BRIEF REPORTS

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Soliton formation in nuclear matter

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The propagation of a density disturbance in nuclear fluid is considered for the case when the nuclear equation of state is derived from a Skyrme-type effective nucleon-nucleon potential. It is shown that the velocity-dependent terms of this potential are responsible for the possible formation of solitary waves in nuclear matter. These solitons are rarefaction waves and not compressional as previously suggested. Their amplitudes increase with increasing temperature, which makes them a possible mechanism for nuclear multifragmentation.

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The possibility of soliton formation in nuclear matter in the course of nucleus-nucleus collisions was first studied by Fowler *et al.* [1] and further investigated in a number of subsequent publications (e.g., [2–4]). According to these authors, when a light nucleus with velocity approximately equal to the speed of propagation of sound in nuclear matter is incident on a heavy nucleus, the projectile and target fuse into a compound nucleus in which density oscillations are set up. The propagation of these density disturbances in the nuclear medium is described in a fluid dynamical framework. In solving the hydrodynamical equations, they use an expansion of the velocity potential that plays the role of a heat function (enthalpy) in terms of the gradients of the density ρ:

$$\phi=c_1\rho'/\rho_0+c_2\nabla^2\rho'/\rho_0+\cdots , \tag{1}$$

where $\rho'=\rho-\rho_0$, ρ_0 is the equilibrium density, and c_1,c_2 are parameters. Glassgold *et al.* [5], who are probably the first to use this expansion in a nuclear fluid-mechanical problem, retain only the first-order term. In this approximation, the density disturbance propagates as a sound wave. Fowler *et al.* [1] have realized that the density oscillations in a compound nucleus formed in the heavy-ion reaction are not expected to be of first order of smallness. They go beyond the first-order term in the expansion of the velocity potential and take into account the density Laplacian term, implicitly assuming that the constant coefficient of this term is positive. They obtain a solitary solution for the hydrodynamic equations which describe a stable pulse of higher density that propagates in the longitudinal direction.

In this paper, we show that the equation of state of nuclear matter for nucleons interacting through a Skyrme-type effective interaction [6] leads to a negative coefficient for the Laplacian of the density which allows the formation of rarefaction rather than compressional solitary waves. However, this conclusion must not be tied to the Skyrme forces except that it can be used as an example for calculating the coefficients of the expansion (1). The negative sign is, in fact, much more general than the Skyrme model, as can be seen from the energy density formalism of Wilets [7]. The energy per unit volume has a term equal to $\rho(-\gamma\nabla^2\rho)$, which after partial integration is $\gamma|\nabla\rho|^2$, where γ is in general a function of ρ. The quantity γ has to be positive [and hence c_2 in (1) negative]; otherwise nuclear matter would be unstable against rapid oscillations.

The Skyrme interaction has been intensively used in nuclear structure calculations, especially since the work of Vautherin and Brink [8] who showed that it is possible in nuclear Hartree-Fock calculations to obtain the binding energies and other gross properties of nuclei to a satisfactory degree of precision with a relatively small number of parameters. Specifically, this potential is

$$\begin{aligned}V(\mathbf{r}_1,\mathbf{r}_2)=&\,t_0(1+x_0P_\sigma)\delta(\mathbf{r}_{12})\\&+\tfrac{1}{2}t_1[k^2\delta(\mathbf{r}_{12})+\delta(\mathbf{r}_{12})k^2]+t_2\mathbf{k}\delta(\mathbf{r}_{12})\mathbf{k}\\&+\tfrac{1}{6}(1+P_\sigma)t_3\rho(R)\delta(\mathbf{r}_{12}) ,\end{aligned} \tag{2}$$

where P_σ is a spin-exchange operator, $\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2$, $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$, and $\mathbf{k}=(\nabla_1-\nabla_2)/2i$, while x_0 and t_i are parameters adjusted by studying the ground-state properties of nuclei. For equal proton and neutron densities, and ignoring the spin- and isospin-dependent terms, the Hartree-Fock theory with the above nucleon-nucleon potential yields the following expression for the energy per nucleon as a function of the nuclear density [8–10]:

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$$E=\frac{3}{10}\frac{\hbar^2}{2m}\left[\frac{3\pi^2}{2}\right]^{3/2}\rho^{2/3}+\frac{3}{4}t_0\rho+\frac{3}{16}t_3\rho^2+\frac{3}{80}(3t_1+5t_2)\left[\frac{3\pi^2}{2}\right]^{2/3}\rho^{5/3}-\frac{1}{32}(9t_1-5t_2)\nabla^2\rho \ .$$

This relation can be simplified when the deviation from the equilibrium density is not large. One may then expand $E(\rho)$ around the equilibrium density ρ_0 and keep terms up to the second order to obtain

$$E(\rho)=E(\rho_0)+\frac{ma_0^2}{2\rho_0^2}(\rho-\rho_0)^2-\gamma\nabla^2\rho \ , \tag{3}$$

where a_0 is the speed of sound in nuclear matter at equilibrium density, which is defined by

$$ma_0^2=\rho_0^2\partial^2 E/\partial\rho_0^2 \ ,$$

and γ is a linear combination of coefficients of the velocity-dependent terms of the Skyrme potential:

$$\gamma=(9t_1-5t_2)/32 \ . \tag{4}$$

We shall now assume that nuclear matter can be considered as an invicid fluid and use the equation of classical hydrodynamics to describe the propagation of a disturbance of its density. For an invicid fluid, in the absence of external forces, the equations of hydrodynamics are the following: (i) the equation of continuity,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho v)=0 \ , \tag{5}$$

(ii) Euler's equation of motion,

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}=-\frac{1}{m\rho}\nabla p \ , \tag{6}$$

where p is the pressure, $\mathbf{v}$ the velocity field at a point in the fluid, and m the nucleon mass.

