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Is there incomplete mixing of states with different K quantum numbers in the neutron resonance region?

B. R. Barrett
Department of Physics, University of Arizona, Tucson, Arizona 85719

R. F. Casten
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

J. N. Ginocchio
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

T. Seligman
Department of Physics, Universidad Autonoma de Mexico, Guemavaca, Mexico

H. A. Weidenmüller
Max-Planck-Institut für Kernphysik, Heidelberg, Germany
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A recent publication claimed incomplete mixing of states with different K quantum numbers in the neutron resonance region. We discuss the theoretical implications of such a claim and show that it leads to serious discrepancies with the statistical model. We, therefore, reexamine the experimental data on which such a claim is based. The totality of the evidence invalidates the claim that K mixing in the resonance region is incomplete.

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In a recent letter, Rekstad, Tveter, and Guttormsen [1] analyzed the degree of mixing of the K quantum number in the neutron resonance region for the target nuclei ^{167}Er and ^{177}Hf. Using the thermal neutron capture data [2] of Refs. [3, 4] and comparing intensities of gamma transitions to final states with the same spin and parity, but different K quantum number, these authors found that the population of final states with $K = 0$ or $K = 1$ was suppressed by roughly a factor three in comparison to states with K values between 2 and 5. This suggests that the compound states in the neutron resonance region which, in the absence of K mixing, have K values between 2 and 5, acquire a $26 \pm 6\,\%$ admixture (in intensity) of compound states with $K = 0$ and $K = 1$. The authors of Ref. [1] conclude that this result contradicts the hypothesis [5] of complete K mixing in the domain of isolated neutron resonances.

In this contribution, we point out that the conclusion of Ref. [1] has significant implications not only for the degree of mixing of the K quantum number, but for the validity of the statistical model itself. Reexamining the arguments presented in Ref. [5], we conclude that it is very difficult, if possible at all, to bring the results of Ref. [1] into line with established results [6] on the level spacing distribution of s-wave neutron resonances. The authors of Ref. [1] have addressed a related problem themselves. They have argued that in the Porter-Thomas distribution of neutron partial widths, incomplete K mixing may be hard to detect. We do not take issue with this statement and focus attention on the level spacing distribution.

To reduce the argument [5] to its essence, let us consider two sets of neutron s-wave resonances, both having the same spin and parity and the same average level spacing $2d$ but differing in K quantum number. One set has $K = K_<$ (standing for $K = 0$, 1), the other, $K = K_>$ (standing for $K \geq 2$). The two sets are mixed by a K-violating interaction V.

We assume that for $V = 0$, the level spacing distribution of each set separately agrees with that predicted by the Gaussian orthogonal ensemble (GOE). Superposition of these two GOE spectra for $V = 0$ does *not* yield the spacing distribution of a single GOE, of course. We ask: How big does V have to be to account for the observed agreement [6] between the level spacing distribution of the nuclear data ensemble (NDE) and that of a single GOE? And is such a strength of V consistent with the results of Ref. [1]?

We denote by v^2 the mean squared matrix element of V between $K_<$ and $K_>$ resonances. The effect of a (weak) mixing of two GOE's on the spectral fluctuation properties has been investigated numerically in Ref. [7] in a different context (isospin mixing); the results are directly applicable also in the present context.

It was found in Ref. [7] that with increasing v^2, the nearest-neighbor spacing distribution changes rapidly from that of a superposition of two GOE's to that of a single GOE. To be quantitative, it is useful to introduce the spreading width $\Gamma^{\downarrow} = 2\pi v^2/d$. For the example in Fig. 3.1 of Ref. [7] (two GOE matrices of dimension 100 each are mixed), the nearest-neighbor spacing distribution becomes indistinguishable from that of a single GOE when $\Gamma^{\downarrow} \geq 2.5\,d$ or $x = v^2/d^2 \geq 0.4$. (Intuitively, this is not too surprising. Indeed, the perturbation series for the mixing of *two* levels, with distance d and mixing matrix element v, diverges when $v^2/d^2 = x \geq 0.5$.) It follows that the agreement of the nearest-neighbor spacing distribution of the NDE with that of a single GOE gives only the weak lower bound $v^2 \gtrsim \frac{1}{2}d^2$.

