Elastic scattering phenomenology by inversion: ^{16}O on ^{12}C at 608 MeV

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Here we introduce a practical approach to optical model phenomenology for heavy ions. The key idea is the determination of a "spline improved McIntyre" S matrix $S(l)$ which is then subjected to $S(l)$-to-$V(r)$ inversion. We apply this to fitting experimental data for ^{16}O on ^{12}C at 608 MeV and obtain potentials which are significantly different from those found from a simple optimization of McIntyre parameters followed by inversion. In particular we find quite different results for such qualitatuve features as W/V in the nuclear surface. The potential found is compared with that obtained by conventional analysis. The two-stage procedure has computational advantages for heavy ions.

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In this paper we demonstrate the feasibility of a new way of getting precise fits to elastic scattering data, reducing these data to local potentials which contain the full information content of the data. The idea involves a particular means of carrying out a procedure advocated at various times [1] and carried out with varying degrees of success [2–5]. We refer to the two-stage procedure whereby the S matrix is determined by a phase-shift search and then subjected to fixed energy $S(l)$-to-$V(r)$ inversion. The recent emergence of a number of practical methods [6–10] for $S(l)$-to-$V(r)$ inversion makes the two-state procedure a feasible alternative to the more usual direct approach, which arguably fails in many cases. The two-stage procedure offers the possibility of gaining insight into the ambiguities which are not fully explicit in direct optical model (OM) searching where the high degree of nonlinearity is a compounding of the nonlinearities of the σ-to-$S(l)$ and $S(l)$-to-$V(r)$ inversions.

Of the above references, the one which is closest to the present work is that of Allen *et al.* [3], who invert from $S(l)$ of McIntyre, Wang, and Becker [11] with parameters previously fitted to the data. Our work differs from that of Allen *et al.* [3] in that the aim is the determination of potentials which give precision fits to elastic-scattering data. To this end we introduce an S-matrix fitting procedure [12], SIM, defined below. The work of Leeb, Fiedeldey, and Lipperheide [4] offers an alternative approach to the same problem we study here, and a comparative evaluation of the procedures should be made in due course.

For our present purposes, the form of $S(l)$ must be free to depart from the restrictive form of McIntyre, Wang, and Becker. There is every reason to believe that an underlying l dependence due to channel coupling, etc., exists and would be represented in an l-independent potential by some degree of nonsmoothness. It is axiomatic that a model with $\chi^2/N \sim 5$ does not work; there will be a vast number of quite different models which will work equally badly. Arguably, the most objective test for a theory (coupled channels, for example) is to reduce it by inversion to a local potential, which can then be compared with phenomenological local potentials. Phenomenology is useful if and only if *either* $\chi^2/N \sim 1$ is achieved *or* one can conclude from the analysis that systematic errors make $\chi^2/N \sim 1$ impossible with physically reasonable models; we shall expand upon this in a paper in preparation. Here we present a method that not only fits data for which there are no published exact fits, but motivates experiments of greater precision and angular range by showing how the information contained in such data can be extracted.

The problem of ambiguities in phase-shift fitting is well known to be profound in principle, but there does seem to be an approach that for high-energy heavy ions give consistent results. By starting the search at a local minimum characterized by a very smooth $S(l)$, it seems that for very large numbers of partial waves a good fit can be obtained, retaining this smoothness. The first step is to fit the data with a five-parameter McIntyre-Wang-Becker form:

$$|S^M(l)| = \frac{1}{1+\exp[(l_g - l)/\Delta]} \tag{1}$$

and

$$\arg S^M(l) = \frac{2\mu}{1+\exp[(l-l_g')/\Delta']} \, . \tag{2}$$

The second step is to add, to the complex $S^M(l)$ so determined, a correction which in our case is expanded in spline functions of l:

$$S(l) = S^M(l) + S^c(l) \, , \tag{3}$$

where $S^c(l)$ is a spline correction chosen in such a way that $|S(l)|$ does not exceed unity at the knots of the spline interpolation procedure. The searching is actually performed on $|S|$ and arg S separately to this end. In practice, there is no problem with unitarity being broken elsewhere. We henceforth refer to $S(l)$ obtained in this way as spline-improved McIntyre (SIM).

To obtain the potentials which correspond to the SIM $S(l)$, we use the IP (inversion procedure) [9,10]. This yields potentials which precisely reproduce $S(l)$ down to values of l corresponding to radii much lower than those which are of significance in the light of current experimental data. The potentials are unique down to such radii. More details will be given in Ref. [13], where we shall present such potentials for ^{12}C on ^{12}C for energies as low as 161 MeV, well below the energies at which WKB inversion works. Here we confine ourselves to describing a single case of considerable interest, the scattering of 608 MeV ^{16}O from ^{12}C [14].

