RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 45, NUMBER 4 APRIL 1992

$p+\vec{d}\rightarrow {}^3\mathrm{He}+\gamma$ reaction with realistic three-nucleon wave functions

S. Ishikawa
Institut für Theoretische Physik II, Ruhr-Universität Bochum, 4630 Bochum, Germany and Department of Physics, Tohoku University, Sendai 980, Japan

T. Sasakawa
University of Library and Information Science, Tsukuba 305, Japan

(Received 1 August 1991)

We study the proton-deuteron radiative capture reaction at low energies ($E_\gamma \cong 10$ MeV) with the three-nucleon Faddeev wave functions for various combinations of available realistic nucleon-nucleon potentials and the two-pion exchange three-nucleon potential. The calculated values of tensor analyzing power $A_{yy}(\theta)$ are found to be sensitive to a different choice of realistic potentials and introduction of the three-nucleon potential. The comparison with available data shows the necessity of the three-nucleon potential.

PACS number(s): 21.30.+y, 25.10.+s, 25.40.Lw

The D-state components in the wave functions of light nuclei (^{2}H, ^{3}H, ^{3}He, ^{4}He, ...) have attracted considerable attention both experimentally and theoretically [1,2]. As the D-state admixture is mainly due to a tensor component in the nuclear force, we can expect to get some information on nuclear tensor interaction by investigating the D-states effect in light nuclei. The proton-deuteron (p-d) radiative capture reaction, $p+\vec{d}\rightarrow {}^3\mathrm{He}+\gamma$, is one of such reactions that is suitable to this study.

In the last decade, some experimental and theoretical works were done for the p-d radiative capture reaction to study a sensitivity of the result to the D state of ^{3}He. King *et al.* [3] measured angular distributions of the reaction $^2\mathrm{H}(p,\gamma)^3\mathrm{He}$ for E_p^{lab} from 6.5 to 15 MeV, and fitted by a Legendre series,

$$\sigma(\theta)=A_0\left[1+\sum_{k=1}^{4} a_k P_k(\cos\theta)\right]. \qquad (1)$$

They performed an effective two-body direct-capture calculation, in which they treated the reaction as a p-d two-body problem. An effective two-body p-d wave function for the final trinucleon bound state is obtained by taking an overlap between ^{3}He and ^{2}H wave functions generated for separable nucleon-nucleon (N-N) interactions [4]. The initial p-d motion was assumed to be the plane wave or a scattering state generated from an optical potential. Using various separable interactions, they found that the coefficient a_2 is sensitive to D-state probability in the ^{3}He wave function.

The tensor analyzing powers (TAP's), $T_{2q}(\theta)$, were thought to be sensitive to the D state, because no TAP is expected in the simplest $p+d$ model of ^{3}He with only an S-state component. Vetterli *et al.* [5] measured $T_{20}(\theta)$ for the $^1\mathrm{H}(\vec{d},\gamma)^3\mathrm{He}$ reaction at deuteron energy of 19.8 MeV, corresponding to $E_p^{\mathrm{lab}}=9.9$ MeV. (Hereafter, we express the energy of the three-nucleon system by E_p^{lab} even for a deuteron projectile experiment.) The obtained $T_{20}(\theta)$ data were compared with results of the effective two-body direct-capture calculations as in Ref. [3]. These calculations show that $T_{20}(\theta)$ is actually sensitive to D-state components in ^{3}He wave functions. Jourdan *et al.* [6] measured $A_{yy}(\theta)$, which is given by

$$A_{yy}(\theta)=-\frac{1}{\sqrt{2}}T_{20}(\theta)-\sqrt{3}T_{22}(\theta), \qquad (2)$$

for $\theta_{\mathrm{lab}}=90°$ at $E_p^{\mathrm{lab}}=14.6$ and 22.65 MeV. In Ref. [6], a consistent calculation with three-nucleon wave functions solved for a realistic N-N interaction (soft-core potential of Reid, RSC [7]) has been performed for the first time. They found that (1) the use of scattering solution for the p-d incident channel instead of the plane wave makes $A_{yy}(\theta)$ decrease by about 50% and (2) the $E2$ and $M1$ effects on $A_{yy}(\theta)$ inside the angular range of 40°–150° are very small. However, as they use only RSC potential, it is worthwhile to perform calculations for other potentials, also including the three-nucleon potential.

