# Can the quadrupole form factors of the $N \rightarrow \Delta$ transition be determined model independently?

Robert W. Lourie

Department of Physics and Institute of Nuclear and Particle Physics, University of Virginia, Charlottesville, Virginia 22901 (Received 16 September 1991)

This work addresses the question of how well, in principle and in practice, the electric and Coulomb quadrupole amplitudes for the  $N \rightarrow \Delta$  transition can be determined. The analysis is motivated by the significantly greater number of observables available in polarized electroproduction that will become accessible at the new generation of electron accelerators. The influence of several common assumptions, such as restriction to s and p waves only, is investigated. Results are presented for low  $[0.12 (\text{GeV}/c)^2]$  and high  $[2.5 (\text{GeV}/c)^2]$  four-momentum transfers.

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### I. INTRODUCTION

A central issue in baryon physics is the presence, or absence, of l = 2 (d-state) admixtures in the quark wave functions of the nucleon and  $\Delta$  isobar. More than a decade ago, Glashow [1] and Vento, Baym, and Jackson [2] showed that the discrepancies encountered when  $G_A/G_V$ , the SU(3) decay ratio (D+F)/(D-F), and the  $\pi N\Delta$  coupling constant are calculated using spherically symmetric quark wave functions can be resolved if a sizable d-state admixture were present in the nucleon and isobar wave functions. An immediate consequence of these admixtures would be nonzero Coulomb and electric quadrupole ( $S_{1+}$  and  $E_{1+}$ ) multipoles in the electromagnetic  $N \rightarrow \Delta$  transition. [These multipoles vanish if the quarks are in the  $(1s)^3$  configuration as required by the SU(6) symmetric quark model. Then the transition is a pure magnetic dipole  $(M_{1+})$  spin-flip of a single quark.] Physically, d-states (and other mixed-symmetry components) can arise from the color hyperfine interaction [3,4] resulting from QCD-motivated one gluon exchange (color magnetism) [5]. Relativistic light-cone quark models can also produce quadrupole components even in the absence of a tensor force [6]. There is an obvious high intrinsic interest in determining the quadrupole components (ideally, over a large range in  $Q^2$ ) and thereby performing tests of microscopic theories of hadron structure.

However, unambiguous determination of  $S_{1+}(Q^2)$  and  $E_{1+}(Q^2)$  is hampered by (1) their small magnitude (a few percent of  $M_{1+}$ ) and (2) the presence of many other contributions to the experimental observables, such as nonresonant terms and tails of higher resonances. The problem is especially difficult for  $Q^2 \gtrsim 1$  (GeV/c)<sup>2</sup> since the  $\Delta$  falls off with  $Q^2$  considerably faster than either the other resonances or the background, both of which have (approximately) the  $Q^2$  dependence of the nucleon elastic form factor [7]. Previous attempts [8] to experimentally determine the quadrupole components from angular distributions in unpolarized electroproduction have always assumed a specific form for the electromagnetic observables, namely, the form that results (1) when only s and p waves in the  $\pi N$  system are retained and (2) when the

only influence of multipoles other than  $M_{1+}$  is via their interference with the  $M_{1+}$  multipole.

In this work, I examine the degree to which the quadrupole form factors may be determined in a modelindependent manner; i.e., I ask: Is there, in principle, sufficient information in polarized electroproduction to allow their determination directly from the experimental observables? Such an analysis is timely in light of the preparation for the next generation of electroproduction experiments at the new high-intensity, high-duty-factor electron accelerators. In contrast to previous measurements, the new experiments will exploit the polarization degrees of freedom by combining polarized electron beams, polarized targets, and/or detection of recoil polarization. This results in a significantly greater number of experimental observables which, one hopes, will provide enough redundancy to reduce or eliminate the previous ambiguities [9] in extracting the quadrupole form factors.

While several authors [10]-[12] have recently shown the large sensitivity of various electromagnetic response functions to the presence of small quadrupole components, this sensitivity is demonstrated by holding all other physics ingredients constant and then turning on or off the quadrupole components. The questions of *sensitivity* and *determination* are different. I will examine the case of "maximal" (in the sense of separating the largest number of response functions) coplanar electroproduction, namely, the  $p(\vec{e}, e'\vec{p})\pi^0$  reaction, and investigate the influence of various physics input on the extracted quadrupole form factors. These include the standard assumptions of only including s and p waves and the influence of higher nucleon resonances and the nonresonant nucleon pole (Born) terms.

