuclear disassembly reactions (see also Ref. [7]).

It is the aim of the present work to investigate the intermittency patterns using different nuclear multifragmentation models and to find out whether they are intrinsic to the models or arise from particular details of the phenomenological description. In fact, we will show that these patterns can appear either because of improperly binning together events which correspond to different excitation energies, or when the multifragmentation model itself predicts a very broad multiplicity distribution.

Let us first consider a large system that is in thermal equilibrium. In calculating the fragment size distribution for such a system the grand canonical approach is rather appropriate and yields a Poisson distribution for fragment sizes N_i [8]

$$P(N_i)=\frac{\langle N_i\rangle^{N_i}}{N_i!}e^{-\langle N_i\rangle}. \tag{2}$$

Using the fact that the sum of two Poisson distributions also follows a similar distribution law, one can easily demonstrate that the logarithm of the factorial moments (1) vanishes for all values of k and resolution sizes Δ. This means that in this case not only is there no intermittency, but also that all factorial moments are equal to one irrespective of the system size or excitation energy. In fact, it is important to realize that even the mixture of fragments arising from the decay of nuclear systems having a diversity of excitation energies would have, in the grand canonical limit, Piosson-like multiplicity distributions and, as a consequence, factorial moments equal to 1, i.e., no signs of intermittency should be present. Departures from unity factorial moment values may thus be traced back to the fact that we are dealing with a relatively small system, for which the Poisson distribution inherent to the grand canonical approach is not the most adequate one.

Let us now consider another fragment distribution function. The so-called "cold fragmentation model" [9], assumes that all possible partitions of the nucleus have the same probability. In this case the multiplicity distribution is rather broad, as illustrated by the solid line in Fig. 1. In fact, its variance σ^2 is larger than its average value $\langle M \rangle$, which makes it broader than a Poisson distribution. For comparison we also employ another distribution where fragments with the same size are suppressed by weighting the probability of a given partition $P(\{N_i\})$ by the factor $W(\{N_i\}) = 1/\Pi N_i!$. These probabilities lead to the much narrower multiplicity distribution shown by the dashed line in Fig. 1.

The factorial moments associated with these two distributions have completely different behaviors. In fact, as shown in Fig. 2, the original cold fragmentation model [9] (solid lines) leads to positive factorial moments, $\ln[F_k(\Delta)] > 0$ which strongly increase with the resolution $-\ln(\Delta)$ and exhibit the characteristic pattern for intermittency. In other words, in the example chosen, the intermittency behavior is an immediate consequence of the assumption that all partitions of the nucleus have the same probability, i.e., any kind of conservation laws such as energy or momentum conservation which would make the distribution much narrower are disregarded. Taking, for example, the narrow distribution due to including the suppression factors into account, the factorial moments take negative values (dashed lines) and the pattern resembling intermittency disappears. This particular behavior can easily be demonstrated by looking at the second factorial moments which can be expressed in terms of the mean multiplicity $\langle M \rangle$ and the spread σ^2 of the distribution function according to

$$\ln F_2(\Delta = Z_0) \approx \frac{1}{\langle M \rangle}\left[\frac{\sigma^2}{\langle M \rangle} - 1\right] \approx \begin{cases} -1/\langle M \rangle, & \langle M \rangle \gg \sigma^2 \\ \sigma^2/\langle M \rangle^2, & \sigma^2 \gg \langle M \rangle , \end{cases} \tag{3}$$

suggesting that the appearance of intermittency is connected to broad multiplicity distributions.

