PHYSICAL REVIEW C VOLUME 45, NUMBER 1 JANUARY 1992

# Elastic scattering of 5 GeV/$c$ pions from $^4$He

David M. DeMuth, Jr.* and Joseph S. Chalmers
*Physics Department, University of Louisville, Louisville, Kentucky 40292*
(Received 19 August 1991)

The elastic scattering of negative pions with a momentum of 5 GeV/$c$ from $^4$He is analyzed using a single-scattering optical potential in a relativistic integral scattering equation. The potential is computed using several nuclear and nucleon form factors as well as both phase-shift and exponential representations of the pion-nucleon amplitude. Predicted scattering observables are compared with experiment. The effect on the calculated observables due to an increase of effective nucleon size is also shown.

PACS number(s): 25.80.Dj, 24.10.Ht, 21.10.Ft, 27.10.+h

## INTRODUCTION

Bedeček *et al.* [1] (BCFH) obtained extensive, high-precision data for the elastic scattering of negative pions from $^4$He at 5 GeV/$c$. The four-momentum-transfer data fall in the range $0.12<|t|<0.90$ (GeV/$c$)$^2$ and exhibit a diffraction minimum and secondary maximum. In this experiment, the helium nucleus is probed by a strongly interacting particle at rather high momentum transfer while still close to the pion-nucleon energy shell. This permits investigation of nuclear and nucleon matter distributions as well as pion-nucleon interactions under conditions in which multiple-scattering calculations of low order are expected to be reliable.

In BCFH, the data were compared with a Glauber calculation using an averaged pion-nucleon amplitude together with Gaussian shapes for the nuclear and nucleon densities. As observed by those authors, the main features of the data were not well reproduced by the calculation.

In this work, we will compute the differential scattering cross section using a relativistic integral scattering equation with a single-scattering optical potential. The single-scattering potential will be constructed from various combinations of pion-nucleon amplitudes and nuclear form factors. The two-body amplitude will be represented by phase-shift representations as well as by the exponential form used in BCFH. The nuclear form factor will be obtained from nuclear charge densities and nucleon form factors as obtained from electron scattering. The effect of increasing the effective nucleon size on nuclear matter will also be investigated.

## ANALYSIS

The partial-wave amplitudes for pion-nuclear scattering are obtained from the integral scattering equation,

$$R_l=V_l-(2/\pi)P\int_0^\infty \frac{V_l(k,k')R_l(k',k_0)k'^2dk'}{E(k')-E(k_0)} , \quad (1)$$

where $R_l$ and $V_l$ are the partial-wave components of the reaction matrix and the optical potential, respectively. The single-scattering optical potential is given by

$$V(q)=A\tau(q)F(q) , \quad (2)$$

where $A$ is the nucleon number, $\tau(q)$ is the averaged pion-nucleon transition operator, and $F(q)$ is nuclear form factor.

The exponential form of the pion-pion amplitude, used in BCFH, is given by

$$f=(\sigma k/4\pi)(i+\alpha)\exp(bt/2) , \quad (3)$$

with $\sigma=27.6$ mb, $\alpha=-0.22$, and $b=7.0$ (GeV/$c$)$^{-2}$. Parameters for the phase-parameter representation are taken from Arndt [2].

$F(q)$ is obtained from the electron scattering data for the nucleus and nucleons by means of

$$F=F_{\text{nuclear}}/F_{\text{nucleon}} , \quad (4)$$

where the quantities on the right are the charge form factors of the nucleon and nucleus as determined by electron scattering.

The helium charge density was described by McCarthy, Sick, and Whitney [3] with a three-parameter Fermi (3pF)

$$\rho(r)=\rho_0[1+w(r^2/c^2)]/\{1+\exp[(r-c)/z]\} , \quad (5)$$

and a shape which leads to the modified Gaussian (MG) form factor,

$$F(q)=[1-(aq)^{12}]\exp(-b^2q^2) \quad (6)$$

with parameters as given in Ref. [3].

The nucleon form factor used to describe the proton in BCFH is the simple Gaussian

$$F_p(q)=\exp(-\langle r^2\rangle q^2/6) \quad (7)$$

with an rms radius of 0.87 fm.

In addition, we also used the simple dipole shape [4]

*Current address: Physics Dept., University of Minnesota, Minneapolis, MN 55455.

 

$$F_p(q)=1/(1+\langle r^2\rangle q^2/12)^2 \tag{8}$$

with an rms radius of 0.81 fm.

The neutron shape, which is more poorly known, may be represented as a point neutron with $F_n=0$, or more accurately as an extended charge, with the ansatz

$$F_n(q)=\beta q^2 F_p(q) , \tag{9}$$

where $\beta$ has been determined [5] to be 0.0199 fm$^2$.

