S-matrix poles and phase shifts for $N\Delta$ scattering

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Threshold behavior of S-wave phase shifts is studied for $N\Delta$ scattering with a nearby S-matrix pole. The phase $\delta_{N\Delta}$ for $pp(^1D_2)\rightarrow N\Delta(^5S_2)$ obtained by experiments for $pp\rightarrow np\pi^+$ are explained on the basis of the pole of unstable bound state type or broad dibaryon, ($M=2144$ MeV, $I=1$, $J^P=2^+$), by taking into account instability of the Δ, NN final-state interaction for $N\Delta\rightarrow np\pi^+$ and a factor i due to p-wave pion emission. This pole explains also the $^1D_2$ phase parameters for $pp\rightarrow pp$ around 2144 MeV in $\sqrt{s}$.

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Dibaryons have been suggested by a number of analyses of the pp scattering data [1-3]. Recently, Shypit *et al.* have reported results of a partial-wave analysis (PWA) for $\vec{p}\,\vec{p}\rightarrow n\pi^+p$ with a conclusion against the existence of dibaryons [4,5]. A criticism of this analysis has been given by Ryskin and Strakovsky [6] and by Lee [7]. As the PWA is the only source of detailed information on $N\Delta$ scattering, we will examine the reported phase solution and try to explain it on the basis of an S-matrix pole of an $N\Delta$ bound state type or broad dibaryon. The case for an $N\Delta$ resonance will be discussed separately.

The reaction $pp\rightarrow np\pi^+$ contains two dominant processes, $pp(^1D_2)\rightarrow N\Delta(^5S_2)$ and $pp(^3P_1)\rightarrow NS(^3S_1)$, in the low-energy region. Here, S stands for a πN S wave. In the following, we will mainly consider the first process and try to find the reason why the S-wave $N\Delta$ phase shift of Ref. [4] (SHY-88) is $\sim 30°-40°$ near the $N\Delta$ threshold and falls off with energy.

We will present a simple S-matrix formalism to observe the threshold behavior of an S-wave phase shift for unstable particle scattering when a nearby S-matrix pole of bound or virtual state type exists. We will then study important effects on the phases for $pp\rightarrow np\pi^+$ not taken into account in SHY-88 and Ref. [5] (SHY-89): the NN final-state interaction (NN FSI) and the problem of a factor i due to p-wave pion emission. Our main results will be given, we will discuss effects of the NN FSI peak cut and πd channel, and another treatment of NN FSI will be given.

The phase shift $\delta_{N\Delta}$ for $N\Delta(^5S_2)\rightarrow N\Delta(^5S_2)$ near the threshold. We first discuss S-wave $N\Delta$ scattering without coupling to pp. We define the Δ momentum $\tilde{p}$ relative to N by smearing in mass m^* the relative momentum $k(s,m^*)$ of the Δ considered as stable and having the mass m^* with a weight $\phi^2(m^*)$ of the Breit-Wigner type normalized to 1. In the narrow width approximation, we obtain $\tilde{p}=k(s,M_\Delta-i\Gamma_\Delta/2)=\tilde{k}+i\tilde{k}_I$ with $(\tilde{k},\tilde{k}_I)=\{[(q^4+\delta_0^4)^{1/2}\pm q^2]/2\}^{1/2}$, where $q=k(s,M_\Delta)$ is the relative momentum of the stable Δ of mass M_Δ [8], $\delta_0^2=\mu\Gamma_\Delta$, with μ being the reduced mass of N and the stable Δ. The $+$ $(-)$ sign is for $\tilde{k}$ $(\tilde{k}_I)$. In the scattering-length approximation, the phase shift for $N\Delta\rightarrow N\Delta$ scattering is given by

$$\cot(\delta_{N\Delta}-\delta_\infty)=\frac{\tilde{k}_I-\kappa}{\tilde{k}}\,, \tag{1}$$

where the background phase shift δ_∞ has been subtracted and κ is the inverse of the scattering length. Equation (1) is derived from the S-matrix element for $N\Delta\rightarrow N\Delta$:

