RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 45, NUMBER 1 JANUARY 1992

Effect of the neutron skin on collective states of nuclei

P. Van Isacker, M. A. Nagarajan, and D. D. Warner

Science and Engineering Research Council, Daresbury Laboratory, Daresbury, Warrington WA4 4AD, England

(Received 26 July 1991)

The effect of a neutron skin on the collective states of medium and heavy-mass nuclei is discussed. We consider two well-known collective modes which correspond to the motion of neutrons with respect to protons, namely, the $M1$ (scissors) and the isovector $E1$ (giant dipole) resonance. For nuclei with a sufficient excess of neutrons the behavior of the "pygmy resonance" is also discussed.

PACS number(s): 21.60.Ev, 21.10.Re, 21.10.Gv, 21.10.Ft

One of the ideas which is currently dominating discussions of the future of nuclear physics concerns the provision of facilities to accelerate unstable nuclei. The aim would be to allow access to vast new regions of the nuclear chart and, in particular, to reach or approach the nucleon drip lines, thus probing nuclear structure at the very limits of nuclear stability. For neutron-rich nuclei, there is already an indication of the new phenomena which may occur. Detailed studies of ^{11}Li have revealed an anomalously large matter radius and a very small two-neutron binding energy [1]. These features can be understood in terms of a "neutron halo" consisting of two neutrons in an extended surface region around a core of ^{9}Li. More generally, calculations [2,3] suggest the development of an increasing "neutron-skin" thickness as the neutron excess increases. The presence of such a mantle of dominantly neutron matter will obviously affect the properties of collective modes which involve the out-of-phase motion of neutrons against protons. Moreover, it can give rise to new modes generated by motion of the core against the mantle. Indeed, evidence consistent with a "pygmy resonance" of this type has already been found [1] in ^{11}Li.

The purpose of this paper is to contribute to this discussion by exploring the effect of the neutron-skin thickness on the $M1$ and isovector $E1$ collective states of medium- and heavy-mass nuclei. We use here a simple model couched in terms of classical oscillations of the neutron and proton densities. The neutron and proton densities are assumed to have Fermi distributions and the effective neutron-proton (np) interaction is assumed to be a zero-range force, $v(\mathbf{r}_n,\mathbf{r}_p)=K\delta(\mathbf{r}_n-\mathbf{r}_p)$, where the strength K of the interaction has dimension MeV fm^3. The $M1$ collective mode under consideration here is commonly referred to as the "scissors" mode (see Ref. [4] and references therein) and is described in our model as an angular vibration of a deformed neutron density against a deformed proton density around a common axis of symmetry. The energy of the state is obtained by evaluating the restoring force and the moment of inertia. In the case of the isovector giant dipole mode we use the Goldhaber-Teller model [5] where the neutron and proton densities perform an out-of-phase vibration around their common center of mass. Its energy is determined from the restoring force and the reduced mass of the np system. The appropriate restoring force in each case is obtained by evaluating the potential energy as a function of the angular or linear displacement, by folding the neutron and proton density distributions with the np interaction. A basic premise of our work is therefore that it is the np interaction which dominates the dynamics of these oscillations.

We first discuss the isovector giant dipole resonance (GDR). The neutron and proton densities are assumed to have spherical symmetry and are parametrized as

$$\rho_i(r;R_i)=\rho_{i0}\{1+\exp[(r-R_i)/a_i]\}^{-1}, \tag{1}$$

where $i=n,p$, and R_n and R_p are the radii of the distributions with a_n and a_p representing the corresponding diffuseness. We shall assume equal diffuseness parameters, $a_n=a_p=a$, and adopt a constant mass-independent value. We now shift the centers of the neutron and proton density distributions along the positive and negative z direction. The shifted densities can be expanded in a Taylor series as

$$\rho_i(\mathbf{r}+z\mathbf{e}_z;R_i)\simeq\left[1+z\frac{d}{dz}+\frac{1}{2}z^2\frac{d^2}{dz^2}\right]\rho_i(r;R_i), \tag{2}$$