Recent progress in theoretical treatment of the three-body problem allows us to solve the three-nucleon Faddeev equations accurately not only for bound states, but also for scattering states with an available realistic N-N potential (2NP) even with a three-nucleon potential (3NP). In this paper, we report our calculations of the low-energy p-d radiative capture reaction for different combinations of available 2NP's and 3NP's, and discuss how the nuclear tensor interaction together with the 3NP effect influence observables.

An amplitude for a transition from an initial state with proton and deuteron whose z components of spin are m_s and M_d, respectively, and relative momentum $\mathbf{p}$, to a final state with a photon of momentum $\mathbf{k}_\gamma$ and the polarization vector $\boldsymbol{\epsilon}_\mu$ ($\mu=\pm 1;\mathbf{k}_\gamma\cdot\boldsymbol{\epsilon}_\mu=0$), is written in the following way in first-order perturbation theory:

$$T(M_t,\mu,\mathbf{k}_\gamma;M_d,m_s,\mathbf{p})=\langle\Psi_{M_t}^{B}|H_\mu(\mathbf{k}_\gamma)|\Psi_{M_d m_s}^{S}(\mathbf{p})\rangle. \qquad (3)$$

Here, $|\Psi_{M_d m_s}^{S}(\mathbf{p})\rangle$ represents a three-nucleon continuum state initiated from the p-d state with the quantum numbers $(M_d,m_s,\mathbf{p})$, and $|\Psi_{M_t}^{B}\rangle$ the ^{3}He ground state with M_t.

$H_\mu(\mathbf{k}_\gamma)$ stands for a photonuclear interaction Hamiltonian

$$H_\mu(\mathbf{k}_\gamma)=-\frac{e}{c}\int \mathbf{J}(\mathbf{r})\cdot \mathbf{A}_\mu(\mathbf{r},\mathbf{k}_\gamma)d\mathbf{r}\,, \tag{4}$$

where $\mathbf{A}_\mu(\mathbf{r},\mathbf{k}_\gamma)$ is a vector potential:

$$\mathbf{A}_\mu(\mathbf{r},\mathbf{k}_\gamma)=\left[\frac{2\pi\hbar c}{k_\gamma}\right]^{1/2}\epsilon_\mu^* e^{-i\mathbf{k}_\gamma\cdot\mathbf{r}}\,, \tag{5}$$

and $\mathbf{J}(\mathbf{r})$ is a nuclear current operator.

We followed the same procedure as in Refs. [8,9], in which we used the continuity equation

$$\nabla\cdot\mathbf{J}(\mathbf{r})+\frac{i}{\hbar}[H_N,\rho(\mathbf{r})]=0\,, \tag{6}$$

where $\rho(\mathbf{r})$ is a nuclear charge density operator and H_N is the nuclear Hamiltonian. After the long-wavelength approximation, we get an expression for $H_\mu(\mathbf{k}_\gamma)$ as follows:

$$H_\mu(\mathbf{k}_\gamma)=\frac{2\pi\hbar e}{(\hbar c k_\gamma)^{1/2}}\sum_{\lambda=1}^{\infty} i^\lambda\sqrt{2\lambda+1}\sum_{\sigma=-\lambda}^{\lambda} D^\lambda_{\sigma\mu}(\pi,\pi-\theta,0)\times T^{\mathrm{el}}_{\lambda\sigma}(k_\gamma)\,, \tag{7}$$

where $D^\lambda_{\sigma\mu}(\alpha,\beta,\gamma)$ is the rotation matrix and $T^{\mathrm{el}}_{\lambda\sigma}(k_\gamma)$ the electric multipole operator,

$$T^{\mathrm{el}}_{\lambda\sigma}(k_\gamma)=-\frac{k_\gamma^{\lambda-1}}{\hbar}\left[\frac{\lambda+1}{\lambda}\right]^{1/2}\frac{1}{(2\lambda+1)!!}\times\int r^\lambda Y_{\lambda\sigma}(\hat{r})[H_N,\rho(\mathbf{r})]d\mathbf{r}\,. \tag{8}$$