#### **II. POLARIZED ELECTROPRODUCTION**

The cross section for the  $p(e,e'p)\pi^0$  reaction, allowing for a polarized electron beam and either a polarized nucleon target or detection of the recoil proton polarization, contains 18 electromagnetic response functions [13]. (In what follows, I only consider recoil polarization; the results for polarized targets are essentially the same.) In the recoil polarization case, the following response func-

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tions may be separated in coplanar experiments:  $R_L$ ,  $R_T$ ,  $R_{LT}$ ,  $R_{LT}^{\prime\prime}$ ,  $R_{LT}^{\prime\prime}$ ,  $R_{LT}^{\prime\prime}$ ,  $R_{T}^{\prime\prime}$ ,  $R_{TT}^{\prime\prime}$ ,  $R_{TT}^{\prime\prime}$ ,  $R_{TT}^{\prime\prime}$ ,  $R_{TT}^{\prime\prime}$ ,  $R_{TT}^{\prime\prime}$ ,  $R_{TT}^{\prime\prime}$ , and the combination  $2\varepsilon R_L^n - \varepsilon R_{TT}^n$ , where  $\varepsilon$  is the virtual photon polarization parameter, the subscripts L, T, LT, and TT denote longitudinal, transverse, longitudinal-transverse interference, and transverse-transverse interference terms, respectively, and the prime denotes the electron-helicity-dependent response functions. The superscripts l, n, and t denote the components of the proton polarization vector in a coordinate system with  $\hat{l}$  along the proton momentum,  $\hat{n}$  normal to the reaction plane, and  $\hat{t} = \hat{n} \times \hat{l}$ . To see how the quadrupole terms of interest enter, one performs a multipole decomposition of the response functions [13]:

$$R_{\rm L} = 4|S_{1+}|^{2}(1+P_{2}) + 4\operatorname{Re}(S_{1-}^{*}S_{1+}),$$

$$R_{\rm T} = 2|M_{1+}|^{2} + 2\operatorname{Re}(E_{0+}^{*}M_{1+})P_{1}$$

$$-[|M_{1+}|^{2} + 2\operatorname{Re}(M_{1-}^{*}M_{1+} - 3E_{1+}^{*}M_{1+})]P_{2},$$

$$R_{\rm TT}^{\prime l} = \frac{5}{3}|M_{1+}|^{2} - 2\operatorname{Re}(E_{1}^{*}M_{1+} + \frac{2}{3}M_{1-}^{*}M_{1+})$$

$$+ 2\operatorname{Re}(E_{0+}^{*}M_{1+})P_{1}$$

$$-[\frac{2}{3}|M_{1+}|^{2} - \operatorname{Re}(8E_{1+}^{*}M_{1+} + \frac{2}{3}M_{1-}^{*}M_{1+})]P_{2},$$

$$R_{\rm TT}^{\prime \prime} = \sin\theta \{\operatorname{Re}(E_{0+}^{*}M_{1+})\}$$

+ [
$$|M_{1+}|^2$$
 + Re( $6M_{1+}^*E_{1+} + M_{1-}^*M_{1+}$ )] $P_1$ },

$$R_{LT}^{\prime l} = -\sqrt{2} \sin\theta \operatorname{Re}[2S_{0+}^{*}M_{1+} + (S_{1-}^{*}M_{1+} + 10S_{1+}^{*}M_{1+})P_{1}],$$

$$R_{LT}^{\prime \prime} = \sqrt{2} \operatorname{Re}[\frac{5}{3}S_{1-}^{*}M_{1+} - \frac{4}{3}S_{1+}^{*}M_{1+} + S_{0+}^{*}M_{1+}P_{1} - \frac{2}{3}(S_{1-}^{*}M_{1+} - 8S_{1+}^{*}M_{1+})P_{2}],$$