In Fig. 1 we compare $|S(l)|$ and $\arg S(l)$ for the McIntyre-Wang-Becker and SIM fits. The most conspicuous features are the long tail on $\arg S(l)$ and a bend in $|S(l)|$, whereby a generally sharper slope toward unity with increasing l crosses the McIntyre-Wang-Becker curve to give a more gradual approach to unity for $l \geq 75$. The behavior for $l < 20$ is not well established, and the fits shown must be considered artifacts of the McIntyre-Wang-Becker starting form. The long tail on $\arg S(l)$ is a most interesting physical feature and lies beyond the scope of the McIntyre-Wang-Becker form. It is, however, essential for a fit to the data, the quality of fit being greatly improved by the objective measure of χ^2/N as well as by more subjective measures of appearance. The tail seems particularly essential for a correct representation of the cross section at the Fraunhofer diffraction minima at 7° and 11° (see Fig. 2 introduced below.) Below, we present our fits graphically in terms of the potentials inverted from the $S(l)$, but we quote here the quality of fit for the σ-to-$S(l)$ fitting state. For the McIntyre-Wang-Becker $S(l)$, we obtain $\chi^2/N=3.023$, and for SIM, we obtain $\chi^2/N=0.897$, substantially better.

Whether we start the IP inversion iterations from a

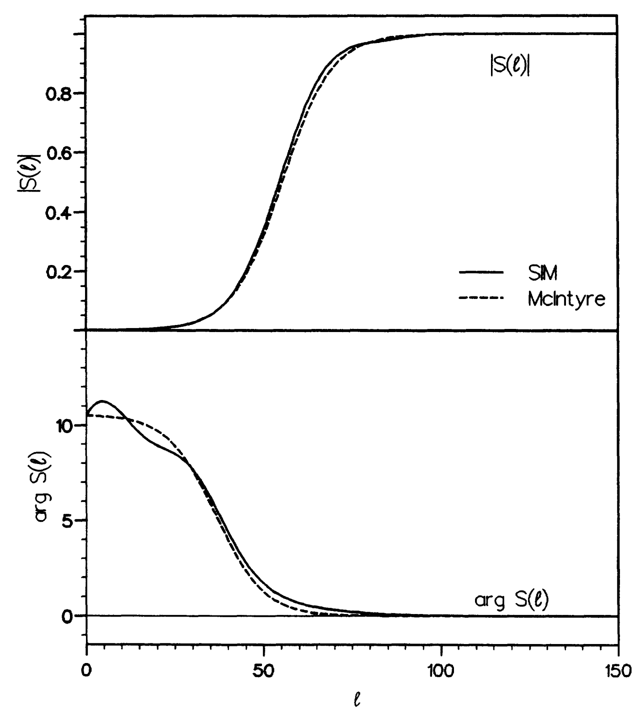

FIG. 1. For 608 MeV ^{16}O incident on ^{12}C, the modulus and argument of the S matrix as found by two alternative procedures. The dashed line is $S(l)$ as determined by optimizing the parameters of a McIntyre-Wang-Becker form, the solid line being the SIM fit (see text).

Woods-Saxon or from a *zero* potential made no difference to the final potential beyond 1 fm. The quality of the inversion is reflected in the fact that the final χ^2/N for the inverted potential was 0.951, close to the value quoted above for the SIM $S(l)$. For the potential obtained by inverting the McIntyre-Wang-Becker $S(l)$, we have $\chi^2/N=3.014$ compared with 3.023 for the McIntyre-Wang-Becker $S(l)$ directly.

One of the key qualitative features characterizing a heavy-ion potential is the ratio of the imaginary to real potential in the nuclear surface, here denoted by W/V. In this respect the difference between the McIntyre-Wang-Becker and SIM potentials is notable—W/V in the surface is entirely different for the two cases. At the strong absorption radius [SAR, defined using $|S(l)|^2=0.5$], we found, for SIM, $W/V=0.70$ (SAR is at 6.73 fm), whereas for McIntyre-Wang-Becker $W/V=1.09$ (SAR is at 6.82 fm). The disparity rapidly increases with radius (see Table I discussed below.) Thus the SIM potential may be classified as surface transparent, but the potential derived from the McIntyre-Wang-Becker $S(l)$ is not. As a result, there is a large difference between the McIntyre-Wang-Becker and SIM differential cross sections regarding the shape of the cross section in the Fraunhofer interference region. This is responsible for a large measure of the improvement of the fit. We have examined the far/near decomposition following the formalism of Fuller [15] and find that, in going from McIntyre-Wang-Becker to SIM, the far component is enhanced in this region and the near component depressed. The enhancement of the far component, at least, would seem to tally with the greater surface transparency of the SIM potential.