$$S=e^{2i\delta_{N\Delta}}=\frac{-\kappa+i\tilde{p}^*}{-\kappa-i\tilde{p}}e^{2i\delta_\infty}\,, \tag{2}$$

where $\tilde{p}^*=\tilde{k}-i\tilde{k}_I$. We see that $i\kappa$ is a bound state pole in Δ-momentum plane if $\kappa>0$, and a virtual state pole if $\kappa<0$. The pole position in the energy plane is obtained from $\tilde{p}^2/2\mu=-\kappa^2/2\mu$, i.e., $\sqrt{s}=M_N+M_\Delta-i\Gamma_\Delta/2-\kappa^2/2\mu$.

At the $N\Delta$ threshold, $s=s_{\rm th}$, we have $q=0$, $\tilde{k}=\tilde{k}_I=\delta_0/\sqrt{2}=0.161$ GeV. (We use $M_\Delta=1.211$ GeV.) From Eq. (1) we get $(\delta_{N\Delta}-\delta_\infty)_{s_{\rm th}}>45°$ $(<45°)$ for a bound (virtual) state pole, being different from the one for stable particle scattering of $\delta=\pi(0)$ for a bound (virtual) state pole. Furthermore, if $\delta_0/\sqrt{2}>\kappa>0$, the $\delta_{N\Delta}-\delta_\infty$ gradually passes 90° at $\tilde{k}_I=\kappa$, i.e., $\sqrt{s}=M_N+M_\Delta+(\delta_0^4-4\kappa^4)/8\mu\kappa^2$. This is another feature for unstable particle scattering when $\delta_0/\sqrt{2}>\kappa>0$. Note that if the narrow-width approximation is not used, the value of 45° becomes smaller.

In the following, we will see how to explain the $N\Delta$ phase shifts [4,5] on the basis of a coupled version of Eq. (1). But, before doing this, we need to examine two important effects not included in SHY-88 and SHY-89.

NN FSI for $N\Delta\rightarrow np\pi^+$. The interaction range for np is ~ 2 fm, being twice as large as the one for πp. Thus, even after the decay of the Δ, a proton and a spectator neutron interact with each other. The c.m. energy squared of the NN pair in the $np\pi^+$ system is given by the Dalitz relation, $s_{NN}=s+2M_N^2+m_\pi^2-2s_{\pi N}$. By energy conservation, $\sqrt{s}=M_N+M_\Delta+(q^2/2\mu)$ with a stable Δ. If we insert the relation $q^2=\tilde{k}^2-\tilde{k}_I^2$ here, we have $\sqrt{s}=M_N+M_{\pi N}+\tilde{k}^2/2\mu$, where $M_{\pi N}=M_\Delta-\tilde{k}_I^2/2\mu$. We set $s_{\pi N}=M_{\pi N}^2$ for an unstable Δ. Then the NN FSI gives

$T_{\rm lab}^{pp}$ (MeV)	492	574	643	729	796
$\delta_{np}(^3S_1)$	103°	81°	61°	38°	27°
$\delta_{\pi^+p}(3,3)$	30°	58°	73°	80°	84°

Here, the corresponding P-wave $\pi p(3,3)$ phase-shift values have been given in comparison. We see that δ_{np} is larger than $\delta_{\pi}p$ below 600 MeV.

This effect has not been subtracted in SHY-88 and SHY-89. So, their phase shift values contain the NN FSI effect.

The phase $\frac{1}{2}\pi$ due to emission of a p-wave pion in $\Delta^{++}\to p\pi^+$. The definition of the $N\Delta$ phase shift $\delta_{N\Delta}(^5S_2)$ in SHY-88 is different from the one in Ref. [2] (EDW-80). In the case of SHY-88, the $\delta_{N\Delta}(^5S_2)$ is defined as the phase shift for $pp(^1D_2)\to np\pi^+$ subtracted by the pp phase shift $\delta_{pp}(^1D_2)$ and the $\delta_{\pi^+p}(3,3)$, while in the case of EDW-80, it is defined as the phase shift for $N\Delta$ scattering [the phase shift for $pp(^1D_2)\to N\Delta(^5S_2)$ subtracted by the $\delta_{pp}(^1D_2)$]. Putting aside the np FSI for the moment, the T matrix for $pp\to np\pi^+$ is given, in a two-step approximation [9], by