$$R_{LT} = -\sqrt{2} \sin\theta \operatorname{Re}(S_{0+}^{*}M_{1+} + 6S_{1+}^{*}M_{1+}P_{1}),$$

$$R_{LT}^{n} = -\sqrt{2} \operatorname{Im}(S_{1-}^{*}M_{1+} + S_{0+}^{*}M_{1+}P_{1} + 4S_{1+}^{*}M_{1+}P_{2}),$$

where  $\theta$  is the proton angle with respect to q in the  $N\pi$  center-of-mass system and  $P_1$  ( $P_2$ ) is the first (second) Legendre polynomial of argument  $\cos\theta$ . Each response function depends on  $Q^2$ ,  $\theta$ , and the invariant mass of the hadronic system W. Solely for compactness, I have written the multipole expansion assuming that (1) only s and p waves contribute and (2) terms that do not contain the (large)  $M_{1+}$  amplitude are negligible (except in  $R_L$  where  $M_{1+}$  does not occur). When higher partial waves, Born terms, etc., are to be included, it is more convenient to use a helicity basis for the response functions. Their representation in this basis is given in a recent paper [14].

# **III. RESULTS AND DISCUSSION**

Clearly, the LT-type responses offer the best opportunity to learn about the  $S_{1+}$  amplitude since it enters linearly and with a large coefficient. The  $E_{1+}$  term occurs both linearly and quadratically in the T- and TTtype response functions. [The quadratic terms do not appear explicitly in Eq. (1) because of assumption (2) above. They are, however, retained in the full calculation of the response functions.] To isolate the  $E_{1+}$  terms requires removing the dominant  $|M_{1+}|^2$  terms. Now consider the following differences of longitudinal-transverse response functions:

$$\begin{split} \widetilde{\Delta}_{LT}^{(1)} &= \frac{R_{LT}^{\prime \prime} - 2R_{LT}}{2\sqrt{2}\sin\theta\cos\theta} ,\\ \widetilde{\Delta}_{LT}^{(2)} &= \frac{2R_{LT}^{\prime \prime}\sin\theta + R_{LT}^{\prime \prime}P_1}{2\sqrt{2}(\cos\theta)(3\cos^2\theta - 4)} , \end{split}$$
(2)  
$$\tilde{\Delta}_{LT}^{(3)} &= \frac{R_{LT}^{\prime \prime}\sin\theta + R_{LT}P_1}{2\sqrt{2}\sin\theta(\cos^2\theta - 2)} \end{split}$$

and of transverse-transverse ones:

$$\widetilde{\Delta}_{TT}^{(1)} = \frac{3R_{TT}^{\prime l} \sin\theta P_1 - (5 - 2P_2)R_{TT}^{\prime l}}{36} ,$$

$$\widetilde{\Delta}_{TT}^{(2)} = \frac{3R_{TT}^{\prime l} (2 - P_2) - (5 - 2P_2)R_T}{12} ,$$

$$\widetilde{\Delta}_{TT}^{(3)} = \frac{R_{TT}^{\prime l} (2 - P_2) - R_T \sin\theta P_1}{12} .$$
(3)

If the above multipole decomposition were truly valid, these differences would allow one to isolate  $\operatorname{Re}(S_{1+}^*M_{1+})$ and  $\operatorname{Re}(E_{1+}^*M_{1+})$  in several independent ways:

$$\operatorname{Re}(S_{1+}^{*}M_{1+}) = \frac{\tilde{\Delta}_{LT}^{(1)}}{P_{1}} = \tilde{\Delta}_{LT}^{(2)} = \tilde{\Delta}_{LT}^{(3)} , \qquad (4)$$

$$\operatorname{Re}(E_{1+}^{*}M_{1+}) = \frac{\tilde{\Delta}_{TT}^{(1)}}{P_{1}(P_{2}-1)\sin\theta} = \frac{\tilde{\Delta}_{TT}^{(2)}}{(2P_{2}-P_{2}^{2}-1)} = \frac{\tilde{\Delta}_{TT}^{(3)}}{(1-P_{2})P_{1}\sin\theta} .$$