up to second order in the displacement z, $\mathbf{e}_z$ being the unit vector in the z direction. To ensure that the shift of the neutron and proton densities does not correspond to an overall shift of the system, the constraint $Nz_n=Zz_p$ must be satisfied where z_n and z_p are the displacements of the neutron and proton densities, respectively, N is the number of neutrons, and Z is the number of protons in the nucleus. (This is the well-known Tassie condition [6].) The change in np-interaction energy resulting from this shift equals

$$\Delta V_{\rm GDR}(z;R_n,R_p)=V_{\rm GDR}(z;R_n,R_p)-V_{\rm GDR}(z=0;R_n,R_p), \tag{3}$$

where

$$V_{\rm GDR}(z;R_n,R_p)=K\int\rho_n(\mathbf{r}+z_n\mathbf{e}_z;R_n)\times\rho_p(\mathbf{r}-z_p\mathbf{e}_z;R_p)d\mathbf{r}. \tag{4}$$

In (4) $z=z_n+z_p$ is the relative displacement of the centers of the neutron and proton densities and $d\mathbf{r}$ represents the three-dimensional volume element.

One can evaluate the integrals (4) for densities of the type of (1) neglecting terms of the order $\exp(-R_i/a)$ and its powers. This approximation is good for medium- and heavy-mass nuclei where the ratio R_i/a is significantly greater than 1. The change in potential energy can be evaluated analytically up to second order in z and is given by

$$\Delta V_{\rm GDR}(z;R_n,R_p) \simeq -\tfrac{2}{3}\pi K\rho_{n0}\rho_{p0}F(R_n,R_p)z^2, \quad (5)$$

where

$$F(R_n,R_p) = -\alpha_n\alpha_p\left[\frac{(\alpha_n+\alpha_p)(C_n-C_p)}{a^2(\alpha_n-\alpha_p)^3} + \frac{C_n'+C_p'}{a(\alpha_n-\alpha_p)^2}\right], \quad (6)$$

$$F(R,R) = \lim_{\epsilon\to 0} F(R+\epsilon,R) = \frac{3R^2+(\pi^2-6)a^2}{18a},$$

with $\alpha_i = \exp(-R_i/a)$, $C_i = (R_i^3+\pi^2a^2R_i)/3$, and $C_i' = R_i^2+\pi^2a^2/3$. Classically, the change in the potential energy is $\Delta V = \mu\omega^2z^2/2$, where μ is the reduced mass of the neutron and proton systems ($\mu = NZm_n/A$) and ω is the frequency of oscillation of the dipole mode. The expression (5) thus gives a simple estimate of the excitation energy $\hbar\omega$ of the GDR. The strength K can be estimated from the volume integral of the $M3Y$ potential [7], multiplied by 1.5 to account for the isospin-dependent part of the interaction [8], giving $K = -555$ MeV fm^3. Using this value of K, together with the experimental rms radius, reproduction of the empirical energy of the GDR in, for example, ^{90}Zr requires a diffuseness of $a = 0.38$ fm. The energy is dependent only on the total mass number A (and not on N and Z separately) mainly as $A^{-1/6}$. The empirical mass dependence has been found [9] to lie between $A^{-1/6}$ and $A^{-1/3}$. There is, however, evidence that the effective *np* interaction varies with mass number [10].

For nuclei with a neutron skin we may define the average radius $\bar{R} = (R_n+R_p)/2$ and the difference $y = R_n - R_p$, and expand the potential energy in powers of y. We obtain from (5)

$$\frac{\Delta V_{\rm GDR}(z;\bar{R},y)}{\Delta V_{\rm GDR}(z;\bar{R},y=0)} \simeq 1 - \frac{y^2}{10a^2},$$
$$\frac{E_{\rm GDR}(\bar{R},y)}{E_{\rm GDR}(\bar{R},y=0)} \simeq 1 - \frac{y^2}{20a^2} + \frac{3y^2}{8\bar{R}^2}. \quad (7)$$

The second equation expresses the dependence on skin thickness of the energy of the GDR in a series of *isobaric* nuclei. (We consider isobaric nuclei in order to isolate the skin-thickness effect from the overall mass dependence.) The exact dependence of the energy is shown in Fig. 1(a). We find that the energy of the GDR is only slightly affected by the neutron skin; when y becomes equal to a, the energy is lowered by about 5%. This insensitivity of the GDR to the neutron skin is understandable because this collective mode corresponds to the bulk motion of all neutrons against all the protons of the nucleus and hence a small neutron excess is unlikely to affect it. We have assumed, in addition, that the *np* interaction is a zero-range force. If a more realistic finite-range force were to be used, one would expect the GDR energy to become even less sensitive to the neutron skin.

