Determination of F spin symmetry in deformed nuclei

Joseph N. Ginocchio
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

Serdar Kuyucak
School of Physics, University of Melbourne, Parkville, Victoria 3052, Australia
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The purity of a neutron-proton symmetry called F spin is estimated in collective nuclei. Two simple formulas are shown to provide a quick and accurate estimate for F spin admixtures in the ground band. Conclusions are also drawn about quadrupole effective charges in collective nuclei.

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F spin is an SU(2) symmetry between valence neutron and proton pairs which originated in the interacting boson model of nuclei (IBM) [1,2] but could also apply to other models of collective motion. If we consider a system of n bosons for neutrons and protons, $b^\dagger_{\rho i}$, $b_{\rho i}$, where $i=1,\ldots,n, \rho=\pi$ (protons) or ν (neutrons), and $[b_{\rho' i}, b^\dagger_{\rho j}]=\delta_{i,j}\delta_{\rho',\rho}$, the F spin generators are

$$\hat{F}_+=\sum_i^n b^\dagger_{\pi i}b_{\nu i},\quad \hat{F}_-=\sum_i^n b^\dagger_{\nu i}b_{\pi i},\quad \hat{F}_0=\tfrac{1}{2}(\hat{N}_\pi-\hat{N}_\nu)\ , \tag{1}$$

where $\hat{N}_\rho=\sum_i^n b^\dagger_{\rho i}b_{\rho i}$ and $\hat{N}_\rho$ counts the number of neutron ($\rho=\nu$) or proton pairs ($\rho=\pi$), which is one-half the number of valence neutrons or protons. For IBM, usually only monopole ($J^\pi=0^+$) and quadrupole ($J^\pi=2^+$) pairs are kept, and hence, $b^\dagger_{\rho i}=(s^\dagger_\rho, d^\dagger_{\rho\mu}, \mu=-2,\ldots,2)$ and $n=6$.

These three operators in (1) form an SU(2) Lie algebra analogous to spin. In a given nucleus, with fixed N_π, N_ν, all states have the same value of $F_0=\frac{1}{2}(N_\pi-N_\nu)$, while the allowed values of the F spin quantum number F range from $|F_0|$ to $F_{\max}\equiv\frac{1}{2}(N_\pi+N_\nu)$ in unit steps. F spin measures the extent to which the states are symmetric in the neutron and proton degrees of freedom. The states with maximum F spin, $F_{\max}$, have the highest symmetry and are the lowest in the energy spectrum in general.

F spin is very different physically than isospin because, for heavy nuclei, the raising operator $\hat{F}_+$ changes a neutron pair in one major shell into a proton pair in a *different* major shell producing a low-lying state in the neighboring nucleus. On the other hand, isospin changes a neutron pair into a neutron-proton pair in the same major shell, thereby producing an excited state, the isobar analog state, in the neighboring nucleus. Hence F spin produces multiplets which relate how-lying states of nuclei, as opposed to isospin which produces multiplets of excited states. Furthermore, F spin is not understood on a fermion level.

The IBM intrinsic state which produces the ground-state rotational band [3] can, in general, be expanded in terms of states of well-defined F spin, as $|c_a\rangle=\sum_F\alpha_F|F\rangle$, where $|F\rangle$ is the component with the F spin quoted and α_F is the amplitude of that component in the intrinsic state $|c_a\rangle$. In a recent paper [4] it was shown that the amount of admixtures of F spin symmetry in the ground-state band is determined by the difference in the neutron and proton deformations of the intrinsic state and not by whether the Hamiltonian is an F spin scalar. An upper and lower limit and an approximate formula were given for the F spin admixtures in the intrinsic state in terms of the IBM deformations. Previously, however, an exact formula for the F spin decomposition of the intrinsic state had been derived using projection techniques [5]. This formula, which applies to arbitrary (e.g., *sd*, *sdg*, etc.) boson models, is given by [5]

$$\alpha^2_{F_{\max}-k}=\frac{N-2k+1}{N-k+1}\frac{\binom{N}{k}}{\binom{N}{N_\pi}}[1-(\mathbf{x}_\pi\cdot\mathbf{x}_\nu)^2]^k \times\sum_i\binom{N_\pi-k}{i}\binom{N_\nu-k}{i}(\mathbf{x}_\pi\cdot\mathbf{x}_\nu)^{2i}\ , \tag{2a}$$

where $N=N_\pi+N_\nu$, $k=0,1,\ldots,F_{\max}-|F_0|$, $\binom{n}{l}$ are binomial coefficients, and $\mathbf{x}_\rho$ are the normalized mean fields to be associated with the deformation parameters, e.g., in the *sd* boson model

$$(\mathbf{x}_\pi\cdot\mathbf{x}_\nu)^2=\frac{(1+\beta_\pi\beta_\nu)^2}{(1+\beta_\pi^2)(1+\beta_\nu^2)}\ . \tag{2b}$$