We assumed the one-body operator for $\rho(\mathbf{r})$ (the Siegert's hypothesis),

$$\rho(\mathbf{r})=\rho^{(1)}(\mathbf{r})=\sum_{i=1}^{3}\frac{1+\tau_z(i)}{2}\delta(\mathbf{r}-\mathbf{r}_i)\,. \tag{9}$$

The expression of Eq. (8) with this assumption is convenient for our calculation, because (1) effects of meson exchange currents are implicitly taken into account, and (2) we will take matrix elements with eigenfunctions of H_N both for the initial and final states, and thus H_N can be replaced by the eigenvalues. We use only electric dipole ($E1$) and electric quadrupole ($E2$) terms, i.e., $\lambda=1$ and 2 in Eq. (7). We should mention that we did not include Coulomb force, which was known to give little effect by the effective two-body direct-capture calculations [10].

Wave functions for the initial and final states are obtained by solving the three-nucleon Faddeev equations numerically. In our method, we express the Faddeev equation as a set of coupled integral equations in the coordinate representation, and solve it by an iterative method called the method of continued fractions [11-13].

In the Faddeev-type calculations for three-body systems, there exist two types of truncations for the partial-wave expansion: for total angular momentum of the two-body subsystem (J) and of the three-body system (J_0). The former corresponds to a truncation of a space of the nuclear Hamiltonian and the latter to a truncation of a wave function of the three-body system. (For the three-nucleon bound state, only the former truncation exists because J_0 is fixed to be $\frac{1}{2}$.) At the moment, our computer facility enable us to calculate three-nucleon continuum states with maximum value of J ($J_{\max}$) to be 2. The use of the Siegert's hypothesis demands that the Hamiltonian for the initial and final states should be the same. Thus, we also set $J_{\max}$ to be 2 for bound-state calculations.

As stated previously, the use of only $E1$ (1^-) and $E2$ (2^+) photonuclear multipole operators is a good approximation to the calculation of low-energy p-d radiative capture. Since spin parity (π) of the final state (^{3}He) is $\frac{1}{2}^+$, we need to calculate p-d states with J_0^π of $\frac{1}{2}^-$, $\frac{3}{2}^+$, $\frac{3}{2}^-$, and $\frac{5}{2}^+$. This contrasts with calculations of the nucleon-deuteron elastic or three-body breakup reactions, in which we need at least to calculate up to $\frac{15}{2}$ for J_0 to get convergent results.

In the present calculation, we used six kinds of realistic 2NP: the Argonne V14 model (AV_{14}) [14], the super-soft-core potential models of de Tourreil-Sprung (models A, B, and C) (dTS-A, B, and C) [15], and de Tourreil-Rouben-Sprung (dTRS) [16], and the r-space OBEP version of the Bonn potential (BONN) [17]. Among them, the first five potentials are known to underbind the triton by the order of 1 MeV compared to the experimental value of 8.48 MeV. In order to explain these differences, we should introduce 3NP to a nuclear Hamiltonian. For 3NP, we used the two-pion exchange model with a parametrization of a Brazilian group [18]. The calculated values of the triton binding energy with 3NP strongly depend on the πN-N form factor and its cutoff mass Λ_π. We choose the monopole form factor with the cutoff mass $\Lambda\pi=700$ MeV (BR_{700}), which is a suitable value to reproduce the triton binding energy. On the other hand, among 2NP's that we used, only BONN can yield the binding energy nearly equal to the experimental one without 3NP. However, it is pointed out [19] that BONN cannot reproduce a $^3S_1$-$^3D_1$ mixing parameter (ϵ_1) of N-N phase-shift analysis because of its weak tensor force, and at the same time this very fact explains why BONN can obtain a large value of B_3.