The situation is different for the Δ_3 statistic. On the one hand, the Δ_3 statistic of the NDE agrees [6] with that of a single GOE over at least 25 level spacings. (Data for larger intervals were not published, but were reported "to be consistent with the GOE" [6].) On the other hand, Fig. 3.3 of Ref. [7], calculated for the same situation as Fig. 3.1 to which we referred earlier, shows that the Δ_3 statistic of two mixed GOE's coincides with that of a single GOE only over a finite interval. For the concrete example of Ref. [7], this interval is roughly $7.5\,d$ or about $3\Gamma^{\downarrow}$. Applying this result to the NDE with an interval $\geq 25\,d$, we estimate $\Gamma^{\downarrow} \geq 8\,d$ or $x \geq 1.3$. This lower bound is somewhat sharper than what we found from the nearest-neighbor spacing distribution. Moreover, it can be improved, at least in principle.

A word of caution is necessary at this point. First, the example of Ref. [7] is numerical and based on two matrices of dimension 100 each. It is not clear how the result scales with the dimensions of the matrices. In applications to the NDE, we are interested in matrices of dimension 10^8 or so, corresponding to the number of configurations in a major shell. Such large dimensions are not accessible numerically. And the analytical treatment of the mixing of two GOE's is an open theoretical problem [8]. As a result, the precise values of the lower bounds ($x \geq \frac{1}{2}$ and $x \geq 1.3$) deduced above may be subject to discussion. However, physical intuition shows that $x \geq 1$ is required for the attainment of GOE fluctuation properties over distances of a few level spacings. For lack of better values we will use $x \geq 1$ in the sequel. While the estimates and figures we deduce in this way will not be quantitatively reliable, they will certainly indicate the general trend. The second proviso concerns our model of mixing two GOE's. In the actual nuclei under consideration, more than two K values are possible and will be present. Given all other uncertainties, it does not seem worthwile at this point to allow for this additional complication in the modeling.

We now show that it is difficult to reconcile a value of $x = 1$ with the $26\pm6\,\%$ admixture (in intensity) of the $K_<$ states to the totality of K states found in Ref. [1]. We use the following perturbation-theoretic argument. For $x = 0$ and a given level with $K = K_>$, we have $K_<$ levels located, on average, at distances $\pm d$, $\pm 3d$, $\pm 5d$, ... With $x_{\pm 1}$, $x_{\pm 3}$, $x_{\pm 5}$, ... denoting the squares of the associated K-mixing matrix elements in units of d^2, we use perturbation theory to estimate the total $K_<$ admixture as $\frac{\sigma}{1+\sigma}$, with $\sigma = (x_1 + x_{-1}) + \frac{1}{9}(x_3 + x_{-3}) + \frac{1}{25}(x_5 + x_{-5}) + \cdots$. With $x_{\pm(2j+1)} = 1$ for all j, this yields $\frac{\sigma}{1+\sigma} = 0.71$, much in excess of the value 0.26 of Ref. [1]. (This argument is not tenable, of course, since $x = 1$ obviously lies outside the radius of convergence of the perturbation series. It nonetheless shows that $x = 1$ is not consistent with weak K mixing.) To reduce σ (and to get back into the domain of convergence of the perturbation series), it is obviously essential to reduce $x_{\pm 1}$ and $x_{\pm 3}$. This amounts to saying that the observation of Ref. [1] is not an effect which is typical of the NDE, but instead is a statistical fluctuation. To estimate the probability of the occurrence of such a fluctuation, we observe that in keeping $x_{\pm(2j+1)} = 1$ for $j \geq 2$, we must have $y = (x_1 + x_{-1}) + \frac{1}{9}(x_3 + x_{-3}) \leq 0.22$ to reduce $\frac{\sigma}{1+\sigma}$ to or below the value 0.26. Now, $x_{\pm(2j+1)}$ are squares of uncorrelated Gaussian distributed random variables with mean zero and variance unity. It is easy to see that the probability to have $y \leq 0.22$ is less than 4%. This would suggest that the result of Ref. [1] is a random event, not typical for s-wave resonances in general, and rather untypical. [Indeed, an analysis of the average resonance capture (ARC) data in Refs. [3] and [4] does not show the effect reported in Ref. [1].] Alternatively, if the result of Ref. [1] is taken to be typical, there exists a serious question about the validity of the statistical model.

A second problem arises when we use the fact [9, 10] that the spreading width $\Gamma^{\downarrow}$ for a symmetry-breaking interaction is invariant under changes of excitation energy, at least within factors of order unity. The arguments given above, even if only semiquantitative, certainly do show that the results of Ref. [1] are incompatible with values of $\Gamma^{\downarrow}$ much in excess of $10d \cong 100$ eV. Using $\Gamma^{\downarrow} \cong 100$ eV in the low-lying part of the spectrum of heavy nuclei, where typical level spacings are 100 keV or more, we find for v^2 the value 1.6 keV^2. This is 2–3 orders of magnitude smaller than the value deduced from experiment in Ref. [11].