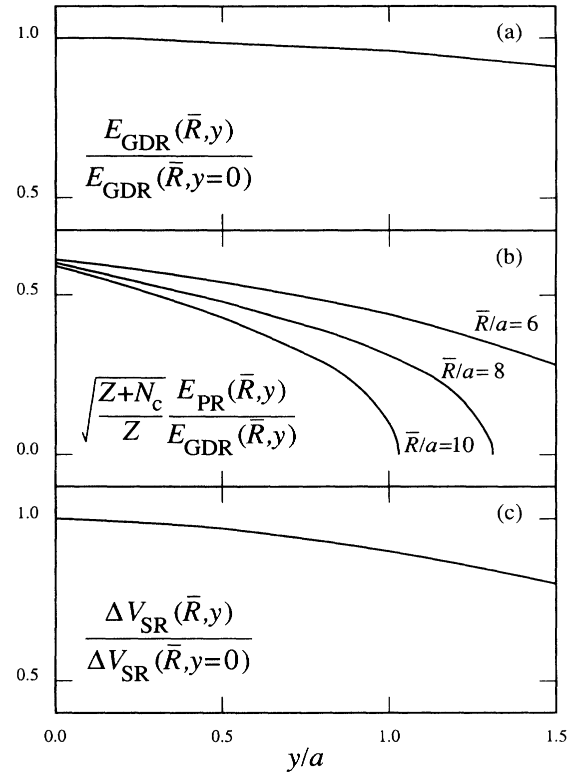

FIG. 1. The quantities (a) $E_{\rm GDR}(\bar{R},y)/E_{\rm GDR}(\bar{R},y=0)$, (b) $[(Z+N_c)/Z]^{1/2}E_{\rm PR}(\bar{R},y)/E_{\rm GDR}(\bar{R},y)$, and (c) $\Delta V_{\rm SR}(\bar{R},y)/\Delta V_{\rm SR}(\bar{R},y=0)$ as a function of the ratio y/a of skin thickness over diffuseness. The curves are obtained on the basis of the expressions (5), (9), and (12). The results in (a) and (c) are only weakly dependent on the ratio $\bar{R}/a$ of average radius over diffuseness and here $\bar{R}/a=8$ is shown; for (b) the three cases $\bar{R}/a=6$, 8, 10 are shown.

The effect of the neutron skin is likely to be far greater in the case of pygmy resonances (PR) in medium- and heavy-mass nuclei. So far, possible evidence for such a resonance only exists in light neutron-rich nuclei. In this model we assume a core comprised of the protons of the nucleus and the bulk of the neutrons and a surface which consists of the remaining weakly-bound neutrons. We thus define the "surface density" by

$$\rho_s(r;R_n,R_p) = \rho_{n0}(\{1+\exp[(r-R_n)/a]\}^{-1} - \{1+\exp[(r-R_p)/a]\}^{-1}). \quad (8)$$

The surface radius necessarily coincides with the neutron radius R_n and we have made the additional assumption that the core radius is R_p. As with the GDR we consider the potential energy as we shift the two densities in opposite directions. The resulting change in potential energy is given by

$$\Delta V_{\rm PR}(z;R_n,R_p) \simeq -\tfrac{2}{3}\pi K\rho_{n0}\rho_{p0} \times [F(R_n,R_p)-F(R_p,R_p)]z^2. \quad (9)$$

The classical frequency of this oscillation is again determined from $\Delta V = \mu\omega^2z^2/2$ but with μ the reduced mass of the core and surface nucleons.