In Table I, we show calculated values of triton binding energy (B_3) and probability of the D state in the triton [$P_D(^3\mathrm{H})$] without (2NP) and with ($2\mathrm{NP}+\mathrm{BR}_{700}$) Brazilian 3NP together with the probability of the D state in the deuteron [$P_D(^2\mathrm{H})$] for all potential models which we are using. Note that 3NP let $P_D(^3\mathrm{H})$ increase for all 2NP's except dTS-B and BONN, both of which give rather small $P_D(^2\mathrm{H})$. Qualitatively, it seems that the effect of 3NP on $P_D(^3\mathrm{H})$ is larger for 2NP with stronger tensor

TABLE I. Results of two- and three-nucleon bound-state calculations for various models used in this article.

		2NP		$2\mathrm{NP}+\mathrm{BR}_{700}$	
2NP	$P_D(^2\mathrm{H})$ (%)	B_3 (MeV)	$P_D(^3\mathrm{H})$ (%)	B_3 (MeV)	$P_D(^3\mathrm{H})$ (%)
AV_{14}	6.08	7.58	8.90	8.42	9.14
dTRS	5.92	7.49	8.55	8.49	8.72
dTS-A	4.43	7.62	6.53	8.34	6.56
dTS-B	4.25	7.70	6.22	8.44	6.20
dTS-C	5.45	7.50	7.95	8.32	8.03
BONN	4.81	8.24	7.01	9.00	6.91

component. Effects of this behavior on the *p-d* radiative capture observables will be discussed below.

Our results of nucleon-deuteron elastic scattering have been reported elsewhere [13], and were comparable with results by other authors. For example, for dTRS *N-N* potential, we found a good agreement with those of momentum space calculation in Ref. [20].

Using three-body bound- and scattering state wave functions, we can calculate the transition amplitude for the *p-d* radiative capture, and thus, various related observables.

First, we investigate the coefficients a_k $(k=1,2,3,4)$ of the Legendre expansion of the angular distribution [see Eq. (1)]. Contrary to the result of the effective two-body direct-capture calculations, we found that not only calculated values of a_2 but also those of a_1, a_3, and a_4 are insensitive to different choice of 2NP and existence of 3NP. For example, in Table II, we show average values ($\langle a_k \rangle$) and standard deviations (s.d.) of calculated values of a_k's for 2NP (first column) and 2NP+BR_{700} (second column) at $E_p^{\text{lab}}=14.6$ MeV, together with experimental values [21,22] at $E_p^{\text{lab}}=14.8$ MeV (third column). In this table, difference in the values of $\langle a_k \rangle$ between 2NP and 2NP+BR_{700} should express the effects of BR_{700}-3NP. On the other hand, the values of standard deviations should express the effects of the different choice of 2NP. From this table, we can see that both effects are very small, at most only 1%. Further, we found that calculated values of a_k's are in good agreement with available experimental values at energies around $E_p^{\text{lab}}=10$ MeV.

Next, we investigated TAP of the reaction ${}^1\text{H}(\vec{d},\gamma){}^3\text{He}$. In Fig. 1, we plot the calculated values of A_{yy} for $\theta_{\text{c.m.}}=96°$ (corresponding to $\theta_{\text{lab}}=90°$) at $E_p^{\text{lab}}=14.6$ MeV vs $P_D({}^2\text{H})$ for used 2NP. In this figure, results for 2NP are shown by squares and those for 2NP+BR_{700}-3NP by crosses, respectively. The range of experimental value (0.0282 ± 0.0016) [6] is shown by the dotted area. We can see that all 2NP's yield 15%–30% smaller values than the experimental result. In Ref. [6], the effect of $E2$ multipole operator on $A_{yy}(\theta_{\text{c.m.}})$ for $\theta_{\text{c.m.}} \approx 90°$ was reported to be very small. In fact, when we neglect the $E2$ multipole operator, the values of $A_{yy}(\theta_{\text{c.m.}}=96°)$ decrease by only 2%–3%, which are small enough compared to the above difference.

