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RAPID COMMUNICATIONS

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Identical bands in widely dispersed nuclei

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Low spin identical bands are found in sets of widely dispersed nuclei, spanning as much as 24 mass units. Moreover, they occur in regions of rapid structural and shape changes. A correlation ansatz to identify such nuclei, based on the concepts of N_pN_n and F spin, is discussed.

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The data of nuclear physics are so rich that a phenomenon identified in one context is often subsequently recognized, or recalled, in another. This can be important because the generality of a phenomenon impacts the avenues taken to understand it. An exotic feature found only in certain states of a particular nucleus, or a narrow class of states, may often reasonably be interpreted in a quite microscopic way in terms of peculiarities of orbits or interactions specific to the case at hand. In contrast, a rather widespread phenomenon demands an explanation that arises from more general features of atomic nuclei, shell structure, and interactions.

Such comments are relevant to the rather startling recent discovery [1] of "identical rotational bands" in adjacent even- and odd-mass superdeformed nuclei with energy spacings that are nearly equal and much closer than expected. Explanations of varying degrees of specificity have been advanced. An assessment of them depends critically on an understanding of how widespread the identical band phenomenon is. Earlier studies by Espino and Garrett [2] showed similar moments of inertia at spins of $J \sim 22$ in a number of rare-earth nuclei. The recent comments that both even-even actinides [3] and adjacent even- and odd-mass rare-earth nuclei [4] near $A = 170$ and 180, both in regions of stable (saturated) deformation, display a similar effect at *low* spin now become quite significant.

It is the purpose of this Rapid Communication to demonstrate that identical bands are quite common at low spin, that they occur even in noncontiguous, widely dispersed nuclei (spanning as much as 24 mass units), that they occur even in regions of rapidly changing structure, and that there is a simple ansatz that can guide the search with such bands. The identification of such a "correlation scheme," couched in terms of a specific view of the evolution and determinants of structure, may provide clues to understanding the identical band phenomenon itself.

Our work is based on the concept of nuclear "multiplets," or ensembles of nuclei expected to have similar properties. The two concepts we invoke are those of N_pN_n and F spin [5,6]. Since we focus on heavy rare-earth nuclei, the two concepts merge into virtual identity. We will show that nuclei in such multiplets can display rotational bands with nearly identical spacings over large mass regions. The present suggestion is different from but has links to remarks by Iachello [7] and Barrett [8] on spacings in superdeformed bands near $A = 190$ and normal bands in actinide nuclei.

To compare spacings in similar rotational bands, it is useful to have a convenient standard for comparison. It has become customary [1,3,4,9] to use, for this purpose, the mass dependence of the rigid rotor moment of inertia, $I \sim A^{5/3}$, although this is a conservative (lower limit) estimate of how rotational spacings should change since it presumes a region of stable structure: In nuclei in shape/phase transitional regions, the differences in spacings can be far greater than this. To quantify the comparison to the rigid rotor, it is useful to define the ratio

$$R_\gamma(J)=\frac{\Delta E_\gamma(J)/E_{\gamma_2}(J)}{(A_2/A_1)^{5/3}-1} \quad (1)$$

relating level spacings in nuclei with masses A_1 and A_2 ($A_2>A_1$). In Eq. (1), $E_{\gamma_2}(J)$ is the transition energy $E(J)-E(J-2)$ in nucleus A_2 and $\Delta E_\gamma(J)=E_{\gamma_1}(J)-E_{\gamma_2}(J)$. R_γ is thus the fractional change in transition energies divided by the fractional change in $A^{5/3}$. If the spacings are identical, $R_\gamma=0$. If they scale as $A^{5/3}$, $R_\gamma=1$, and if, as is common in transitional regions, they change faster than $A^{5/3}$, R_γ can be much greater than unity. In superdeformed states in the $A=150$ and 190 regions [1,9], transition energies of ~ 1 MeV in adjacent nuclei differ by ~ 1-2 keV, giving $R_\gamma \sim 0.1$-0.2: The differences in rotational spacings are 5-10 times less than expected for a rigid rotor. In recent work [3] on even-even actinides, $\langle R_\gamma \rangle$ values of ~ 0.12 and ~ 0.37 were found. A recurring phenomenon in deformed nuclei is the saturation of collectivity or deformation near midshell [10]. The small R_γ values in the actinides [3] are an excellent example of identical bands in regions of nearly constant deformation and (saturated) collectivity. We shall focus below on two examples of identical bands in rare-earth nuclei, one of which is also in a saturation region while the other is decidedly not.

