Reconstruction of the spin dependence of one-nucleon-transfer spectroscopic sums from incomplete information

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We investigate a technique to reconstruct spectroscopic sums for the transfer of the even type of nucleon from an odd-even nucleus, given an orbital occupancy in the target and a limited amount of additional information taken from the analog transfer reaction. The technique is applied to $f_{7/2}$ neutron transfer on ^{45}Sc, and $f_{7/2}$ proton transfer on ^{49}Ti.

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Following French [1] we construct the spherical multipole operators for a nucleon orbit $(j\,t_3)$:

$$U^{JM}_{jj}(t_3) \;=\; [J]^{-1/2}[a^\dagger(j\,t_3)\,b^\dagger(j\,t_3)]^{JM}, \qquad (1)$$

where we use $[J]=(2J+1)$ throughout, and the hole creation operator $b^\dagger$ is related to the particle annihilation operator a by

$$b^\dagger(j\,m\,t_3) \;=\; (-1)^{j+m}a(j\;-m\,t_3). \qquad (2)$$

We also define [2] partial spectroscopic sums $S^-_{J_\alpha}(j\,t_3)$ and $S^+_{J_n}(j\,t_3)$ for the transfer of a $(j\,t_3)$ nucleon from a target ground state, spin J_r, to final pickup states α, J_α, and final stripping states n, J_n:

$$S^-_{J_\alpha}(j\,t_3) \;=\; \sum_{\alpha,J_\alpha\ \mathrm{fixed}}\left(C^{T_\alpha\ \ 1/2\ T_r}_{T_r-t_3\ t_3\ T_r}\right)^2 S_\alpha(j\,t_3) \qquad (3)$$

and

$$S^+_{J_n}(j\,t_3) \;=\; \sum_{n,J_n\ \mathrm{fixed}}\frac{[J_n]}{[J_r]}\left(C^{T_r\ 1/2\ T_n}_{T_r\ \ t_3\ T_r+t_3}\right)^2 S_n(j\,t_3), \qquad (4)$$

where C^2S_α and $[J_n]C^2S_n/[J_r]$ are the spectroscopic quantities extracted directly from one-nucleon-transfer experiments by a distorted-wave analysis, and C^2 is the square of the appropriate isospin Clebsch-Gordan coefficient.

The expectation values of the operator U^J_{jj} for the target ground state are directly related to the spectroscopic sums $S^-_{J_\alpha}$ by

$$\langle J_r||U^J_{jj}(t_3)||J_r\rangle = [J_r]\sum_{J_\alpha}(-1)^{J+J_\alpha+J_r+j}\begin{Bmatrix}J_r & j & J_\alpha\\ j & J_r & J\end{Bmatrix}S^-_{J_\alpha}(j\,t_3). \qquad (5)$$

Substituting for the $S^-_{J_\alpha}$, using spin-dependent sum rules [3]

$$\sum_{J'}(-1)^{J_r+j+J'}\begin{Bmatrix}J_r & j & J'\\ j & J_r & J\end{Bmatrix}[S^+_{J'}(j\,t_3)+(-1)^J S^-_{J'}(j\,t_3)] = [j]^{1/2}[J_r]^{-1/2}\delta_{J0} \qquad (6)$$

yields

$$\langle J_r||U^J_{jj}(t_3)||J_r\rangle = [j]^{1/2}[J_r]^{1/2}\delta_{J0} - [J_r]\times\sum_{J_n}(-1)^{J_r+J_n+j}\begin{Bmatrix}J_r & j & J_n\\ j & J_r & J\end{Bmatrix}\times S^+_{J_n}(j\,t_3). \qquad (7)$$

