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Second-forbidden unique β decays of ^{10}Be, ^{22}Na, and ^{26}Al

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A shell-model study is made of second-forbidden unique β decay of ^{10}Be, ^{22}Na, and ^{26}Al. Calculations were made with a p-shell interaction for ^{10}Be and the sd-shell interaction of Wildenthal for ^{22}Na and ^{26}Al. The resulting second-forbidden unique matrix elements are consistently larger than experiment. The ratio is discussed in terms of two effects: the "collective suppression" discussed in 1970 by Warburton, Garvey, and Towner and in-medium quenching of the spin operator. Comparison is made to similar effects for $M3$ transitions.

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The presently available experimental information on the second-forbidden unique β decays of ^{10}Be, ^{22}Na, and ^{26}Al are collected in Table I. The beta moment B_2 for this process is defined experimentally by [1]

$$B_2 = [M_2]^2 = \lambda^2 \langle G_2 \rangle^2 = 6.166 \times 10^{15} \frac{\lambda_{Ce}^4}{f_2 t} \text{ fm}^4 , \quad (1)$$

where λ is the ratio of the axial to vector coupling constants ($\equiv C_A/C_V$) which is taken to be 1.2605, and λ_{Ce} = 386.159 fm is the Compton wavelength of the electron. The second-forbidden unique Fermi integral f_2 is evaluated for $\beta^{\pm}$ decay as described in Ref. [1]. For electron capture, f_2 is calculated using the procedures and parameters given by Bambynek *et al.* [2]. Theoretical evaluations of the beta moments of Table I were presented in detail in 1970 by Warburton, Garvey, and Towner [1] — hereafter referred to as WGT. There are several reasons why a reconsideration of these decays is of interest. First, the experimental values have changed somewhat. In 1970, the available experimental information led to B_2 values of 45(8), 8.0(20), 1.24(6), and 24(2) fm^4 for the decays of ^{10}Be and ^{22}Na, and the two decays of ^{26}Al, respectively. Note the sizable changes in the first and last. (These changes are due to a new half-life measurement for ^{10}Be [3], new branching ratios for ^{26}Al decay [4], and changes in C_A/C_V.) Second, the shell-model calculations for the ^{22}Na and ^{26}Al decays involve rather large dimensions in the appropriate $0d_{5/2}0d_{3/2}1s_{1/2}$ model space (the 2^+ states of ^{24}Mg have a J dimension of 4500) so that the 1970 calculation was done in a $0d_{5/2}1s_{1/2}$ space. A calculation with the highly successful $0d_{5/2}0d_{3/2}1s_{1/2}$ USD interaction of Wildenthal [5, 6] is now possible and certainly in order. The third reason involves the interesting question of in-medium effects on these transitions which we now consider.

The unique second-forbidden matrix element M_2 has the same operator as the isovector part of the spin term which dominates the $M3$ operator [7]. It is assumed to be a sum over all nucleons of a one-body operator [1]

$$\left[\frac{2\pi}{3}\right]^{1/2} \tau_q [Y_2(\hat{\mathbf{r}}) \times \boldsymbol{\sigma}]^{(3)}_m r^2 , \quad (2)$$

where τ_q is a component of a spherical isospin tensor. The evaluation of the matrix elements of this operator is described in detail by WGT. The presence of $Y_2(\hat{\mathbf{r}})r^2$ and of σ in Eq. (2) suggests a relationship to the $E2$ and $M1$ operators — Zamick [8] proposed that M_2 be thought of crudely as $E2 \otimes M1$. The Gamow-Teller operator can be thought of formally as the unique zeroth-forbidden operator. From the nuclear structure point of view a major difference between the zeroth- and second-forbidden operators is that the former has no first-order contributions from $n\hbar\omega$ admixtures in the wave functions, while the second-forbidden operator has first-order contributions from $2\hbar\omega$ admixtures very similar to those occurring in $E2$ transitions. However, there is an important difference between the β and $E2$ processes: The effect on $E2$ transitions is to enhance the isoscalar matrix element while the usual effect on the matrix element of Eq. (2) is to decrease the matrix element, hence this latter effect was termed "collective suppression" by WGT. The first evaluation of this collective suppression of M_2 was that of Kurath [9] who used procedures akin to those of Nilsson [10] to estimate the effects of deformation, these effects being primarily the admixing of $\Delta Q = \pm 2$ orbits, where Q is the number of quanta in a major shell. The methods introduced by Kurath were used by WGT to express the collective suppression in terms of the nuclear deformation and thus to estimate it for the cases of Table I. The collective suppression of the $M3$ matrix element was rediscovered in a series of studies by Arima and colleagues [11,12], Zamick [8], and Brown *et al.* [13]. (These authors appeared to be unaware of Kurath's work.) These studies bring forth new approaches to the collective suppression and it is of interest to compare the two approaches. This will be done after the impulse approximation evaluation of the B_2 is presented. One might note that it is now possible to calculate the decay of ^{10}Be in a model space including all orbits up through the $0f, 1p$ shells. Thus the $\Delta Q = 2$ admixtures which presumably are the main cause for the collective suppression could be included in the diagonalization and an evaluation of the effect can be made nonperturbatively. The reason why this is not done is the great difficulty of obtaining meaningful wave functions for the $\Delta Q = \pm 2$ components in $(0+2)\hbar\omega$ calcula-