One can evaluate the energy of the PR, E_{PR}, relative to that of the GDR, E_{GDR}, thereby eliminating the strength K of the *np* interaction. Again, we may express this ratio in terms of an average radius $\bar{R}$ and a difference y and expand in powers of y:

$$\frac{E_{PR}(\bar{R},y)}{E_{GDR}(\bar{R},y)} \simeq \left[\frac{Z}{Z+N_c}\frac{3\bar{R}^2(\bar{R}^2+\pi^2a^2)}{[3\bar{R}^2+(\pi^2-6)a^2](3\bar{R}^2+\pi^2a^2)}\right]^{1/2}\left[1-\frac{\bar{R}y}{20a^2}\right]. \tag{10}$$

The factor $[Z/(Z+N_c)]^{1/2}$ (where N_c is the number of neutrons in the core) arises because we expect both the neutrons and the protons in the core to contribute to the reduced mass while only the protons are assumed to have an impact on the restoring force. Equation (10) predicts a ratio of $\sim[Z/3(Z+N_c)]^{1/2}$ for small values of skin thickness over diffuseness, $y/a \ll 1$, and a substantial decrease in this ratio as the skin thickness increases. This is confirmed by a calculation [see Fig. 1(b)] on the basis of the exact expressions (5) and (9). The figure shows the quantity $[(Z+N_c)/Z]^{1/2}E_{PR}/E_{GDR}$ as a function of y/a for three values of $\bar{R}/a$. As the skin thickness increases, the overlap of the core and the surface density tends to become a maximum not where their centers coincide but at a finite separation of the centers and an approach assuming the nucleus to be spherical will no longer be valid. Hence, as one approaches the value of y/a where the restoring force becomes small, the limitations of our model have to be borne in mind.

One can show that the density-vibration models we use to describe the GDR and PR satisfy classical sum rules. Also the ratio of the PR to GDR sum rule can be derived and is given by $(N-N_c)Z/(Z+N_c)N$, a result similar to that obtained by Suzuki, Ikeda, and Sato [11].

We can now treat the angular oscillations of the scissors resonance (SR) with the same approach. In this case we assume axially symmetric densities (i.e., R_i in (1) is replaced by $R_i[1+\beta_iY_{20}(\theta)]$) which, for small deformations β_i, can be approximated as

$$\rho_i(r,\theta;R_i,\beta_i) \simeq \left[1+R_i\beta_iY_{20}(\theta)\frac{d}{dR_i}+\tfrac{1}{2}R_i^2\beta_i^2Y_{20}^2(\theta)\frac{d^2}{dR_i^2}\right]\rho_i(r,\theta;R_i,\beta_i=0). \tag{11}$$

The change in the potential as a function of θ_s, the angle between the axes of symmetry of neutron and proton distributions, is

$$\Delta V_{SR}(\theta_s;R_n,R_p) \simeq -2K\rho_{n0}\rho_{p0}R_nR_p\beta_n\beta_pF(R_n,R_p)\theta_s^2, \tag{12}$$

up to second order in the deformation parameters β_n and β_p. This result is closely related to the corresponding expression (5) for the GDR. Again, comparison with the classical expression, $\Delta V = \mathcal{J}\omega^2\theta_s^2/2$, where $\mathcal{J}$ is the "reduced moment of inertia" $[\mathcal{J}=\mathcal{J}_n\mathcal{J}_p/(\mathcal{J}_n+\mathcal{J}_p)]$ and ω is the frequency of oscillation, leads to an estimate of the excitation energy $\hbar\omega$ of the SR. A prediction of the dependence of the SR energy on mass number and deformation can only be obtained assuming some model approximation for the moments of inertia $\mathcal{J}_i$. For instance, assuming rigid-body moments of inertia, the dependence is found to be mainly $|\beta|(1+\beta\sqrt{5/16\pi})^{-1/2}A^{-1/6}$. As is the case for the GDR, the variation with mass number of the effective *np* interaction will modify this A dependence.

The effect of the neutron skin can be estimated in the same way as for the GDR and leads to

$$\frac{\Delta V_{SR}(\theta_s;\bar{R},y)}{\Delta V_{SR}(\theta_s;\bar{R},y=0)} \simeq 1-\frac{y^2}{10a^2}-\frac{y^2}{4\bar{R}^2}. \tag{13}$$

The corresponding expression for the SR energy is more complicated due to the presence of the reduced moments of inertia. The behavior of expression (13) is illustrated in Fig. 1(c) which shows that ΔV_{SR} is only mildly affected by the neutron skin—a feature which remains true for $E_{SR}(\bar{R},y)$.