Equations (5) and (7) may be inverted to obtain

$$S^-_{J_\alpha}(j\,t_3) = \frac{[J_\alpha]}{[J_r]}\sum_J(-1)^{J_\alpha+J+J_r+j}[J]\begin{Bmatrix}J_r & j & J_\alpha\\ j & J_r & J\end{Bmatrix}\times\langle J_r||U^J_{jj}(t_3)||J_r\rangle \qquad (8)$$

and

$$S^+_{J_n}(j\,t_3) = \frac{[J_n]}{[J_r]}\left[1-\sum_J(-1)^{J_n+J_r+j}[J]\begin{Bmatrix}J_r & j & J_n\\ j & J_r & J\end{Bmatrix}\times\langle J_r||U^J_{jj}(t_3)||J_r\rangle\right]. \qquad (9)$$

Whenever t_3 refers to the even type of nucleon in an odd-even target the prevalence of pairing and quadrupole forces in nuclei ensures that the multipole moments with $J\neq 0,2$ are relatively unimportant[1] [3]. Therefore all spectroscopic sums can be calculated using just these two moments in Eqs. (8) and (9). Now

[1]The vanishing of the odd-J moments then leads to a generalization of the well-known $(2J+1)$ rule, or equivalently to approximate sum rules [2], which have been shown to hold to a high degree of accuracy for a range of nuclei in the *sd* and *fp* shells [4].

TABLE I. Experimental values [5] of spectroscopic sums to final states of spin J, S_J^- for $f_{7/2}$ proton pickup on ^{45}Sc and S_J^+ for $f_{7/2}$ neutron stripping on ^{49}Ti.

J^π	S_J^-	S_J^+
0^+	0.45 ± 0.05	0.72 ± 0.07
2^+	0.41 ± 0.04	0.25 ± 0.03
4^+	0.14 ± 0.01	0.03 ± 0.00

$$< J_r \| U^0_{jj}(t_3) \| J_r > = n_p(j\,t_3)[J_r]^{1/2}/[j]^{1/2}, \tag{10}$$

where $n_p(j\,t_3)$ is the occupancy of the orbit $(j\,t_3)$ in the target ground state. For convenience we put

$$< J_r \| U^2_{jj}(t_3) \| J_r > = Q(j\,t_3) \tag{11}$$

and drop the labels $(j\,t_3)$. Thus

$$S^-_{J_\alpha} = \frac{[J_\alpha]}{[J_r]}\left[\frac{n_p}{[j]} + (-1)^{J_\alpha+J_r+j}[2]\begin{Bmatrix} J_r & j & J_\alpha \\ j & J_r & 2 \end{Bmatrix} Q\right] \geq 0 \tag{12}$$

and

$$S^+_{J_n} = \frac{[J_n]}{[J_r]}\left[\frac{n_h}{[j]} - (-1)^{J_n+J_r+j}[2]\begin{Bmatrix} J_r & j & J_n \\ j & J_r & 2 \end{Bmatrix} Q\right] \geq 0, \tag{13}$$

where $n_h = [j] - n_p$ is the hole occupancy in the target ground state. The inequalities follow from the positive-definite nature of the spectroscopic sums.

Necessary conditions for Eqs. (12) and (13) to hold to good approximation are that the spectroscopic data refer to the transfer of the even type of nucleon from an odd-even nucleus and that the data satisfies the spin-dependent sum rules [3]. This is the case for $f_{7/2}$ neutron transfer on ^{45}Sc, and $f_{7/2}$ proton transfer on ^{49}Ti [5], both targets having spin $J_r = j = \frac{7}{2}$. Sum-rule analyses for these targets [5] give occupancies close to the simple shell-model values, in agreement with theoretical calculations in this mass region [6]. Thus, taking $n_p = 4$ for $f_{7/2}$ neutrons in ^{45}Sc and $n_p = 2$ for $f_{7/2}$ protons in ^{49}Ti, Eqs. (12) and (13) yield

$$-0.80 \leq Q \leq +0.80 \quad \text{and} \quad -0.40 \leq Q \leq +0.76 \tag{14}$$

for ^{45}Sc and ^{49}Ti, respectively. Using the analytic form for the six-J coefficient [7]