TABLE I. Summary of experimental information on the five second-forbidden unique transitions under consideration. The quantities in parentheses are the uncertainties in the last significant figure; those in square brackets are powers of ten. The branching ratio (B) is for the designated process as well as the final state in question. Q is the energy difference between the neutral atoms for the initial and final states in question.

Transition	Half-life (s)	B (%)	Q (keV)	$\log f_2$	B_2 (fm^4)
^{10}Be$(0^+) \xrightarrow{\beta^-} {}^{10}B(3_1^+)$	4.77(19)[13]	100.0	556.0(4)	14.11(7)	89.0(35)
^{22}Na$(3^+) \xrightarrow{\beta^+} {}^{22}Ne(0_1^+)$	8.233(5)[7]	0.056(14)	2842.0(5)	14.91(11)	8.0(20)
^{26}Al$(5^+) \xrightarrow{\beta^+} {}^{26}Mg(2_1^+)$	2.33(9)[13]	82.1(25)	2195.33(15)	15.73(2)	1.16(6)
^{26}Al$(5^+) \xrightarrow{EC} {}^{26}Mg(2_2^+)$	2.33(9)[13]	2.7(2)	1065.63(20)	14.61(3)	15.3(13)

tions as discussed, e.g., by Hoshino, Sagawa, and Arima [14] and by Millener, Hayes, and Strottman [15].

The shell-model calculations were done with the computer code OXBASH [16]. Results are given in Table II. The USD interaction referred to above was used for the $A = 22$ and 26 calculations. The result given for ^{10}Be$(\beta^-)^{10}$B in Table I uses a p-shell effective interaction due to Millener [17] similar to the classic Cohen-Kurath interaction [18] but resulting from a least-squares fit to 48 energy levels in $A = 10$–15 nuclei. This interaction gives a significantly better fit to these 48 levels than the Cohen-Kurath interaction, which results from a fit to energy levels in the larger range of $A = 6$–16 or 8–16 nuclei. The M_2 were calculated with Woods-Saxon (WS), Hartree-Fock (HF), and harmonic-oscillator (HO) radial wave functions. The Woods-Saxon radial wave functions utilized the parameters of Streets, Brown, and Hodgson [19] for $A = 22$ and 26 and parameters derived by Millener for the p shell from a careful consideration of the spectroscopy of $A = 9$–19 nuclei [20]. The Hartree-Fock wave functions were calculated using the Skyrme SK11 interaction of van Giai and Sagawa [21] as described by Brown, Bronk, and Hodgson [22] with the same orbit occupancies as in the WS calculation. The agreement with experiment for the known root-mean-square (rms) charge and matter radii is better for the HF than WS calculations in $A = 9$–10 and 22–26 nuclei and the HF results are adopted. However, the difference in the M_2 values between these two results was small. Also the HO results were close to those obtained in the HF and WS calculations as long as the oscillator parameter was chosen to reproduce the average rms charge radius of the initial and final nuclei [23].