We first consider the nuclei ^{156}Dy, ^{160}Er, ^{164}Yb, ^{168}Hf, ^{172}W, and ^{180}Os. These cover 24 mass units and are in the midst of a rapid shape transition: Isotopic sequences display quite different ground band energies. For example, in $^{156-166}$Er, shown in Fig. 1 (top), $E(2_1^+)$ is 244, 192, 126, 102, 91, and 81 keV, respectively. Normally, one would hardly seek out such a region for similar band sequences.

Nevertheless, the lower part of Fig. 1 shows that the ground band energies [11] of the ^{156}Dy-^{180}Os nuclei are indeed remarkably close to one another across the entire multiplet. Figure 2 presents examples of the 15 possible

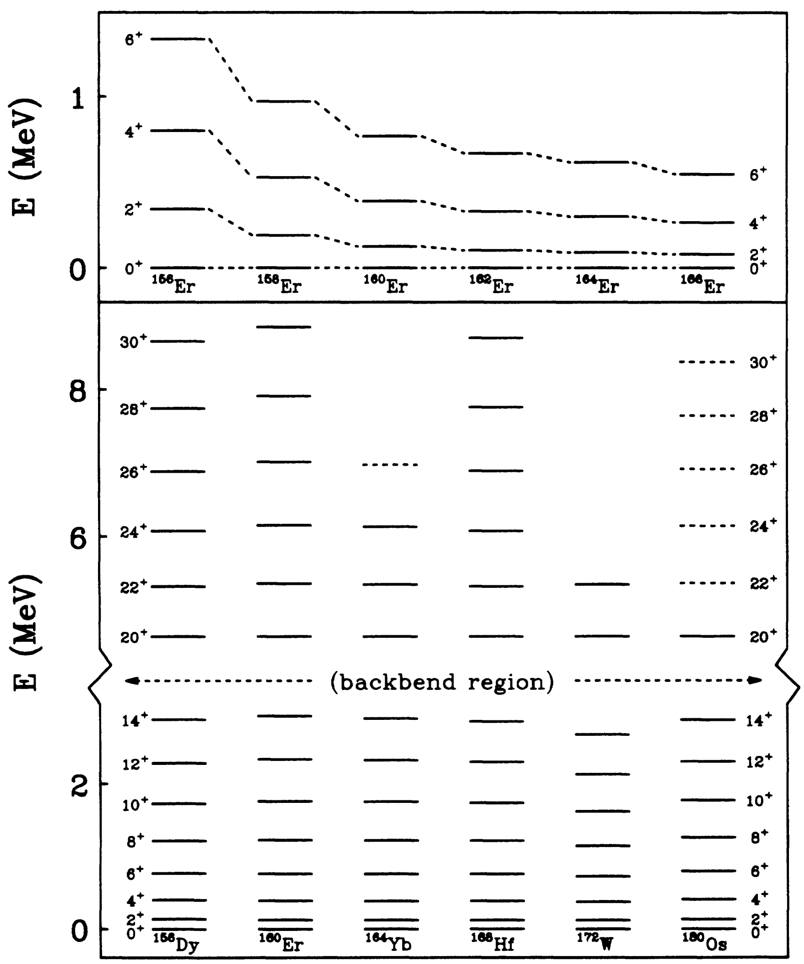

FIG. 1. Top: Low spin yrast levels [11] of $^{156-166}$Er showing a shape transitional region. Bottom: Levels of the multiplet ^{156}Dy-^{180}Os showing the occurrence of nearly identical band spacings. The backbending region, of course, differs in detail in these nuclei and is excluded from the comparison. Above the backbend, the energies are normalized to ^{156}Dy at spin 20^+ to emphasize the rotational spacings.