(The various angular functions are removed to avoid trivial singularities at  $\theta = 0^{\circ}, 90^{\circ}$ , and 180°). Of course, the simple assumptions used above describe only part of each response function. However, the important point is that any additional physics input enters differently in each of the  $\tilde{\Delta}_{LT}^{i}$ 's and this might allow the "beyond quadrupole" contribution to each to be characterized model independently. What the above procedure isolates is the entire quadrupole amplitude; it does not a priori indicate how much is resonant, i.e., there may be contributions from the  $T=\frac{1}{2}$  channel and other nonresonant physics that also have  $S_{1+}$  and  $E_{1+}$  multipoles. If, as predicted [15], there is a significant  $T = \frac{1}{2}$  contribution to  $S_{1+}$ , it will have a dramatic influence on  $R_{LT}^n$  since the  $\operatorname{Im}(S_{1+}^*M_{1+})$  interference would vanish if the two multipoles both come from the  $\Delta$ , i.e., have the same phase. Indeed, Ref. [10] showed that this response function is much more sensitive to the interference with the (real) Born terms than to the  $\Delta$  quadrupole amplitudes.

To explore the sensitivity of the  $\tilde{\Delta}_{LT}^i$ 's to the physics input, I calculated the response functions for several cases. The base-line case is that of an isolated  $\Delta$  resonance, i.e., only the  $S_{1+}$ ,  $E_{1+}$  and  $M_{1+}$  resonant amplitudes were present. Then the nucleon pole terms were (coherently)

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added, then all  $l \leq 1$  nucleon resonances, and, finally, all of the major resonances up to l=4:  $D_{13}(1520)$ ,  $D_{15}(1675), F_{15}(1688), S_{31}(1620), S_{11}(1700), F_{37}(1950),$ and  $G_{17}(2190)$ . The resonant amplitudes were taken from the dispersion-relation-based parametrization of Devenish and Lyth [16] for the  $\Delta$  and higher nucleon resonances. They tabulate only the resonance contributions to the dispersion integral; to this must be added the (real) background terms. For this purpose, I utilize the results of von Gehlen [17]. Although the primary goal of the present work is to investigate the stability of the  $\tilde{\Delta}_{LT}^{i}$ 's as one varies the model input rather than to provide detailed predictions, I note that the numerical results for the response functions agree at the 20% level with the more sophisticated microscopic model of Nozawa et al. [12] and that this model provides a reasonable description of the existing data.

Instead of the  $\tilde{\Delta}_{LT}^{i}$ 's (the experimentally accessible quantities), I present the results as

$$D_{LT}^{i} = \frac{\widetilde{\Delta}_{LT}^{i} - \widetilde{\Delta}_{LT}^{i}(0)}{\operatorname{Re}(S_{1+}^{*}M_{1+})},$$

$$D_{TT}^{i} = \frac{\widetilde{\Delta}_{TT}^{i} - \widetilde{\Delta}_{TT}^{i}(0)}{\operatorname{Re}(E_{1}^{*}M_{1+})},$$
(5)

where  $\tilde{\Delta}_{IJ}^{i}(0)$  refers to the base-line case of an isolated  $\Delta$  resonance and the  $E_{1+}$  and  $M_{1+}$  multipoles are from the Devenish-Lyth [16] parametrization. The  $D_{IJ}^{i}$ 's are, then, either the fractional deviation from the base-line case or that deviation times a known function of  $\theta$ .

The results are shown in Fig. 1 for  $Q^2 = -0.12$  $(GeV/c)^2$  and an invariant mass W=1.232 GeV (where the  $\Delta$  amplitudes are purely imaginary). The solid curve is the result for an isolated  $\Delta$  resonance and no Born terms. The  $S_{1+}$  and  $E_{1+}$  amplitudes are -5% and -2% of the  $M_{1+}$  amplitude. (In the parametrization of Ref. [16],  $E_{1+}$  has the same  $Q^2$  dependence as  $M_{1+}$  while  $S_{1+}$  contains an additional kinematic factor to ensure proper threshold behavior.) The long-dashed curve shows the  $\Delta$ -plus-Born-terms result while the further inclusion of the  $l \leq 1$   $(l \leq 4)$  resonances is shown by the short-dashed (dot-dashed) curve. As one might expect, at low  $Q^2$  there are large regions of  $\theta$  where most of the  $D_{IJ}^{i}$ 's exhibit rather little model dependence. In particular, not only does the  $\Delta$  resonance dominate the electroexcitation spectrum compared to other resonances (at this  $Q^2$ ), but also any interference between the Born terms and the  $\Delta$  amplitudes is, on resonance, purely imaginary and hence cannot contribute to the response functions (there are, however, both LT- and TT-type in-