The theoretical predictions of Table II for M_2 are consistently larger than the experimental values. The ratio in the last column is interpreted as a measure of the combined effects of "collective suppression" and quenching of the axial current, i.e., in an obvious notation, as

$$1 - \delta_{cs}\delta_A. \tag{3}$$

For the time being we neglect any in-medium effects on the spin operator and assume $\delta_A \sim 1$. "Collective suppression" has been a much-studied quantity in both beta decay [1] and $M3$ transitions [9, 11, 8, 12, 13]. It is realized that the shell-model calculations have inherent uncertainties that might not be too much smaller than the deviation of this measure of δ_{cs} from unity. Nevertheless, taken *in toto*, the comparison gives strong support for "collective suppression". We now consider these decays individually. Initially we consider the Nilsson model estimate of δ_{cs}. Then the approaches to the evaluation of this suppression factor of Refs. [11, 13] are considered.

^{10}Be decay. — The present p-shell calculation for ^{10}Be decay is quite similar to the previous one which was discussed in detail by WGT and which was also adopted in a study by Szybisz [24]. It has only one contributing orbital transition $\nu 0p_{3/2} \rightarrow \pi 0p_{3/2}$. M_2 is 89% of the value for pure $p_{3/2}^6$ initial and final states. The HF radial integral $\langle 0p_{3/2}|r^2|0p_{3/2}\rangle$ is 7.61 fm^2. The expected value of δ_{cs} is $\frac{1}{3}\delta$, where δ is the Nilsson deformation parameter and is ~0.48 in this case [1]. The resulting estimate for δ_{cs} of 0.16 is in essentially perfect agreement with the value of 0.15 from Table II.

TABLE II. Comparison of experiment and theory for the four decays of Table I. The theoretical values are for the p shell for ^{10}Be and the sd shell for the other three, i.e., "collective suppression" is not included.

Transition	M_2 (fm^2) Expt	Theory	$\frac{M_2(\text{expt})}{M_2(\text{theory})}$
^{10}Be$(0^+) \xrightarrow{\beta^-} {}^{10}B(3_1^+)$	9.43(19)	11.09	0.85
^{22}Na$(3^+) \xrightarrow{\beta^+} {}^{22}Ne(0_1^+)$	2.82(35)	4.07	0.69
^{26}Al$(5^+) \xrightarrow{\beta^+} {}^{26}Mg(2_1^+)$	1.08(3)	1.14	0.95
^{26}Al$(5^+) \xrightarrow{EC} {}^{26}Mg(2_2^+)$	3.91(17)	5.60	0.70

^{22}Na decay. — The *sd*-shell transitions have five possible orbital transitions, $0d_{5/2} \rightarrow 0d_{5/2}$, $0d_{3/2} \leftrightarrow 0d_{5/2}$, and $0d_{5/2} \leftrightarrow 1s_{1/2}$. The ^{22}Na transition is highly coherent with all but a weak $\pi 1s_{1/2} \rightarrow \nu 0d_{5/2}$ contribution in phase. Thus the result is insensitive to variations in the USD wave functions. The matrix element is dominated by the $\pi 0d_{5/2} \rightarrow \nu 0d_{5/2}$ contribution. As for ^{10}Be, the Nilsson model result for δ_{cs} is $\frac{1}{3}\delta$ and since $\delta \sim 32$ for ^{22}Na [1], $\delta_{cs} \sim 0.11$ is expected as opposed to 0.31 from Table II. The agreement is poor.

^{26}Al decay. — The weakness of the decay to the 2_1^+ state of ^{24}Mg is due to poor overlap of the initial and final states and cancellation between comparable $0d_{5/2} \rightarrow 0d_{5/2}$ and $0d_{5/2} \leftrightarrow 1s_{1/2}$ contributions [25]. The decay to the 2_2^+ state in contrast has highly coherent contributions from all five possible orbital transitions with only the very weak $0d_{3/2} \rightarrow 0d_{3/2}$ contribution out of phase. Thus this M_2 is also insensitive to variations in the USD wave functions. The transition is dominated by the $\pi 0d_{5/2} \rightarrow \nu 1s_{1/2}$ contribution. As discussed by WGT, the Nilsson model does not provide a quantitative estimate for δ_{cs} for these transitions. However, since the deformation is small, a small value of δ_{cs} is expected, say, ~ 0.05, again in poor agreement with Table II.