However, the values from Ref. [11] may not be representative for the value of v^2 since they relate only to the octupole vibrational bands. An estimate which is both more realistic and more conservative is obtained as follows. From Chap. 8.2 of Ref. [12] and the Nilsson model, it is easy to show that in heavy, deformed, odd-A nuclei, $v^2 \cong 9$ keV^2. This value, an average over *all* Nilsson states, is obtained under the assumption that the average K is $\frac{3}{2}$, that $J = K + 1$ for a low-lying band member, with a pairing reduction of 0.5 and a further attenuation of 0.75. Taking into account only states with equal parity, we find that v^2 increases to 36 keV^2. Allowing for uncertainties in the estimate, we conclude that for odd-A nuclei v^2 should lie in the interval between 30 and 40 keV^2 and thus be much bigger than the value of 1.6 keV^2 deduced above. This estimate will be lowered for even-even nuclei where some matrix elements are forbidden, but the strong discrepancy with the conclusion drawn from Ref. [1] persists.

There is a simple way out of this dilemma. If the combined level density of the $K_<$ states amounts to 35% of the combined level density of the $K_>$ states, then the total admixture of $K_<$ states in *any* of the s-wave neutron resonances is $35/(100+35) \cong 25\%$ even for $x \gg 1$. The counterargument against this (trivial) explanation of the data of Ref. [1] is the following. For the excitation energies relevant for the experiments, the level density is [5] the product of two factors, a level-density factor depending on K and another factor depending on excitation energy. From this expression we find the relative density of $K_<$ states to the total to be about 40%. This number is uncertain within a few percent because of the uncertainty in the value of the moment of inertia around the symmetry axis. The expected admixture of $K_<$ levels is therefore 50% larger than the observed 26 ± 6 %.

This situation has motivated us to redo the analysis of thermal neutron capture data reported in Ref. [7]. In attempting to reproduce the results of Ref. [1], we have encountered several problems.

First, Fig. 1 of Ref. [1] cites a total of 101 primary transitions as comprising their ensemble, yet Refs. [3] and [4], on which the analysis in Ref. [1] is based, show only 91 J=2, 3, 4, 5 levels arranged into rotational bands with assigned K values. Moreover, the authors of Ref. [1] eliminated at least 14 of these levels claiming they showed evidence of strong Coriolis (K) mixing. Further, several of the remaining levels occur as close-lying multiplets in the (n,γ) spectra and it is impossible to properly allocate the intensities separately to the individual levels. In the end, then, there are only 57 levels remaining. Intriguingly, these states are not at all uniformly distributed over either parity or K values (Ref. [1] distinguishes two categories, $K = 0$, 1 and $K = 2, 3, 4, 5$): In particular, the final set of states available for analysis contains *no* low K ($K = 0, 1$) negative parity states in *either* nucleus. Moreover, *all* the low K states fed by $E1$ primary transitions occur in ^{178}Hf and *all* those fed by $M1$ transitions occur in ^{168}Er. It would seem that the validity of a combination of such different distributions, from two nuclei, into one ensemble, as is done in Ref. [1], should be examined carefully before strong conclusions are drawn.

An additional difficulty arises when the intensities of the 57 transitions are analyzed according to the prescriptions of Refs. [1,13]. We find none of the intensities with $x > 2.0$ given in Fig. 1 of Ref. [1] for $M1$ transitions to $K = 2 \cdots 5$ levels and neither of those above $x = 4.0$ for $E1$ transitions. Moreover, we find an extra case of large x ($x > 3$) for $M1$ transitions to $K = 0$, 1 states.

The upshot of these changes is to significantly reduce the number of states forming the ensemble studied in Refs. [1, 13]. Moreover, our analysis of the intensities of primary transitions in this ensemble, for the two nuclei ^{168}Er and ^{178}Hf, shows virtually no distinction in intensities of transitions to the lower or higher K states assigned: Indeed, the ratio of the average intensity to the $K_<$ and $K_>$ states is 0.92.

Our analysis leads us to the conclusion that a $K_<$ admixture into the total of about 35–40% as required by the theoretical arguments given above is consistent with the data of Refs. [3, 4].

In conclusion, we have presented evidence which invalidates the claim of Ref. [1] that K mixing in the neutron resonance region is incomplete.

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