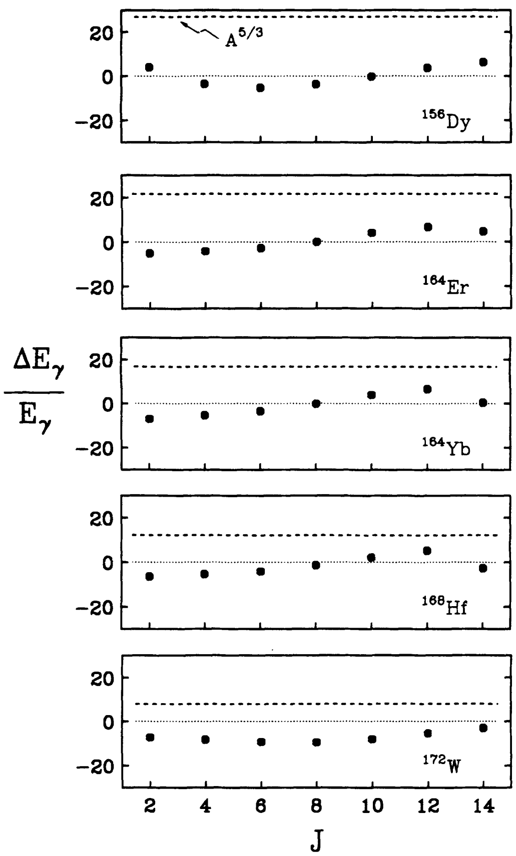

FIG. 2. $\Delta E_\gamma/E_\gamma$ against J for each nucleus in the ^{156}Dy-^{180}Os multiplet compared with ^{180}Os. The dashed horizontal lines give the rigid rotor $A^{5/3}$ guideline.

differences in rotational spacings for this set, namely, for ^{180}Os and each of the other members of the multiplet. In each case (except ^{172}W), these differences are clearly much less than expected from the $A^{5/3}$ guideline. The $\langle R_\gamma \rangle$ values (for $J < 16^+$) for these nuclear pairs are 0.14 (^{156}Dy-^{180}Os), 0.18 (^{160}Er), 0.23 (^{164}Yb), 0.32 (^{168}Hf), and 0.91 (^{172}W). All except ^{172}W are in the range of $\langle R_\gamma \rangle$ found previously in identical superdeformed and normal bands in stably deformed regions [1,3,9]. Figure 3 summarizes all the results for the ^{156}Dy-^{180}Os nuclei in tabular matrix form: Above the diagonal, the average R_γ values are given for each pair of nuclei while the numbers below are the average ratios $\langle |\Delta E_\gamma|/E_\gamma \rangle$ in percent. Most of the energy spacings differ by a few percent or less compared to differences in $A^{5/3}$ ranging from 4–8% for adjacent nuclei to about 27% for the most distant pair. These close rotational spacings are reflected in $\langle R_\gamma \rangle$ values that are nearly always $\ll 1$. In 12 of 15 pairs $\langle R_\gamma \rangle \lesssim 0.4$ and there are five cases with $\langle R_\gamma \rangle < 0.2$. For all 15 pairs, $\langle R_\gamma \rangle$ averages 0.36 (without the anomalous nucleus ^{172}W, the average of $\langle R_\gamma \rangle$ is 0.26). In contrast, the average $\langle R_\gamma \rangle$ ($J = \leq 12$) for the Er isotopes in Fig. 1 (top) is 8.4: This is 25–30 times greater than for the ^{156}Dy-^{180}Os set. Finally, we note that other bands (e.g., negative parity) in the ^{156}Dy-^{180}Os set also show $\langle R_\gamma \rangle$ values in the same range as for the ground bands.

The smaller matrix in Fig. 3 shows the results for another multiplet. This one, comparing ^{162}Dy-^{166}Er-^{172}Yb with $E(2_1^+) \sim 80$ keV, is, like the actinide case [3] and unlike our first example, an example of identical bands in a region of saturated collectivity and deformation. Again, $\langle R_\gamma \rangle \ll 1$, lying between 0.17 and 0.34.

We now turn to a discussion of the rational linking the

$\langle R_\gamma \rangle$ and $|\Delta E_\gamma|/E_\gamma$ Values for Sets of Identical Bands

$\langle R_\gamma \rangle$ \ $\frac{\lvert\Delta E_\gamma\rvert}{E_\gamma}$	^{156}Dy	^{160}Er	^{164}Yb	^{168}Hf	^{172}W	^{180}Os
^{156}Dy		0.64	0.39	0.17	0.37	0.14
^{160}Er	2.7 b		0.15	0.16	0.35	0.18
^{164}Yb	3.4 b	0.7 d		0.25	0.41	0.23
^{168}Hf	2.3 c	1.4 d	1.0 d		0.79	0.32
^{172}W	6.6 c	4.5 e	3.4 e	3.2 e		0.91
^{180}Os	3.8 a	4.0 a	3.8 a	3.9 a	7.2 a	

$\langle R_\gamma \rangle$ \ $\frac{\lvert\Delta E_\gamma\rvert}{E_\gamma}$	^{162}Dy	^{166}Er	^{172}Yb
^{162}Dy		0.33	0.17
^{166}Er	1.3		0.34
^{172}Yb	1.7	3.3	