In order to obtain further information on the possible intermittency in fragmenting nuclear systems, we have calculated the factorial moments using the statistical multifragmentation model based on a microcanonical treatment [8,10], which is similar in spirit to those of Koonin and Randrup [11] and Gross and co-workers [12]. The microcanonical approaches take into account energy and momentum conservation for the different partitions and consider the indistinguishability of fragments with the same neutron and proton number. The resulting multiplicity distributions have Gaussian-like shapes, whereby the variances are considerably smaller than the corresponding average values. Obviously, these distribution functions turn out to be rather narrow and, in accordance with the considerations in formula (3) there should be no sign of intermittency. To quantify this we show in Fig. 3(a) (dashed lines) the calculated factorial moments when disassembling a gold nucleus with an average excitation energy of $E^* = 6$ MeV per nucleon. One clearly sees that the factorial moments are less than one and no sign of intermittency is observed. These results do not qualitatively change when the cooling of the hot primordial fragments due to evaporation is disregarded. Since both the variance and the average values of the multiplicity distribution depend on the excitation energy E^*, we calculated them for a wide range of E^* values. The result was that the variances were always significantively smaller than the corresponding average values. As a consequence, irrespective of the adopted initial excitation energy, the factorial moments turned out to be rather small with $\ln(F_k) < 0$ and insensitive to the bin size chosen. In this connection it should be noted that equilibrium fragmentation models indeed show criticality at the onset of multifragmentation that is signaled by widely different fragment sizes within each event, i.e., by a big value of the parameter γ_2 accord-

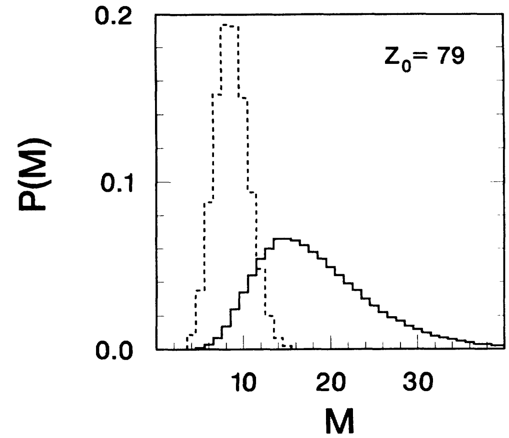

FIG. 1. Multiplicity distribution for two versions of the "cold fragmentation model." The solid line corresponds to the case [9] where all charge partitions are equally probable. The dashed curve takes into account quantum suppression of identical fragments.

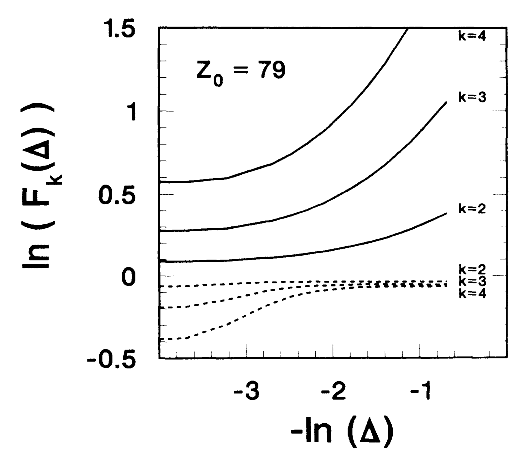

FIG. 2. Factorial moments labeled by the order parameter k as a function of the resolution $-\ln(\Delta)$ for the two fragmentation models discussed in Fig. 1.