TABLE II. Average values ($\langle a_k \rangle$) and standard deviations (s.d.) of the calculated a_k's [Eq. (1)] at $E_p^{\text{lab}}=14.6$ MeV, and the experimental values at $E_p^{\text{lab}}=14.8$ MeV.

a_k	2NP	2NP+BR_{700}	Expt.
$\langle a_1 \rangle$	0.340	0.339	0.33(1)
s.d.	0.002	0.002	
$\langle a_2 \rangle$	−0.919	−0.915	−0.89(1)
s.d.	0.003	0.004	
$\langle a_3 \rangle$	−0.335	−0.334	−0.29(2)
s.d.	0.002	0.002	
$\langle a_4 \rangle$	−0.029	−0.028	−0.08(1)
s.d.	0.000	0.000	

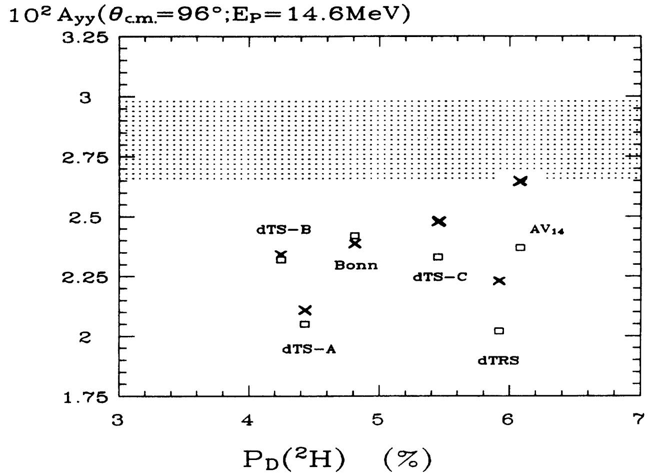

FIG. 1. $A_{yy}(\theta_{\text{c.m.}}=96°)$ at $E_p^{\text{lab}}=14.6$ MeV vs $P_D({}^2\text{H})$ for 2NP (squares) and 2NP+BR_{700} (crosses) calculations. The dotted area indicates the range of experimental value.

It should be noted that BONN, whose tensor force is relatively weak, yields the largest value for A_{yy} among the 2NP's that we used. This shows that A_{yy} cannot be determined simply by $P_D({}^2\text{H})$.

The introduction of 3NP makes the calculated value of A_{yy}'s increase for almost all 2NP's except BONN. This figure shows that the increase of A_{yy} (ΔA_{yy}) due to the inclusion of 3NP is large for 2NP with a strong tensor component.

In Fig. 2, we plot $\Delta A_{yy}(\theta_{\text{c.m.}}=96°)$ due to the introduction of BR_{700}-3NP at $E_p^{\text{lab}}=14.6$ MeV against those of $P_D({}^3\text{H})$ [$\Delta P_D({}^3\text{H})$]. We found a linear correlation between ΔA_{yy} and $\Delta P_D({}^3\text{H})$. After the least-squares fitting, the correlation is expressed by a straight line (with the

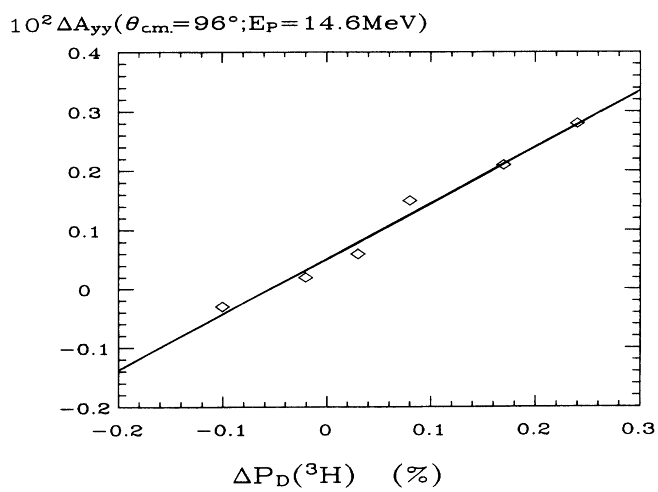

FIG. 2. The increase of $A_{yy}(\theta_{\text{c.m.}}=96°)$ at $E_p^{\text{lab}}=14.6$ MeV vs that of $P_D({}^3\text{H})$ due to BR_{700}-3NP. The straight line is obtained by the least-squares fitting.