$$T(pp\to np\pi^+)=T(pp\to n\Delta^{++})T(\Delta^{++}\to p\pi^+)\Phi(s_\Delta)\,, \tag{3}$$

if $N\Delta$ is produced in the first step. Here $\Phi(s_\Delta)$ is the Δ propagator,

$$\begin{aligned}\Phi&=(M_\Delta^2-s_\Delta-iM_\Delta\Gamma_\Delta)^{-1}\\ &\approx(M_\Delta\Gamma_\Delta)^{-1}\exp[i\delta_{\pi p}(3,3)]\sin[\delta_{\pi p}(3,3)]\,,\end{aligned}$$

for $E_{\pi N}$ close to M_Δ; $T(\Delta^{++}\to p\pi^+)$ is the T matrix for Δ-decay. We see that the first definition is different from the second one by the phase of the $T(\Delta^{++}\to p\pi^+)$, which is $-\pi/2$, since the $T(\Delta^{++}\to p\pi^+)$ is proportional to $-i=e^{-\pi/2i}$ due to the p-wave pion being emitted by the decay, $V=-f(ik_\pi/m_\pi)Y_l^M(\hat{k}_\pi)g$ [10]. Here, f is the coupling constant for $\Delta\to N\pi$ or $N\to N\pi$ and g is a spin-i-spin factor which is real. In the following, we adopt the latter definition, because it fits in the present formalism. Note that this phase correction of 90° to SHY-88 is not required for s-wave pion emission for $pp\to np\pi^+$.

The phase shifts $\delta_{N\Delta}$ for $N\Delta\to N\Delta$. As stated above, the phase shifts for $N\Delta\to N\Delta$ obtained by Shypit *et al.* ($\delta_{N\Delta}^{\rm Shy}$) are to be redefined by

$$\delta_{N\Delta}(^5S_2)=[\delta_{N\Delta}^{\rm Shy}(^5S_2)\pm\tfrac{1}{2}\pi]-\delta_{np}(^3S_1)\,. \tag{4}$$

We take the + sign in Eq. (4) to cancel the irrelevant $\Delta N\pi$ vertex phase for Δ decay in SHY-88. Then, Eq. (4) explains why there was the falling off of the $\delta_{N\Delta}^{\rm Shy}(^5S_2)$ with energy [4,5]. Use of the values of $\delta_{N\Delta}^{\rm Shy}(^5S_2)$ tabulated in SHY-88 gives

$T_{\rm lab}$ (MeV)	492	574	643	729	796
$\delta_{N\Delta}(^5S_2)$	31°	46°	60°	72°	75°

The phase $\delta_{N\Delta}(^5S_2)$ with channel coupling. The behavior of $\delta_{N\Delta}(^5S_2)$ near the $N\Delta$ threshold can be explained at least qualitatively on the basis of a 2×2 coupled version of Eq. (1). We introduce channel coupling of $pp(^1D_2)$ and $N\Delta(^5S_2)$, which makes the S-matrix pole shift to the left in Δ momentum ($\tilde{p}$) plane: $i\kappa\to i\kappa-\lambda$, where $\lambda>0$ [11]. We put λ proportional to k_p^5, where k_p is the proton momentum in pp c.m. system. The mass and width corresponding to the pole are given by

$$M_B=M_N+M_\Delta-\frac{\kappa^2-\lambda^2}{2\mu},\ \ \Gamma_B=\Gamma_\Delta+\frac{2\kappa\lambda}{\mu}\,. \tag{5}$$