Similar formulas had also been derived for the other (excited) bands in Ref. [5]. After some manipulations (2) can be written as

$$\alpha^2_{F_{\max}-k}=\frac{\binom{N_\pi}{k}\binom{N_\nu}{k}}{\binom{N-k+1}{k}}y^k \times F(-N_\pi+k,-N_\nu+k;-N+2k;y)\ , \tag{3a}$$

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where $F(a,b;c,y)$ is a hypergeometric function and

$$y=1-(\mathbf{x}_\pi\cdot\mathbf{x}_\nu)^2=\frac{(\beta_\pi-\beta_\nu)^2}{(1+\beta_\pi^2)(1+\beta_\nu^2)} . \tag{3b}$$

For N_π,N_ν large, this becomes asymptotically

$$\alpha^2_{F_{\max}-k}\simeq e^{-Py/2}\frac{(Py/2)^k}{k!} , \tag{4}$$

where $P=2N_\pi N_\nu/N$.

In Ref. [4] an approximate expression is derived for these admixtures which is

$$\alpha^2_{F_{\max}-k}\simeq e^{-x^2}\frac{x^{2k}}{k!} , \tag{5a}$$

where

$$x^2=\left[\left[\frac{(1+\bar{\beta}^2)}{(1+\beta_\pi\bar{\beta})(1+\beta_\nu\bar{\beta})}\right]^2(1+\beta_\pi^2)(1+\beta_\nu^2)\right]\frac{P}{2}y \tag{5b}$$

and $\bar{\beta}$ is defined by the equation [4]

$$(N_\pi\beta_\nu+N_\nu\beta_\pi)\bar{\beta}^2+N(1-\beta_\pi\beta_\nu)\bar{\beta}-(N_\pi\beta_\pi+N_\nu\beta_\nu)=0 . \tag{5c}$$

For $(\beta_\pi-\beta_\nu)/(\beta_\pi+\beta_\nu)$ small, the quantity in square brackets in (5b) is approximately unity, $x^2\simeq Py/2$, and (4) becomes approximately equal to (5a).

In order to test the validity of the asymptotic and approximate expressions for $\alpha^2_{F_{\max}}$ given in (4) and (5), respectively, we compare them in Fig. 1 with the exact value of $\alpha^2_{F_{\max}}$ as a function of y given in (3) for $N_\pi=6$, $N_\nu=10$, which implies $P=7.5$. Both agree well up to relatively large values of y ($\simeq 0.2$) with the approximate formula (5) giving the better agreement. Although it is not apparent from (5), numerical calculations show that the approximate formula (5), like (3) and (4), is also a function of only y and not of β_π, β_ν separately.

The asymptotic formula (4) and, to a degree, the approximate formula (5) suggest that $\alpha^2_{F_{\max}}$ depends mainly on P, and not on N_π,N_ν separately. In Fig. 2, we compare the exact formula (3) as a function of P for two different functional dependences on N_π,N_ν. The solid line has $N_\pi=N_\nu$ and the dashed line as $N_\pi=20-N_\nu$. The two overlap even for very large values of y, which establishes that $\alpha^2_{F_{\max}}$ depends predominantly on P. This result is also consistent with the observation [6] that collective observables depend primarily on P.

We can use the IBM β_π and β_ν determined [4,7,8] from pion charge exchange on ^{165}Ho to estimate the amount of F spin admixtures in the ground-state rotational band. In this estimation, the ratio of IBM neutron and proton quadrupole effective charges, $R=e_\nu/e_\pi$, must be assumed. The estimation of the F spin admixtures is very sensitive to this ratio R. The larger the value of R taken, the larger the estimation of F spin admixtures because the larger value of R means that the transition operator becomes more like an F spin scalar, and hence, a given difference in measured quadrupole moments must be explained by a larger difference in β_π and β_ν, thereby implying larger F spin admixtures in the ground-state band. Generally very large R's are used in IBM [1], but this may be due to the fact that only electromagnetic transition rates are used to determine R. Two rates are needed to determine R, and one of the transition rates will be less collective and thus sensitive to other degrees of freedom not in the IBM space. A better way to determine R is to measure a collective proton *and* neutron transition rate. This can be done by measuring a collective transition for both π^+ and π^- inelastic scattering. The best determination of R comes from fitting both $\pi^\pm$ scattering to the