We have applied a notch test to determine the minimum radius for which the given data determine the potential; it is about 3 fm. More specifically, a Gaussian notch of width 0.3 fm and of depth equal to 5% of the local real potential increases χ^2/N by a factor of 2 when centered at 3.6 fm.

The χ^2 values quoted above show that the fits for the SIM-derived potential are much better than for the potential derived from the McIntyre-Wang-Becker $S(l)$. We compare our potential with a standard OM potential, exploiting the published parameters of Brandan [16]. In Fig. 2 we compare our SIM fit (solid line) with that of Brandan (dotted line), which we find to give $\chi^2/N=4.72$; the Fraunhofer interference region between 10° and 20° is fitted much better by SIM, with oscillations of greater magnitude. There are weak oscillations in the shape between 20° and 30° which are missed by the conventional potential. We do slightly worse only at the maximum near 6°. Figure 2 also compares the SIM and McIntyre-Wang-Becker (dashed line) fits; the SIM potential gives a very much better fit than the McIntyre-Wang-Becker potential to σ at the Fraunhofer region between 5° and 15°; we mentioned above the origin of this in the relative strengths of far and near amplitudes.

We have not presented figures comparing the differential cross sections calculated directly from McIntyre-Wang-Becker or SIM $S(l)$ with cross sections calculated from the respective inverted potentials; in both

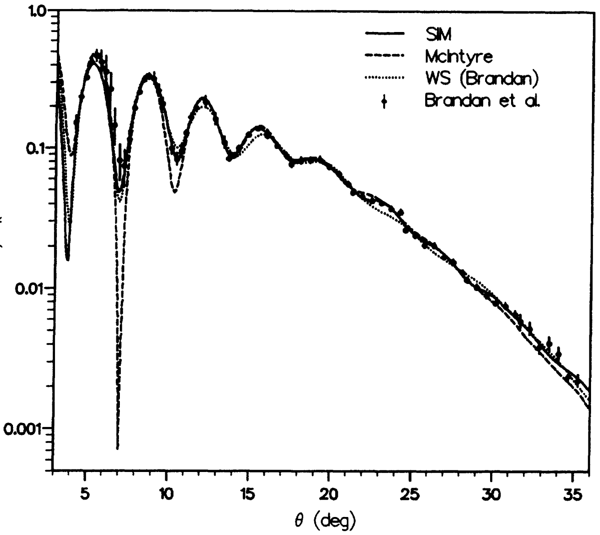

FIG. 2. For 608 MeV ^{16}O incident on ^{12}C comparing the SIM fit to the data (solid line) with the conventional optical model fit of Brandan and Satchler (dotted line). The McIntyre-Wang-Becker fit to the data is given by the dashed line.

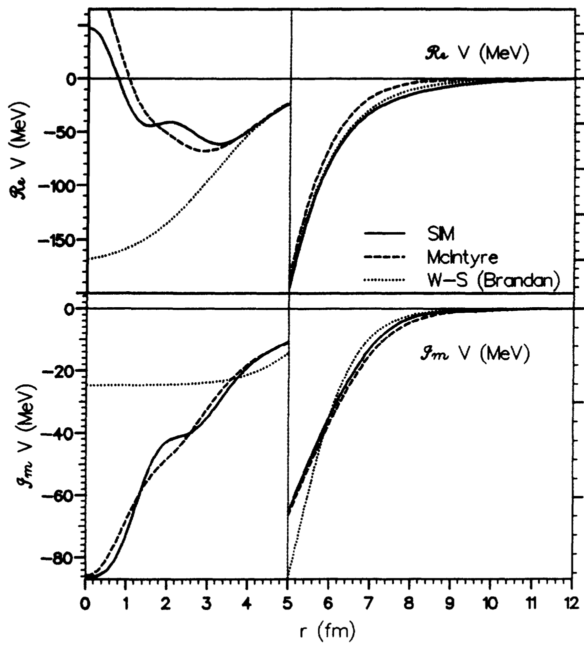

FIG. 3. Potentials corresponding to the fits of Fig. 2. The SIM potential is the solid line, the McIntyre-Wang-Becker potential is the dashed line, and the Brandan-Satchler potential is the dotted line. The scale is expanded in the nuclear surface.

cases they are virtually indistinguishable over the entire angular range where there is experimental data.