The unitary symmetric S-matrix elements are

$$\begin{aligned}S_{jj}&=\eta_je^{2i\delta_j}=\frac{-\tilde{\kappa}-i\tilde{p}+2i\lambda_j}{-\tilde{\kappa}-i\tilde{p}}e^{2i\delta_{j\infty}}\,,\\ S_{jk}&=i\eta_{jk}e^{i(\delta_j+\delta_k+\phi_{jk})}=\frac{2i(\lambda_j\lambda_k)^{1/2}}{-\tilde{\kappa}-i\tilde{p}}e^{i(\delta_{j\infty}+\delta_{k\infty})},\ \ j\neq k\,.\end{aligned} \tag{6}$$

For the present 2×2 case, $j=1$ for pp, $j=2$ for $N\Delta$, $\eta_1=\eta_2=\eta$, $\eta_{12}=\eta_{21}=(1-\eta^2)^{1/2}$, and $\phi_{12}=0$. Further, $\lambda_1=\lambda$, $\lambda_2=\tilde{k}$, $\tilde{\kappa}=\kappa+i\lambda$, and $\tilde{p}=\tilde{k}+i\tilde{k}_l$ as before. $\delta_{2\infty}$ ($\delta_{1\infty}$) is a background phase shift for $N\Delta$ (pp) scattering. Equation (6) is related to the Dalitz-Tuan ($\alpha\beta$) representation [11], generalized for unstable-particle scattering with the $N\Delta$ channel being singled out. We start with the relation between the T and K matrices and smear in mass the $N\Delta$ phase space over the Δ width to obtain the T expressed in terms of $\tilde{p}$ [8]. Next we transform the T into the $\alpha\beta$ form, replace the α (βk_p^5) term by κ (λ), and we have Eq. (6).

The coupled version of Eq. (1) is obtained from Eq. (6) as

$$\begin{aligned}\cot2(\delta_j-\delta_{j\infty})&=\frac{(\tilde{k}_l-\kappa)^2\pm(\lambda^2-\tilde{k}^2)}{2\lambda_j(\tilde{k}_l-\kappa)}\,,\\ \eta&=\left[\frac{(\tilde{k}_l-\kappa)^2+(\tilde{k}-\lambda)^2}{(\tilde{k}_l-\kappa)^2+(\tilde{k}+\lambda)^2}\right]^{1/2}\,,\end{aligned} \tag{7}$$

where the + (−) sign is for $j=2$ (1). The features of unstable particle scattering as stated above are reserved in this channel-coupling case.

We choose $\kappa=0.09$ GeV, $\lambda=0.03$ GeV at $s_{\rm th}$. The pole is of unstable bound state type (UBS), whose mass and width are

$$M_B=2.144\ {\rm GeV},\ \ \Gamma_B=0.11\ {\rm GeV}. \tag{8}$$

As the narrow-width approximation has been made to obtain the explicit form for $\tilde{p}$, the following calculation will be valid in the range of the order of the Δ width around the $N\Delta$ threshold. We will, however, study a bit further beyond the limit in order to see the behavior of the phase parameters qualitatively in a wider energy range [12]. The case of Eq. (8) predicts that the $\delta_{N\Delta}-\delta_{2\infty}$ gradually passes 90°, while $\delta_{pp}-\delta_{1\infty}$ gradually passes 0° at $\sqrt{s}=2.23$ GeV ($T_{\rm lab}^{pp}=0.80$ GeV), at which $\tilde{k}_l=\kappa$, consistent with the results for $\delta_{pp}(^1D_2)$ obtained from pp PSA.

We assume that $\delta_{2\infty}=\tan^{-1}[-\tilde{k}R_\Delta/(1-\tilde{k}_lR_\Delta)]\approx-\tilde{k}R_\Delta$, with $R_\Delta=0.22$ fm, by which $\delta_{2\infty}$ ranges from −6° at 470 MeV to −20° at 790 MeV. Further, we use the one-boson-exchange (OBE) model for $\delta_{1\infty}$ and put $\delta_{1\infty}=\sum_i\delta_i$, where i is summed over π, η, σ, ρ, ω, and δ. We calculate $\delta_{1\infty}$ by using the OBE model parameters determined from NN data below 300 MeV [13]. We use 13 (1) for $g_\omega^2/4\pi$ ($g_\delta^2/4\pi$) instead of 12 (7.5), without destroying the goodness of fit to low-energy experimental data. $\delta_{1\infty}$ runs from 9° at 470 MeV to 5° at 800 MeV. The $\delta_{N\Delta}^{\rm Shy}$, δ_{pp} and the absorption parameter η calculated from Eq. (7) with the pole of (8) are shown in Fig. 1, in agreement with SHY-88 and with the pp PWA [1,3,14].