The relationship to the E2 isoscalar effective charge. — Brown *et al.* [13] showed that in the limit of a delta-function residual interaction, δ_{cs} can be related to the incremental effective isoscalar $E2$ charge δ_0 by

$$\delta_{cs} = -\left(\frac{\Delta E_0(2^+)}{\Delta E_1(3^+)}\right)\frac{\delta_0}{3} , \tag{4}$$

where $\Delta E_0(2^+)$ and $\Delta E_1(3^+)$ are the centroids of the excitation energies of the giant isoscalar $E2$ and giant isovector $M3$ resonances in the $T_Z = 0$ nucleus (parent or daughter). The ratio of energies in Eq. (4) is estimated to be between 0.5 and 1.0 with the lower value favored [13]. Since δ_0 in both the p shell and sd shell is found to be highly state-independent and closely equal to 0.70, this estimate gives $\delta_{cs} \sim 0.12$–0.23 (lower limit favored) for all p-shell and sd-shell second-forbidden unique β transitions and $M3$ transitions and moments. Horikawa *et al.* [11] incorporated the effects of $\Delta Q = \pm 2$ admixtures on $A =$ 17 and 41 closed-shell-plus-one nuclei perturbatively with a G-matrix interaction. They found results consistent with this δ-function interaction estimate.

Comparison of the methods of Kurath and Brown et al. — In his treatment of the ground states of ^{7}Li, ^{9}Be, ^{10}B, and ^{11}B Kurath found a strong state dependence for the effects of $\Delta Q = 2$ admixtures on the $M3$ moments with multiplicative corrections of 1.6, 0.83, 0.36, and 1.19 for these four nuclei, respectively. This very strong state dependence is at variance with the conclusion from Eq. (4) of a relatively constant suppression. Certainly, the role of $\Delta Q = 2$ admixtures on the operator of Eq. (2) cannot be said to be understood until this discrepency is resolved.

Isolation of the axial current quenching? — We assume, again for purposes of discussion, the essential correctness of Eq. (4). This estimate of Eq. (4) is compared to results for the two strong *sd*-shell transitions and the two values from the $A = 24$ and 34 analysis of $M3$ transitions of Brown *et al.* [13] as follows:

A	Process	Empirical estimate of δ_{cs}
22	β	0.31
24	$M3$	0.14
26	β	0.30
34	$M3$	0.12.

It is seen that the estimate based on Eq. (4) is in agreement with the empirical estimates for the two $M3$ cases but not with those for the two cases of β decay. A quite possible reason for this difference is the different behavior of the quenching of the spin operator in the two processes. In analogy with the well-studied comparison of Gamow-Teller and $M1$ transitions [26], it is expected that the quenching of the spin operator due to Δ-isobar currents and higher-order configuration mixing will be roughly the same in the two processes but that meson-exchange currents (MEC) will be strongly enhancing for $M3$ transitions — thus largely offsetting the effects of Δ isobars, etc. — and small for the β decays — thus leaving a net large quenching from Δ isobars, etc. Thus, it is suggested that the difference between the β and $M3$ estimates of δ_{cs} in the table above may well be due to a significant quenching of the axial current with $\delta_A \sim 0.18$ [see Eq. (3)]. If so it would be presumed that the ratio M_2(expt)/M_2(theory) of Table II errs on the high side for ^{10}Be decay.

In summary, the various perturbative estimates of the "collective suppression" of second-forbidden unique β decays and $M3$ transitions give results in qualitatitive agreement with those extracted by comparing experiment and theory. However, a microscopic understanding of this effect has not yet been achieved. The perturbative result of Horikawa *et al.* [11] for a single particle outside a closed shell agrees with the estimate of Eq. (4). An important, as-yet-unanswered question is: "Would a perturbative treatment for the actual decays and transitions in question show the state dependence suggested by the estimates for δ_{cs} listed above?" Horoshino, Sagawa, and Arima [14] have suggested an approximate way to satisfy the Hartree-Fock condition so as to produce realistic $\Delta Q = \pm 2$ admixtures in $(0{+}2)\hbar\omega$ calculations. An interesting study with their method would be to calculate ground-state wave functions for the $A = 7$–11 nuclei considered by Kurath and to see whether the shell model gave results in agreement with his Nilsson model approach.