$$\begin{Bmatrix} J_r & j & J' \\ j & J_r & 2 \end{Bmatrix}$$

it can be shown that its maximal magnitude is attained for $J' = | J_r - j | = 0$ in the present work. Thus, if $n_p \leq n_h$ one of the bounds is obtained from $S_0^- \geq 0$ and the other bound is given either by $S_J^- \geq 0$ with $J \neq 0$ or by $S_0^+ \geq 0$. Similarly, if $n_p \geq n_h$ one bound is given by $S_0^+ \geq 0$ and the other either by $S_J^+ \geq 0$ with $J \neq 0$ or by $S_0^- \geq 0$. In ^{45}Sc $n_p = n_h$ and the bounds are given by $S_0^- \geq 0$ and $S_0^+ \geq 0$. In ^{49}Ti $n_p < n_h$ and the bounds are given by $S_0^- \geq 0$ and $S_5^- \geq 0$. Clearly the ranges specified by Eqs. (14) can be narrowed by using available information on the spectroscopic sums $S^-_{J_\alpha}$ and $S^+_{J_n}$ to increase the lower limits of Eqs. (12) and (13). This could be supplied, for example, by the analog reactions. For the present cases, the latter supply the particularly clear-cut transfer data [5] shown in Table I for $f_{7/2}$ proton pickup on ^{45}Sc and $f_{7/2}$ neutron stripping on ^{49}Ti. We have again assumed the simple shell-model occupancy of 1 in each case, and normalized the data to this value. We assign statistical errors of 10 % in these numbers [5, 8], postponing a discussion of the possibility of missing strength (i.e., an overall error in absolute normalisation) to later in the paper. Given this information, we can extract the corresponding contributions to $T_>$ states for $f_{7/2}$ neutron pickup on ^{45}Sc and $f_{7/2}$ proton stripping on ^{49}Ti. Assuming isospin symmetry the ratio of the spectroscopic factors are given by the square of the ratio of the isospin Clebsch-Gordan coefficients. This gives a value of $[T_r]^{-1}$, where T_r is the target isospin. Thus the corresponding numbers in Table I must be divided by the factors 4 and 6, respectively [2], and result in spectroscopic strengths of only $\sim$ 3% of the total of 8 available for $f_{7/2}$ transfer. Since the total strength is greater than the $T_>$ strength, the lower limits of the $T_>$ strengths deduced from Table I provide new lower bounds on Eqs. (12) and (13). This limited additional information drastically reduces the ranges of Q to

$$0.50 \leq Q \leq +0.80 \quad \text{and} \quad -0.40 \leq Q \leq -0.18 \tag{15}$$

TABLE II. Experimental [5] and predicted values [from Eqs. (12) and (13)] of spectroscopic sums to final states of spin J, S_J^- for pickup and S_J^+ for stripping of a $f_{7/2}$ neutron for ^{45}Sc.

	S_J^-		S_J^+	
J^π	Expt.	Pred.	Expt.	Pred.
0^+	0.16 ± 0.02	0.11 ± 0.01	0.00 ± 0.00	0.01 ± 0.01
1^+	0.31 ± 0.03	0.31 ± 0.03	0.03 ± 0.00	0.06 ± 0.03
2^+	0.45 ± 0.05	0.43 ± 0.01	0.26 ± 0.03	0.19 ± 0.03
3^+	0.42 ± 0.04	0.45 ± 0.01	0.53 ± 0.05	0.42 ± 0.01
4^+	0.39 ± 0.04	0.41 ± 0.07	0.77 ± 0.08	0.72 ± 0.04
5^+	0.45 ± 0.05	0.39 ± 0.07	0.85 ± 0.09	0.98 ± 0.07
6^+	0.49 ± 0.05	0.59 ± 0.05	1.14 ± 0.11	1.03 ± 0.05
7^+	1.32 ± 0.13	1.29 ± 0.08	0.42 ± 0.04	0.58 ± 0.08

TABLE III. Experimental values [5] and predicted values [from Eqs. (12) and (13)] of spectroscopic sums to final states of spin J, S_J^- for pickup and S_J^+ for stripping of a $f_{7/2}$ proton for ^{49}Ti.