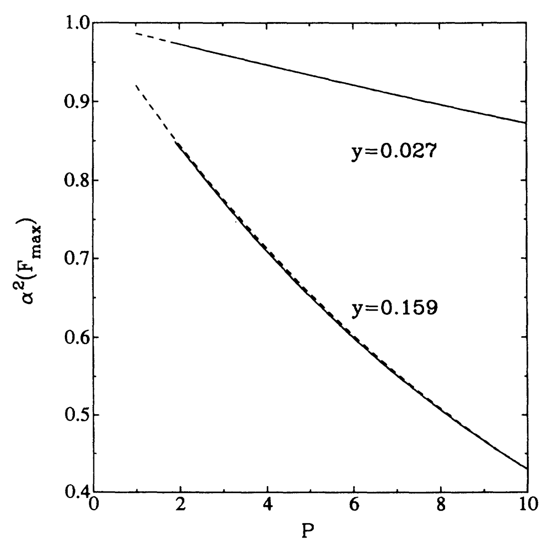

FIG. 1. The exact value of $\alpha^2_{F_{\max}}$ (solid line), the asymptotic value of $\alpha^2_{F_{\max}}$ given in (4) (dashed line), and the approximate value of $\alpha^2_{F_{\max}}$ given in (5) (dash-dotted line) are plotted vs y defined in (3b), where $P=7.5$.

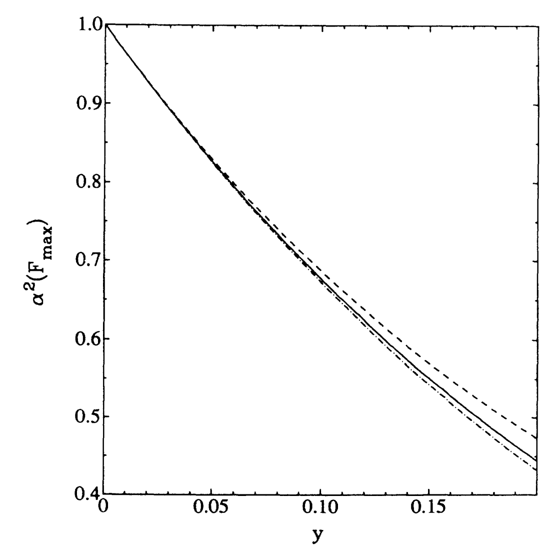

FIG. 2. The exact values for $\alpha^2_{F_{\max}}$ plotted against P for two different values of y. The solid line is for $N_\pi=N_\nu$, the dashed line for $N_\pi=20-N_\nu$. The overlap of the two lines demonstrates P dependence of $\alpha^2_{F_{\max}}$.

first 2^+ state in the Pd isotopes [9]. In this analysis the value $R=0.41$ was extracted in IBM-1, independent of the IBM-1 Hamiltonian. This value is slightly larger than the shell-model value of $R=0.33$, but the use of IBM-1 may have the effect of making R effectively larger. Since we are using IBM-2 in this paper, we expect a value of R more consistent with the shell model to be the correct one to use.

If R is assumed to be close to the shell-model value of 0.33, then $y=0.017$ and there is a 6% admixture of lower F spins with $F<F_{\max}$. If a large value of $R=0.73$ is used, which is in fact used for many IBM calculations [1], then $y=0.14$ and there is a 43% admixture of lower F spins in the ground-state band. This latter estimate is much larger than either that determined by $B(M1)$ measurements [10] or IBM calculations [11].

In conclusions, we have shown that once the IBM deformation parameters for both neutrons and protons are determined, the amount of admixtures of F spin lower than $F=F_{\max}$ can be estimated in the ground-state rotational band. Furthermore, the simple formulas (4) and (5) can be used to determine these estimates. Finally, we conclude that neutron and proton quadrupole effective charges that are nearly equal will produce unrealistically large estimates of F spin admixture, which suggests that the ratio of the neutron and proton boson quadrupole effective charges is more like the ratio of the neutron and proton shell-model effective charges, which is much smaller than the ratio of the boson quadrupole effective charges used in many IBM calculations.

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