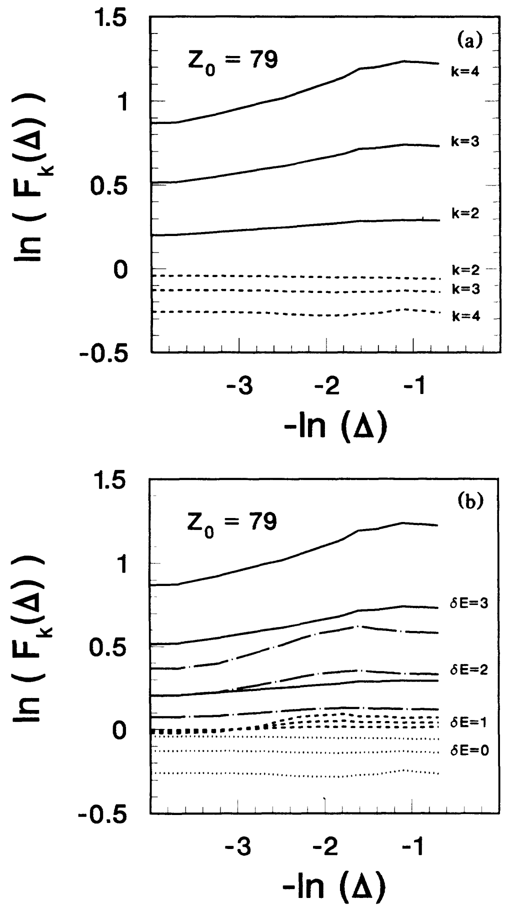

FIG. 3. (a) Factorial moments as predicted by the Copenhagen statistical multifragmentation model [8,10] as a function of the resolution $-\ln(\Delta)$. The dashed curves are calculated for the fragmentation of a gold nucleus with an average excitation energy $E^* = 6$ MeV per nucleon. The solid lines are calculated by mixing equally events corresponding to excitation energies of $E^* = 3$, 6, and 9 MeV per nucleon. (b) Same as (a) taking the excitation energy of the fragmenting nucleus uniformly distributed in the range $[6-\delta E,\ 6+\delta E]$ MeV per nucleon, with $\delta E = 0$, 1, 2, and 3 MeV.

ing to Campi's analysis [13]. For the studied equilibrium model this critical onset of multifragmentation happens at an excitation energy per nucleon of $E^*/A \approx 4$ MeV, where most of the contributing events have $\gamma_2 > 2$. It is important to realize that an intermittency signal is not related to a typical event, but instead is determined by fluctuations from event to event.

The question is then, how can an intermittency pattern emerge when analyzing the data? One source of a behavior resembling intermittency is mixing events associated with different excitation energy E^*. In fact, different excitation energies lead to qualitatively distinct decay patterns of the fragmenting nucleus and may cause correlations which resemble those inferred from the data by applying the intermittency analysis. To demonstrate the effect of event mixing we show in Fig. 3(a) (solid lines) the factorial moments when equally mixing events with $E^* = 3$, 6, and 9 MeV per nucleon. One sees clearly that $\ln(F_k) > 0$ and increases with the resolution $-\ln(\Delta)$. In other words, any kind of event mixing implies that the resulting multiplicity distribution function becomes broader than the original one and as a consequence the factorial moments may significantively increase with $-\ln(\Delta)$. This is illustrated by the results shown in Fig. 3(b), where the factorial moments are calculated as a function of the spreading δE of excitation energies included in the event mixing. It is clearly seen that an increased range of initial excitations δE implies an increased range of linear scaling of $\ln(F_k)$ with $-\ln(\Delta)$. As a consequence, for a sufficiently broad range of initial excitations the factorial moments appear to scale linearly with bin size and therefore seem to signal intermittency.

It seems to us that a signal that may be interpreted as an intermittency pattern may arise because (i) it is directly linked to the larger width of the multiplicity distribution obtained when fragments arising from breakup processes at widely different combinations of energy and mass of the fragmenting system are put together, and also because of (ii) the finiteness of the nuclear system that undergoes multifragmentation, since mixed-up Poisson distributions characteristic for large systems would always lead to vanishing factorial moments.

The intermittency pattern found recently [14] by applying the microcanonical fragmentation to gold nuclei could most likely be traced back to the adopted excitation energy spectrum of the gold nucleus. This has to be seen in line with the results of Ref. [15] in which fragment distributions from breakup processes at different excitation energies have been studied. It is also worthwhile noting that it has recently been shown [16] that linear rate equations for fragment yields, which can be related to simulations of sequential decay processes, are generally unable to reproduce intermittency in the final fragment distribution.

In conclusion the appearance of intermittency patterns in the data of [6] seems to arise from the mixing of different types of fragmentation events. It is importnat to repeat this analysis disentangling the peripheral from the more central processes. In principle this could be done using the method of Ref. [7]. If intermittency still appears for such more exclusive data, it may have important consequences on the validity of the statistical models of fragmentation, e.g., the assumption of thermal equilibrium of the fragmenting system.

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