Effect of the NN FSI *peak cut.* The NN FSI peak region where $M_{NN} < 1890$ MeV has been cut from the analysis of the experiment [5]. Because of this cut, $\langle M_{NN}\rangle_{av}$ increases at a given s and $\langle\delta_{np}(^2S_1)\rangle_{av}$ decreases by $\sim 35\%$. The experimental $\delta_{N\Delta}(^5S_2)$ obtained previously increases to 68° at 492 MeV and to 85° at 790 MeV. However, because of the Dalitz relation, the corresponding high-energy region of the $M_{\pi N}$ distribution is also cut from the analysis of the experiment and this makes $\langle M_{\pi N}\rangle_{av}$ decrease by some amount ε as a function of s. We must set $M_{\pi N} = M_\Delta - [(\tilde{k}_l^2/2\mu) + \varepsilon]$. We need to change $\tilde{k}^2/2\mu$ to $(\tilde{k}^2/2\mu) + \varepsilon$, so as to keep the relation $q^2 = \tilde{k}^2 - \tilde{k}_l^2$ and s unchanged. Then, the theoretical $\delta_{N\Delta}(^5S_2)$ calculated with $\varepsilon = 0$ also changes if $\tilde{k}$ and $\tilde{k}_l$ with ε are employed. At 492 MeV, we have $\varepsilon \sim 12$ MeV and the theoretical $\delta_{N\Delta}(^5S_2)$ increases from 36° to 42°, and at 790- MeV, $\varepsilon \sim 25$ MeV and $\delta_{N\Delta}(^5S_2)$ changes from 91° to 75°. Thus, the net peak-cut effect is weakened.

The remaining difference between the experimental and theoretical $\delta_{N\Delta}$'s (26° at 492 MeV and 10° at 790 MeV) is interpreted, if we put $\delta_{2\infty} = \delta_{2\text{th}} - b(s - s_{\text{th}})$ with $\delta_{2\text{th}} = 18°$ and $b = 25°\ \text{GeV}^{-2}$. The $\delta_{2\infty}$ ranges from 21.6° at 470 MeV to 6.6° at 790 MeV. If the $N\Delta$ background scattering is due to the one-boson-exchange process as in $pp \to pp$, the $\delta_{2\infty}$ is expected to take this form around the threshold.

Effect of the πd channel. The p-wave πd channel is introduced as a third channel coupled to the $pp(^1D_2)$ and $N\Delta(^5S_2)$. The S matrix is given by Eq. (6) with $\eta_{jk} = [(1 + \eta_l^2 - \eta_j^2 - \eta_k^2)/2]^{1/2}$, $j \neq k \neq l$. The extra phase ϕ_{jk} in S_{jk} is given by $\cos 2\phi_{jk} = [\eta_j^2 + \eta_k^2 - (\eta_{jl}^2\eta_{kl}^2/\eta_{jk}^2)]/2\eta_j\eta_k$, $j \neq k \neq l$. We obtain in place of Eq. (4)

$$\delta_{N\Delta}(^5S_2) = [\delta_{N\Delta}^{\text{Shy}}(^5S_2) \pm \tfrac{1}{2}\pi] - [\phi_{12} + \delta_{np}(^3S_1)]\,, \qquad (4')$$

which shows that ϕ_{12} plays a role similar to $\delta_{np}(^3S_1)$, although its magnitude is smaller than $\delta_{np}(^3S_1)$. The phase $\delta_{N\Delta} + \phi_{12}$ is calculated on the basis of Eq. (6) by performing a three-channel analysis for pp, $N\Delta$, and πd. The results are that the value of the pole parameters remains the same as the one obtained by the present two-channel analysis. We note that the pole position has been essentially determined by the pp phase parameters obtained by pp PWA.