FIG. 1. The longitudinal-transverse (left) and transverse-transverse (right) difference functions at  $Q^2 = 0.12$  (GeV/c)<sup>2</sup> and W = 1.232 GeV. The solid curve is the result for an isolated  $\Delta$  resonance, the long-dashed curve includes Born terms, and the short-dashed curve includes Born terms and  $l \leq 1$  resonances while the dot-dashed curve includes Born terms and the dominant  $l \leq 4$  resonances.

terferences from the Born terms alone). Thus, the  $D_{LT}$ 's should allow  $\operatorname{Re}(S_{1+}^*M_{1+})$  to be determined model independently at the 10-15 % level. (They are nonzero even for the base-line case due to a  $\operatorname{Re}(S_{1+}^*E_{1+})$  term in the response functions.) The electric quadrupole term would be best obtained at forward angles from either  $D_{TT}^{(1)}$ or  $D_{TT}^{(3)}$  where the model uncertainties are less than 10% (these functions are not really mirror images as will be apparent at higher  $Q^2$ ). The strong influence of the Born terms on  $D_{TT}^{(2)}$  renders it unsuitable for extracting  $\operatorname{Re}(E_{1+}^*M_{1+})$ . On the other hand, this same sensitivity makes it a potent constraint on other ingredients to model calculations. At this small  $Q^2$ , all of the  $D_{IJ}^i$ 's are insensitive to the presence of the higher spin (l > 1) resonances (although the presence of the other low-lying  $l \leq 1$ resonances, in particular the  $S_{11}$ , has a discernible effect at backward angles).

The situation is dramatically different at the higher  $Q^2$  of 2.5 (GeV/c)<sup>2</sup>. The  $\Delta$  amplitudes are much reduced compared to both the other resonances and background and, indeed, the background at W = 1.232 GeV is nearly as large as the total  $\Delta$  response [7]. This implies that the response functions now result from a combination of many relatively small amplitudes, none of which is truly dominant. Consequently, the interference structures should be both more pronounced and more sensitive to the particular combination of amplitudes considered.

This is borne out by the results displayed in Fig. 2. Large deviations from the base-line case are observed in all the  $D_{IJ}^{i}$ 's. Furthermore, they exhibit less stability than at  $Q^2 = 0.12$  (GeV/c)<sup>2</sup> in that they change significantly as each new ingredient is added. In particular, the inclusion of the higher partial waves now has a large influence. In general, the  $D_{IJ}^{i}$ 's no longer provide as good a modelindependent characterization of the quadrupole components as they did at lower  $Q^2$ . However, it is encouraging that there are regions of  $\theta$  where at least some of them are stable at the 20% level, i.e.,  $D_{LT}^{(2)}$  and  $D_{LT}^{(3)}$  for  $60^\circ \le \theta \le 100^\circ$  and  $D_{TT}^{(1)}$  and  $D_{LT}^{(3)}$  for  $120^\circ \le \theta \le 140^\circ$ . I emphasize that these results represent somewhat of a worstcase scenario for determinating  $E_{1+}$  since, if  $E_{1+}/M_{1+}$ were to increase as a function of  $Q^2$ , its extraction would be correspondingly less uncertain. Just such as increase is predicted in both the relativistic constituent quark model of Weber [18] where  $E_{1+}/M_{1+}$  goes from 2% at  $Q^2=0$  to -10% at  $Q^2=2.5$  (GeV/c)<sup>2</sup> and also by the perturbative QCD calculations of Carlson [19]. (Although dominance of the  $M_{1+}$  amplitude is consistent with the highest  $Q^2$  existing data, these data are also consistent with the perturbative QCD prediction of equality between  $E_{1+}$  and  $M_{1+}$  [9].) Of course, if any given model were able to accurately reproduce all the  $D_{IJ}^{i}$ 's (or the response functions themselves), confidence in such a model would then be justified. This is, however, a different



FIG. 2. The same as Fig. 1 except at  $Q^2 = 2.5 (\text{GeV}/c)^2$ .

state of affairs than truly model-independent knowledge.