Improved OM fits, reaching χ^2/N close to unity, have been obtained by using generalized Woods-Saxon (WS) shapes and spline real potentials [17], but the present comparison serves well to demonstrate one of our conclusions: that an *uncorrected* McIntyre-Wang-Becker $S(l)$ can lead to erroneous qualitative results. In Fig. 3 we compare our SIM potentials with those Brandan and Satchler and with that derived from the McIntyre-Wang-Becker form. For $r \geq 5$ fm the agreement of the real part of the SIM with that of Brandan and Satchler is remarkable, but not complete: We find a long tail that is an evident concomitant of the SIM $\arg S(l)$; the real part of the McIntyre-Wang-Becker potential, also shown in Fig. 3, entirely lacks such a long tail.

The fundamental disagreement between McIntyre-Wang-Becker and SIM potentials is more evident in Table I, which also shows that the SIM and Woods-Saxon potentials have the same general W/V behavior. The disagreement between SIM and McIntyre-Wang-Becker results, the agreement between SIM and Woods-Saxon results, and the much better fit for the SIM potential all taken together suggest that potentials derived from the McIntyre-Wang-Becker $S(l)$ can lead to incorrect conclusions concerning qualitative features such as surface transparency. This is an important point and underlines the essential difference between our work and that of Ref. [3].

The inverted potentials shown in Fig. 3 should not be taken seriously for radii below 3 fm, the repulsion at $r=0$ being a clear artifact of the underlying McIntyre-Wang-Becker form, which the data could not discriminate against. There is evidence to suggest that data over a wider angular range might actually lead to information concerning the behavior of the potential around 3 fm or below. We mentioned above the notch test relating to the effect upon χ^2/N; it turns out that the quantity $\sigma^2 = \sum_l |\Delta(S(l))|^2$ that we use [10] to study the quality of the inversion is sensitive to perturbations in the potential down to about 2 fm.

Further applications. The procedure described here is of wide applicability: Papers are in preparation describing our analysis of ^{12}C on ^{12}C at laboratory energies from 161 to 2400 MeV; unlike the WKB inversion procedure employed by Allen *et al.* [3], our method has no difficulty with precise inversions below 500 MeV and works well for the 161-MeV case. Allen *et al.* remark that the WKB method requires smooth $S(l)$; whether the behavior of the SIM $S(l)$ of this paper around $l=75$ is smooth in their sense, it gives no trouble for the IP inversion, and

TABLE I. Ratio W/V at various radii for the Woods-Saxon potential of Brandan and Satchler [17], the potential corresponding to the McIntyre-Wang-Becker parametrization of the S matrix, and the potential derived from the SIM S matrix.

Radius (fm)	5.5	6.0	6.5	7.0	7.5	8.0	8.5
Woods-Saxon	0.68	0.66	0.58	0.48	0.40	0.31	0.24
McIntyre-Wang-Becker	0.65	0.86	0.96	1.30	1.52	1.69	1.57
SIM	0.57	0.67	0.64	0.66	0.52	0.36	0.22

we have inverted rather less smooth $S(l)$. We are also preparing for publication cases for a wide range of targets and projectiles. Although we have here preferred the SIM $S(l)$ to that of McIntyre, Wang, and Becker, it is probable that in many cases, where the far component of the scattering amplitude is very small, a McIntyre-Wang-Becker parametrization will adequately fit the data.

Implications for experiment. There is no reason to suppose that we could not achieve fits of comparable quality to data of higher precision and wider angular range. The SIM plus inversion phenomenology provides the motivation for pushing elastic-scattering measurements to more backward angles with the assurance that the corresponding information could be extracted. Indeed, even smoothly falling angular distributions provide useful information; in an analysis of ^{12}C on ^{12}C scattering to be published, we find that, particularly where there are systematic errors in the data, forcing a precise fit to the data can lead to divergent behavior beyond the last angle with the data. Further data points to constrain the fit are therefore highly desirable.

Practicality of two-stage SIM elastic-scattering phenomenology. The two-stage method may be the simplest way of finding perfect fits to elastic-scattering data for heavy ions at high energies where the number of partial waves, the number of mesh points, and the number of potential parameters (for model-independent fitting) are all very high. The initial S-matrix fitting is generally straightforward, characterized by considerable linearity, and takes a matter of seconds of CPU time on a VAX. Much the same applies to the $S(l)$-to-$V(r)$ stage; all our calculations were carried out on line with fits and $S(l)$ monitored graphically.

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