We restrict our consideration to the case of isentropic flow. In this case,

$$dp=\rho\, dh \ ,$$

where h is the enthalpy per nucleon, which is related to the energy per nucleon E by

$$h=E+\rho\,\partial E/\partial\rho \ . \tag{7}$$

Then, Eq. (6) reads

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}=-\frac{1}{m}\nabla h \ . \tag{8}$$

Substituting (3) into (7), we obtain, after neglecting $\partial(\nabla^2\rho)/\partial\rho$,

$$h=E(\rho_0)-\frac{ma_0^2}{2\rho_0^2}(3\rho^2+\rho_0^2-4\rho_0\rho)-\gamma\nabla^2\rho \ . \tag{9}$$

We see that the equation of state of nuclear matter involving nucleons interacting by a Skyrme-type potential $\phi=h/m$ yields essentially identical to expression (1), proposed by Glassgold *et al.* [5] and used in [1–4] when the term that contains the density Laplacian has a coefficient

$$c_2=-\gamma\rho_0/m \ .$$

It is important to note that the parameter γ takes positive values (making c_2 negative) for different choices [8–10] of the parameters t_1 and t_2. For example, $\gamma=169$, 126, 210, 256, and 98 MeV fm^5 for the SII-VI potentials, respectively. The coefficient γ is also positive when one takes into account the possible temperature dependence of the effective interaction as a consequence of the decrease of Pauli blocking as temperature increases. Indeed, the values of the coefficients of the velocity-dependent terms of the Skyrme potential

$$t_1=192.5(1+0.423T^2) \ ,$$

$$t_2=148.4(1-0.010T^2) \ ,$$

obtained by Cugnon *et al.* [11] from a comparison of the Brueckner g matrix calculated at different temperatures with the Skyrme potential, give

$$\gamma=31.0+23.1T^2 \ . \tag{10}$$

We now follow the treatment developed in [1] to obtain a solitary wave solution for Eqs. (5) and (8) for the case of a one-dimensional motion in the x direction. We assume that the deviation from equilibrium density is small enough to justify the use of perturbation theory. We expand the dimensionless density $\hat{\rho}=\rho/\rho_0$ and velocity $\hat{v}=v/a_0$ around their equilibrium values 0 and 1, respectively:

$$\hat{\rho}=1+\epsilon\rho_1+\epsilon^2\rho_2 \ , \quad \hat{v}=\epsilon v_1+\epsilon^2 v_2 \ . \tag{11}$$

We then define the new stretched coordinates

$$\xi=\epsilon^{1/2}(x-a_0t)/R \quad \text{and} \quad \tau=\epsilon^{3/2}a_0t/R \ ,$$

where R is the nuclear radius. Substituting (11) into (5) and (8) and equating the coefficients of powers 1 and 2 of ϵ to 0, we obtain a set of equations for ρ_1,ρ_2 and v_1,v_2. Solving them for ρ_1, we obtain the following Korteweg–de Vries equation [12]:

$$\frac{\partial\rho_1}{\partial\tau}+3\rho_1\frac{\partial\rho_1}{\partial\xi}-\frac{\gamma\rho_0}{2ma_0^2R_0^2}\frac{\partial^3\rho_1}{\partial\xi^3}=0 \ , \tag{12}$$

which has a solitary wave solution of the form

$$\rho_1=-\left[\frac{\gamma\rho_0}{2ma_0^2R_0^2}\right]^{1/3}\times b\,\mathrm{sech}^2\left\{\frac{\sqrt{b}}{2}\left[\left[\frac{\gamma\rho_0}{2ma_0^2R_0^2}\right]^{-1/3}\xi+b\tau\right]\right\} \ , \tag{13}$$

where b is an arbitrary positive constant.

From (12), we see that the soliton solution is possible only if $\gamma\neq 0$. We have defined γ as a linear combination of the Skyrme parameters t_1 and t_2 given by (4). Therefore, the soliton solution (13) owes its existence to the velocity dependence of the Skyrme potential, which modes-up finite-range in nucleon-nucleon forces.

We also observe that the solitary solution (13) has a negative sign as long as the parameter γ is positive, which follows for the realistic choices of the Skyrme parameters and can be seen from the energy-density formalism [7] as mentioned above. Thus, it describes a rarefaction wave. It is different from the compressional solitary waves considered in [1–4]. Similar rarefaction waves are met with in the study of wave propagation in liquid ^{4}He as observed in [3].

We finally observe that, according to (10) obtained for the temperature-dependent coefficients of the Skyrme potential [11], the quantity γ increases rapidly with the nuclear temperature. This suggests the importance of solitary rarefaction waves in hot nuclear matter.

Certainly, our conclusions have to be supported by more realistic three-dimensional calculation which takes into account nuclear viscosity and finite size. However, the results of Ref. [3] suggest that the three-dimensional effects lead to a damping of the soliton propagation, which has a limited effect because of the finite size of the nucleus. Moreover, the density variation in the nuclear surface region is shown in Ref. [4] to increase the amplitude of the solitary wave and decrease its width. Combining this result with the finding of the present work suggests that solitonlike cracks resulting from density oscillations amplify and become more defined at the nuclear surface. We thus conclude that the formation of rarefaction solitary waves is a possible mechanism for cracking hot nuclei formed in intermediate-energy nuclear reactions and their subsequent fragmentation [13].

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