J^π	S_J^- Expt.	S_J^- Pred.	S_J^+ Expt.	S_J^+ Pred.
0^+	0.00 ± 0.00	0.01 ± 0.00	0.13 ± 0.01	0.12 ± 0.01
1^+	0.00 ± 0.00	0.04 ± 0.02	0.35 ± 0.04	0.34 ± 0.02
2^+	0.06 ± 0.01	0.10 ± 0.02	0.67 ± 0.07	0.52 ± 0.02
3^+	0.19 ± 0.02	0.21 ± 0.01	0.71 ± 0.07	0.67 ± 0.01
4^+	0.48 ± 0.05	0.35 ± 0.03	0.69 ± 0.07	0.77 ± 0.03
5^+	0.64 ± 0.06	0.47 ± 0.05	0.80 ± 0.08	0.90 ± 0.05
6^+	0.51 ± 0.05	0.50 ± 0.04	1.02 ± 0.10	1.12 ± 0.04
7^+	0.12 ± 0.01	0.31 ± 0.06	1.64 ± 0.16	1.56 ± 0.06

for ^{45}Sc and ^{49}Ti, respectively. We note that it is the transfer to 0^+ states which is crucial in the present cases when the data from the analog reactions are included. Using $n_p = 4$ and $Q = 0.65 \pm 0.15$ for ^{45}Sc, and $n_p = 2$ and $Q = -0.29 \pm 0.11$ for ^{49}Ti, all the spectroscopic sums can be generated by Eqs. (12) and (13) with the excellent results shown in Tables II and III.

We finally consider possible errors in the absolute normalization of the input data in Table I. Such errors could arise if the correctly normalized pickup and stripping strengths to low-lying states, $S_{J_\alpha}^{-'}$ and $S_{J_n}^{+'}$, say, do not exhaust the particle and hole occupancies, respectively [8, 9]. In this case, simultaneous fits [5] to sum rules as well as to the ground-state spins strongly suggest that any unobserved higher-lying strength mimics the spin distribution of the observed low-lying strength. Taking the total unobserved strength as a constant fraction of the available strength [8] then results in the simple proportionalities

$$S_{J_\alpha}^- = (1 + a) S_{J_\alpha}^{-'} \quad S_{J_n}^+ = (1 + a) S_{J_n}^{+'}, \qquad (16)$$

where a is the ratio of the unobserved to observed strength. The interpretation of the numbers in Tables I–III would then be that they all refer to relative values for low-lying strength. Similar arguments also hold for the case [10] where the spin distribution of the unobserved strength is proportional to $(2J + 1)$, where J is the final-state spin.

In conclusion, we have shown that the spin distribution of spectroscopic strength for the transfer of the even type of nucleon from the odd-even nuclei ^{45}Sc and ^{49}Ti can be estimated using an occupancy and a limited amount of additional information from the analog reaction. With 10% errors in the data, our predictions are in overall agreement with experiment to about one standard deviation. It is interesting to note that the largest discrepancy is for proton pickup on ^{49}Ti, and that it is for proton transfer on ^{49}Ti that sum-rule fits have been found to be the least satisfactory of a number of similar fits for neighboring nuclei [5]. The present results thus indicate that a re-examination of the proton pickup data for ^{49}Ti may prove worthwhile. In general, the technique should prove useful whenever the spin distribution of spectroscopic strength for the transfer of the even type of nucleon from an odd-even target is incompletely known. In particular we note that in all such cases the final states belong to odd-odd nuclei, with the consequent problems of resolution because of the higher density of states at low excitation energy. In the present analysis we have taken the information in addition to the occupancy from the analog reaction. However, it is clear that information from any source (e.g., from a subset of final states) will restrict the bounds of Q, thus sharpening the predictions of the spin distribution of the spectroscopic strength based on the above technique.

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