FIG. 3. Matrix tabulation of $\langle R_\gamma \rangle$ (above the diagonals) and $\langle |\Delta E_\gamma|/E_\gamma \rangle$ (in percent) (below the diagonals) for the two multiplets ^{156}Dy-^{180}Os and ^{162}Dy-^{172}Yb. The values of $\langle |\Delta E_\gamma|/E_\gamma \rangle$ expected from the $A^{5/3}$ guidelines range from approximately 4–8% for adjacent nuclei in the matrix to 27% for the ^{156}Dy-^{180}Os pair. In the smaller matrix the $|\Delta E_\gamma|/E_\gamma$ averages (and hence $\langle R_\gamma \rangle$) are taken over all spins $J \leq 10^+$. In the larger matrix the letters in each box in the $|\Delta E_\gamma|/E_\gamma$ section identify the spin range over which the averages were taken according to the following notation: (a) $J = 2^+$-14^+; (b) $J = 4^+$-14^+; (c) $J = 4^+$-12^+; (d) $J = 2^+$-10^+; (e) $J = 2^+$-8^+.

nuclei in these sets. Of course, they were not picked at random but because they comprise multiplets expected, *a priori*, to show similar behavior. Specifically, the sequences constitute N_pN_n multiplets [5], that is, nuclei with similar values of the valence N_pN_n product. The residual valence p-n interaction dominates the evolution of collectivity and the quantity N_pN_n embodies a first-order estimate of its integrated strength [5]. Nuclei in a given region with similar N_pN_n values should be similar in structure. Following the discovery of identical bands [1] and their recognition [3] among "normal" states, the N_pN_n multiplet concept emerges naturally as an ideal ansatz for searching for such bands and, being successful in this role, it may suggest appropriate avenues in which to search for microscopic interpretations. The ^{156}Dy-^{180}Os group has N_pN_n values in the range 128–144. All but the last member also form an F-spin [6] (constant N_p+N_n) multiplet [12] with $F=6$. The second multiplet in Fig. 3 has $N_pN_n=224$–240: The first two nuclei have $F=7.5$ while ^{172}Yb has $F=8$. We note that, in both cases, we have not made selections among available nuclei with these N_pN_n (and F) values. These are the full sets in the major shell quadrant $Z=66$–82 and $N=82$–104. Of course, not all rare-earth N_pN_n multiplets behave as well as those in Fig. 3, although in other cases R_γ values are also well below unity.

Nuclei in an F-spin multiplet have $F_0 \equiv (N_p-N_n)/4$ values from $-F$ to $+F$. Note that nuclei with symmetric $\pm F_0$ values in an F-spin multiplet *also* have identical N_pN_n values. For example, in the ^{156}Dy-^{180}Os case, ^{160}Er and ^{168}Hf have $(N_p,N_n)=(14,10)$ and $(10,14)$, respectively, so that $N_pN_n=140$ and $|F_0|=1$ in both cases. It was suggested in Ref. [13] that such (F_0,N_pN_n) pairs should be particularly similar in structure. It is interesting to apply this test to the identical band idea. In the deformed or transitional rare-earth nuclei with $Z=66$–82 and $N=82$–104, there are a total of six such cases of nuclear pairs with $\pm F_0$ values and identical N_pN_n: ^{156}Dy-^{172}W, ^{160}Er-^{168}Hf, ^{158}Dy-^{170}Hf, ^{162}Er-^{166}Yb, ^{160}Dy-^{168}Yb, and ^{162}Dy-^{166}Er. (Three of these are in the multiplets already discussed.) The $\langle R_\gamma \rangle$ values ($J=2^+$ or 4^+ to 14^+) for these pairs are 0.37, 0.16, 0.12, 0.05, 0.13, and 0.33, respectively, and average only 0.19: the differences in rotational spacings in these pairs (which range from 4–16 mass units apart) are 5 times *less* than expected for a rigid rotor.

In conclusion, we have exhibited "identical bands" in noncontiguous nuclei spanning as much as 24 mass units using the N_pN_n scheme (and F spin) to select sets of widely dispersed nuclei that might be similar. The ratio $\langle R_\gamma \rangle$ is often 0.1–0.3, that is, the rotational energy differences between pairs of nuclei in these multiplets are 3 to 10 times smaller than expected for a rigid rotor. This kind of equality is comparable to that found in superdeformed bands [1] and in the actinides [3]. The equality of transition energies over such large mass regions has never before been demonstrated and is particularly notable in view of the fact that some of these nuclei are not in stable or saturated shape regions but in the midst of rapid shape changes as a function of N and Z. The simple ansatz used to choose these nuclei (namely, N_pN_n), with its grounding in the valence p-n interaction, and the now apparently widespread occurrence of identical band spacings, suggest possible avenues for an appropriate model explanation and encourage an explanation with generality comparable to that of the observed phenomenon.

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