Comparison of the second-forbidden beta decay results with previous studies of $M3$ transitions [13], with analogy to Gamow-Teller and $M1$ transitions, suggests the possibility of isolating MEC effects in second-forbidden β decay. Certainly further studies to better explore this possibility would be welcome. A necessary step in these studies is a reconciliation of the two methods of estimating the "collective suppression".

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[1] E. K. Warburton, G. T. Garvey, and I. S. Towner, Ann. Phys. (N.Y.) **57**, 174 (1970).
[2] W. Bambynek, H. Behrens, M. H. Chen, B. Crasemann, M. L. Fitzpatrick, K. W. D. Ledingham, H. Genz, M. Mutterer, and R. L. Intemann, Rev. Mod. Phys. **49**, 77 (1977); **49**, 961(E) (1977).
[3] F. Ajzenberg-Selove, Nucl. Phys. **A490**, 1 (1988).
[4] E. A. Samworth, E. K. Warburton, and G. A. P. Engelbertink, Phys. Rev. C **5**, 138 (1972).
[5] B. H. Wildenthal, Prog. Part. Nucl. Phys. **11**, 5 (1984).
[6] B. A. Brown and B. H. Wildenthal, Annu. Rev. Nucl. Sci. **38**, 29 (1988).
[7] E. K. Warburton, Phys. Rev. Lett. **1**, 68 (1958).
[8] L. Zamick, Phys. Rev. Lett. **40**, 384 (1978).
[9] D. Kurath, in *Nuclear and Particle Physics*, edited by B. Margolis and C. S. Lam (Benjamin, New York, 1968), pp. 199–272.
[10] S. G. Nillson, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. **29**, No. 16 (1955).
[11] Y. Horikawa, T. Hoshino, and A. Arima, Phys. Lett. **63B**, 134 (1976).
[12] A. Arima, Y. Horikawa, H. Hyuga, and T. Suzuki, Phys. Rev. Lett. **40**, 1001 (1978).
[13] B. A. Brown, S. E. Massen, W. Chung, B. H. Wildenthal, and T. A. Shibata, Phys. Rev. C **22**, 842 (1980).
[14] T. Hoshino, H. Sagawa, and A. Arima, Nucl. Phys. **A481**, 458 (1988).
[15] D. J. Millener, A. C. Hayes, and D. D. Strottman, Phys. Rev. C **45**, 473 (1992), this issue.
[16] B. A. Brown, A. Etchegoyen, W. D. M. Rae, and N. S. Godwin, OXBASH, 1984 (unpublished).
[17] D. J. Millener, private communication (1984).
[18] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).
[19] J. Streets, B. A. Brown, and P. E. Hodgson, J. Phys. G **8**, 839 (1982).
[20] D. J. Millener, private communication (1987).
[21] N. Van Giai and H. Sagawa, Phys. Lett. **106B**, 379 (1981).
[22] B. A. Brown, C. R. Bronk, and P. E. Hodgson, J. Phys. G **10**, 1683 (1984).

[23] Two reasons for making the effort to calculate the $\langle r^2 \rangle$ integrals with more sophisticated radial wave functions than harmonic oscillators are (1) the rms radii are not directly measured for all the nuclei in question, and (2) even if they were, the change in $\langle r^2 \rangle$ between parent and daughter nucleus can only be approximately accomodated with HO wave functions. In contrast the WS or HF treatments allow an extrapolation from, say, ^{23}Na to ^{22}Na, and different values of $\langle r^2 \rangle$ for parent and daughter.
[24] The ^{10}Be decay was considered by L. Szybisz [Z. Phys. A **273**, 277 (1975)]. Szybisz followed the shell-model approach of WGT exactly so that his comparison to experiment differed only in the updated values of the decay half-life and C_A/C_V, and in the use of a somewhat larger radial integral. The significant difference to the present study is that Szybisz regarded the suppression of the beta moment as due to renormalization of the axial vector coupling constant C_A alone.
[25] In the Nilsson model the 2_1^+ and 2_2^+ states of ^{26}Mg belong to $K = 0$ and 2 bands, respectively, while the ^{26}Al ground state is the bandhead of a $K = 5$ band. Thus the transition to the 2_1^+ state is ΔK forbidden and that to 2_2^+ is allowed which gives a rough explanation for the relative sizes of the M_2.

[26] A. Richter, A. Weiss, O. Haüsser, and B. A. Brown, Phys. Rev. Lett. **65**, 2519 (1990).