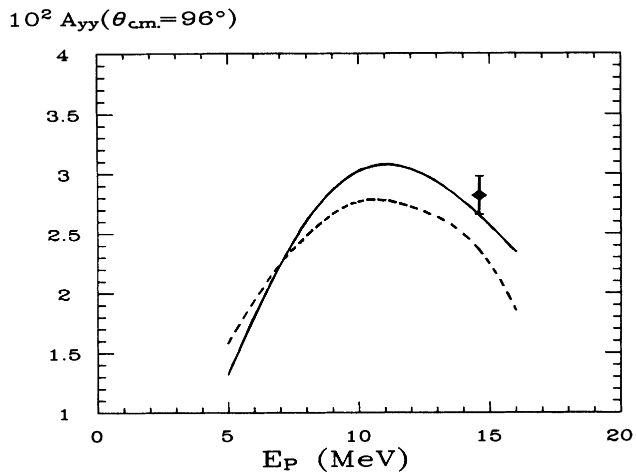

FIG. 3. Energy dependence of $A_{yy}(\theta_{c.m.}=96°)$ for AV_{14} (dashed curve) and $AV_{14}+BR_{700}$-3NP (solid curve). Data are taken from Ref. [6].

correlation coefficient $r=0.991$):

$$\Delta A_{yy}=0.9454\Delta P_D(^3\mathrm{H})+0.0520\,. \qquad (10)$$

Among all models which we used, AV_{14} with BR_{700}-3NP turns out to be the best choice to reproduce the experimental value of $A_{yy}(\theta_{c.m.}=96°)$ at $E_p^{\mathrm{lab}}=14.6$ MeV. Clearly, we need more TAP data at other energies and other angles to draw a definite conclusion. In Fig. 3, we plot energy dependence of A_{yy} at $\theta_{c.m.}=96°$ around $E_p^{\mathrm{lab}}=10$ MeV for AV_{14} and $AV_{14}+BR_{700}$-3NP.

We apologize that at the moment, our computer facility prevents us from extending the calculation to the energy $E_p=22.5$ MeV ($E_d=45.3$ MeV) for which we need much more mesh points to treat oscillating behavior of wave functions correctly.

To summarize, we calculated the angular distribution and tensor analyzing power of the reaction $p+\vec{d}\to {}^3\mathrm{He}+\gamma$. As far as the available experimental data are concerned, the $AV_{14}+BR_{700}$-3NP yields the most favorable result. Also the necessity of 3NP in three-nucleon continuum is found for the first time in the present work. We conclude that the tensor analyzing power $A_{yy}(\theta)$ at different energies around $E_p^{\mathrm{lab}}=10$ MeV and accurately calculated realistic three-nucleon wave functions could provide important information on nuclear interactions as to the tensor force effect and choosing a favorable realistic potential.

The calculation was supported in part by a Grant-in-Aid of the Ministry of Education, Science and Culture of Japan, Research Center for Nuclear Physics, Osaka University, and the Cyclotron Radioisotope Center, Tohoku University.

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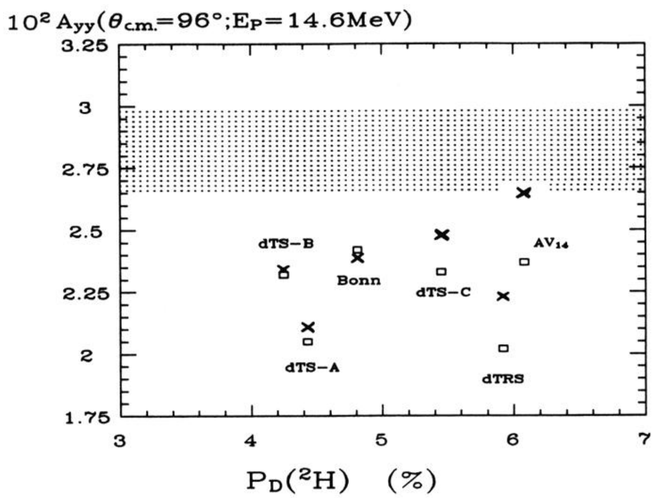

FIG. 1. $A_{yy}(\theta_{c.m.}=96°)$ at $E_p^{lab}=14.6$ MeV vs $P_D(^2H)$ for 2NP (squares) and 2NP+BR_{700} (crosses) calculations. The dotted area indicates the range of experimental value.