Another treatment of np FSI. Equation (4′) shows that the NN FSI can be included in a third-channel $NN\pi$ as the main part [15]. We put $\phi'_{12} = \phi_{12} + \delta_{np}(^3S_1)$. In the low-energy region, the $NN\pi$ channel is dominant, and so, ϕ'_{12} becomes large. In the high-energy side, the $N\Delta$ channel is main and hence ϕ'_{12} is small.

The $NN\pi$ channel can be generalized to represent collectively all open channels coupled to the $pp(^1D_2)$ and $N\Delta(^5S_2)$. We are able to include, e.g., the P-wave NN

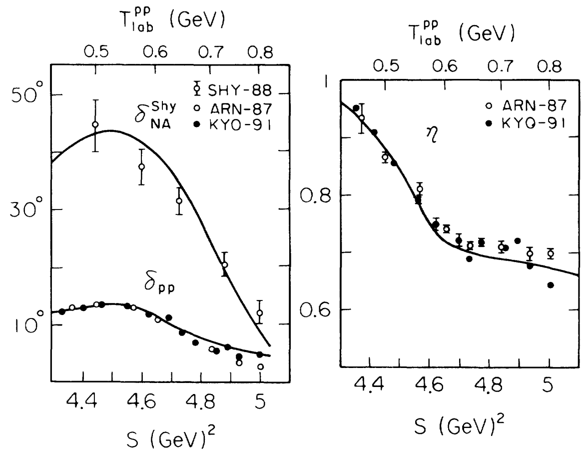

FIG. 1. Plots of $\delta_{N\Delta}^{\text{Shy}}(^5S_2)$, $\delta_{pp}(^1D_2)$, and η vs s. Solid curves are our prediction. PSA values of SHY-88 [4], ARN-87 [3], and Kyoto-91 [14] are given in comparison.

FSI, $\pi(NN)_P$, there. Its effect will weaken by introducing a repulsive background phase $\delta_{2\infty}$ into the $N\Delta$ channel.

Our approximations used in this report are a narrow-width approximation for Γ_Δ, a neglect of np FSI peak cut, πd channel, and an effective range term in $N\Delta$ channel. The last effect was partly taken into account by $\delta_{2\infty}$. Effects of the np FSI peak cut and πd channel on $\delta_{N\Delta}(^5S_2)$ were discussed briefly to show that they do not seriously affect the pole parameters.

The pole of Eq. (8) was assumed to exist, and our calculations were for physical s. If a pole search is performed, the $\tilde{\rho}(s)$ needs to be extended from physical to complex s, and a careful smearing of the phase space is required [2]. The form of $\tilde{\rho}(s)$ becomes complicated, but we will find the pole position similar to Eq. (8). An $N\Delta$ virtual state pole (IVS) is unlikely, because in this case, $\kappa < 0$, and hence $\tilde{k}_l - \kappa > 0$, and we get $\delta_{pp} - \delta_{1\infty} > 0$ by Eq. (7). This contradicts with the fact that pp PWA suggests $\delta_{pp} \lesssim 0$ for T_{lab}^{pp} from 850 to 1000 MeV and $\delta_{1\infty} \gtrsim 0$.

In summary, we have shown that (i) if S-wave $N\Delta$ interaction has a nearby pole of UBS or IVS, the threshold value of $\delta_{N\Delta}(^5S_2)$ is $\delta_{N\Delta} - \delta_\infty > 45°$ ($< 45°$) for the UBS (IVS) case, (ii) the NN FSI effect must be taken into account, and (iii) a difference of $\pi/2$ in defining the phase $\delta_{N\Delta}$ exists in the literature. With these corrections, the threshold value and the energy dependence of the $\delta_{N\Delta}(^5S_2)$ are both explained by the existence of an S-matrix pole of UBS type, $M = 2144$ MeV, in agreement with the results of pp $^1D_2$ resonance of $M = 2140$ MeV obtained from pp PSA.

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