An important practical consideration is that the finite acceptances of any real apparatus result in averaging over a range in all the kinematic quantities. In particular, experiments cannot be performed at a single value of W, e.g., at W = 1.232 GeV where the resonant amplitudes are purely imaginary. Therefore, it is also necessary to know the variation in the measured quantities due to the averaging procedure. To this end, Figs. 3 and 4 show the  $D_{IJ}^{i}$ 's at  $Q^{2}=2.5 (\text{GeV}/c)^{2}$  for W=1.214 and 1.250 GeV  $(\pm 18 \text{ MeV off resonance})$ . The difference functions are highly sensitive to this variable. Although  $D_{LT}^{(2)}$  and  $D_{LT}^{(3)}$ still exhibit a relatively broad region of insensitivity at W=1.214 GeV, they vary dramatically with input at W=1.250 GeV. The ability of  $D_{TT}^{(1)}$  and  $D_{TT}^{(3)}$  to characterize  $E_{1+}$  is no better than 30% at W=1.214 GeV and considerably worse at the higher invariant mass. These results argue for (1) measurements whose W acceptance is asymmetric around the  $\Delta$  peak to deemphasize the high W side and (2) measurements with good resolution in Wso that the averaging effects can be accurately characterized.

All of the results presented above assumed that the individual response functions were known with infinite precision and so comprised an "in principle" investigation. Are the  $\tilde{\Delta}_{IJ}^{i}$ 's useful in practice? This is especially relevant for the  $\Delta_{TT}^{i}$ 's since they involve the difference of two relatively large quantities as one is trying to remove the  $|M_{1+}|^2$  contribution. I recalculated all of the  $D_{IJ}^i$ 's assuming that the uncertainty on each response function was  $\pm 5\%$  (which is somewhat conservative for the next generation of experiments). This produces error bands on the  $D_{IJ}^i$ 's of 10 -25% with the LT ones exhibiting the smaller "error magnification." Thus, this method of extracting information on the quadrupole components can be fruitfully employed in practice.

## **IV. CONCLUSIONS**

This work has shown that the functions  $\tilde{\Delta}_{IJ}^i$ , constructed from differences of the electromagnetic response functions, can provide, particularly at low  $Q^2$ , an accurate, model-independent determination of the (total) quadrupole amplitudes. These may, however, include contributions from processes other than  $\Delta$  production. Within the context of the (realistic) models employed in this work, the  $\Delta$  amplitudes are indeed the dominant contributions to the total quadrupole amplitudes (the influence of the quadrupole components in the Born terms and other resonances are much less than the resonant ones, especially on the LT-type response functions). However, to resolve this in a truly model-independent manner would require (1) an isospin decomposition, i.e., additional measurements with neutron targets, (2) measurement of



FIG. 3. The same as Fig. 2 except at W = 1.214 GeV.



FIG. 4. The same as Fig. 2 except at W = 1.250 GeV.

charged pion production and subsequent detection of the recoil neutron polarization, and (3) a verification that the extracted T=3/2 amplitudes are resonant, i.e., that they are purely imaginary at the  $\Delta$  pole. These technically more challenging measurements were not addressed in this work.

Although there is greater difficulty extracting the quadrupole amplitudes at higher  $Q^2$ , the difference functions introduced here retain much of their utility, especially if the kinematics are selected carefully with respect to the invariant mass. This conclusion for the  $E_{1+}$  multipole is further strengthened if, as predicted [19,18], it increases with  $Q^2$  so that it becomes significantly greater

than 2% of  $M_{1+}$  as was assumed in this work. When precise response function data become available, it should be possible to determine the Coulomb and electric quadrupole components to better than 20% without recourse to models or simplifying assumptions in the data analysis.

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