Finally, we may deduce from (5) and (12) an expression for the SR energy relative to that of the GDR. Assuming equal radii and deformations for neutrons and protons and neglecting in the calculation of the rigid-body moments of inertia corrections due to diffuseness and deformation, we obtain $E_{SR}/E_{GDR} \simeq \beta\sqrt{15/2\pi}$. Thus, for deformations $\beta\sim 0.3$, we predict a scissors state at half the energy of the GDR. This is rather higher than the observed energies of 3–4 MeV in deformed rare-earth and actinide nuclei [4]. This, however, is to be expected and follows from our initial assumption that all nucleons participate in the angular oscillation. More elaborate models of the SR can be constructed by assuming that only part of the nucleons (e.g., the valence nucleons) participate. They would involve a separation into a "core" and a "surface" density, similar to the example of the PR, and possibly could lead to more realistic predictions.

In summary, we have studied the effect of the neutron skin on those modes of nuclei which are mainly dependent upon the *np* interaction assuming these to be described as density vibrations. One would thus expect the model to predict the average properties of these resonances. Whereas small effects of the neutron skin are expected for the giant dipole and scissors resonances, our model predicts the possibility of observing low-lying pygmy resonances in nuclei with a reasonable neutron skin. In addition, it predicts these states to be excited with a $B(E1)$ strength which is a fraction $(N-N_c)ZE_{GDR}/(Z+N_c)\times NE_{PR}$ of the $B(E1)_{GDR}$ strength. Recent investigations of the scissors ($M1$) states in rare-earth nuclei have identified [12] some $E1$ strength in the energy region of 3–5 MeV. Could this strength be associated with pygmy resonances?

One of us (D.D.W.) would like to acknowledge support from NATO research Grant No. 910214.

[1] I. Tanihata, T. Kobayashi, O. Yamakawa, S. Shimoura, K. Ekuni, K. Sugimoto, N. Takahashi, T. Shimoda, and H. Sato, Phys. Lett. B **206**, 592 (1988); T. Kobayashi, O. Yamakawa, K. Omata, K. Sugimoto, T. Shimoda, N. Takahashi, and I. Tanihata, Phys. Rev. Lett. **60**, 2599 (1988); P. G. Hansen and B. Jonson, Europhys. Lett. **4**, 409 (1987).

[2] W. D. Myers, W. J. Swiatecki, and C. S. Wang, Nucl. Phys. A **436**, 185 (1985).

[3] J. A. Sheikh and P. Ring, Phys. Rev. Lett. (to be published).

[4] A. Richter, in *Proceedings of the International Conference on Contemporary Topics in Nuclear Structure Physics,* edited by R. F. Casten, A. Frank, M. Moshinsky, and S. Pittel (World Scientific, Singapore, 1988), p. 127.

[5] M. Goldhaber and E. Teller, Phys. Rev. **74**, 1046 (1948).

[6] L. J. Tassie, Aust. J. Phys. **9**, 407 (1956).

[7] G. Bertsch, J. Borysowicz, H. McManus, and W. G. Love, Nucl. Phys. A **284**, 399 (1977).

[8] G. R. Satchler, *Direct Nuclear Reactions* (Oxford Univ. Press, New York, 1983), p. 470.

[9] B. L. Berman and S. C. Fultz, Rev. Mod. Phys. **47**, 713 (1975).

[10] D. S. Brenner, C. Wesselborg, R. F. Casten, D. D. Warner, and J. Y. Zhang, Phys. Lett. B **243**, 1 (1990).

[11] Y. Suzuki, K. Ikeda, and H. Sato, Prog. Theor. Phys. **83**, 180 (1990).

[12] P. von Brentano, A. Zilges, R. Jolos, A. Richter, R. D. Heil, U. Kneissl, H. H. Pitz, and C. Wesselborg, in *Capture Gamma-Ray Spectroscopy,* Proceedings of the Seventh International Symposium, edited by R. W. Hoff, AIP Conf. Proc. No. 238 (AIP, New York, 1991), p. 234.