The form for $F_{\text{nucleon}}$ in Eq. (4) is

$$F_{\text{nucleon}}=F_p+N/ZF_n , \tag{10}$$

in which $N$ and $Z$ are the neutron and proton numbers.

The partial-wave components of $V$ are obtained by Legendre projection and used in Eq. (1), which is numerically solved by a matrix inversion method [6]. The reaction matrix $R$ is constructed from the partial-wave components and used to obtain the pion-nuclear scattering matrix $T$ from which observables may be calculated. The partial-wave for $T$ is continued to an $l_{\max}$ of 40. Higher partial wave components of $T$ are accounted for by the observation that above $l_{\max}$, the partial-wave components of $T$ converge very strongly to those of $V$. This permits the summation of the partial waves to all orders rather the usual truncation. The resulting $T$ matrix is given by

$$T=4\pi\sum^{l_{\max}}(2l+1)(T_l-V_l)P_l(x)+V . \tag{11}$$

Coulomb effects are found to be negligible in the region of these data.

The expectation is that the single-scattering potential (2) obtained by combining the best two-body amplitude with the best nuclear density should lead to the best fit to the experimental data.

## RESULTS

In order to compare our calculation with that of BCFH, the scattering of 5 GeV pions from $^4$He, we calculate the scattering with a potential obtained from exponential two-body amplitude, as used in BCFH, and Gaussian nuclear and Gaussian nucleon form factors as used in Ref. [1]. As may be seen in Fig. 1(a), the results are similar to those obtained by BCFH, and do not fit the data well. Figure 1(b) shows the result obtained when the Gaussian nuclear form factor is replaced with the modified Gaussian [3] form factor. As expected from Liu and Shakin [7], the modified Gaussian shape for $^4$He fits the pion elastic scattering data much better than does the simple Gaussian. However, when the Gaussian nucleon form factor is replaced with the dipole nucleon form factor, as seen in Fig. 1(c), the quality of the fit is degraded. Since the dipole shape is expected to be superior to the Gaussian shape in representing the proton form factor, we look to the pion-nucleon amplitude as the source of the poor fit.

Figure 2 shows the effect of using a pion-nucleon amplitude expressed in terms of partial-wave amplitudes, in

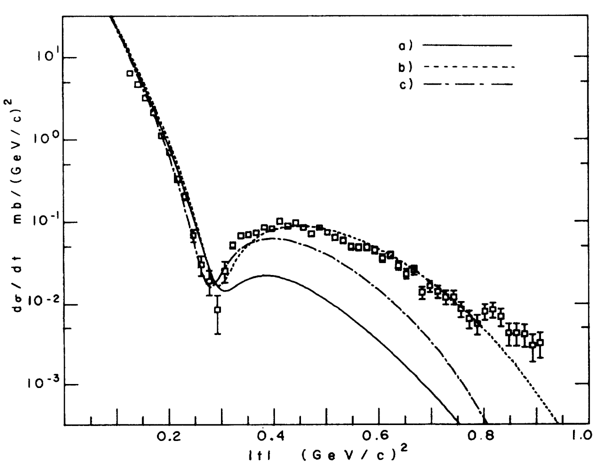


FIG. 1. Comparison of calculation using exponential two-body amplitude with data from Ref. [1]. (a) Gaussian nuclear form factor; Gaussian nucleon form factor. (b) Modified Gaussian, nuclear; Gaussian, nucleon. (c) Modified Gaussian, nuclear; dipole, nucleon.

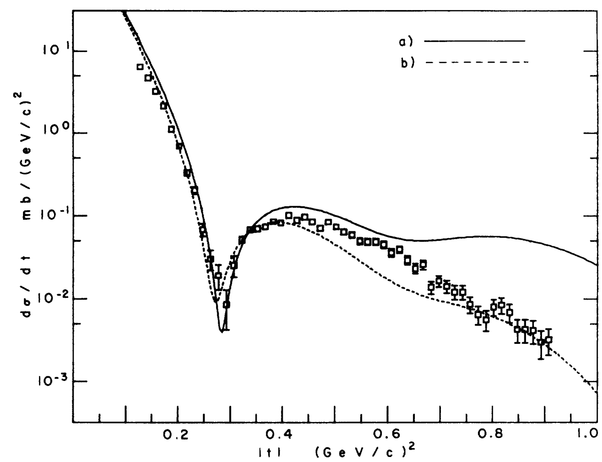


FIG. 2. Comparison of calculation using KH80 phase-parameter representation for the two-body amplitude, and the modified Gaussian nuclear form factor, with (a) Gaussian, and (b) dipole, nucleon form factors.

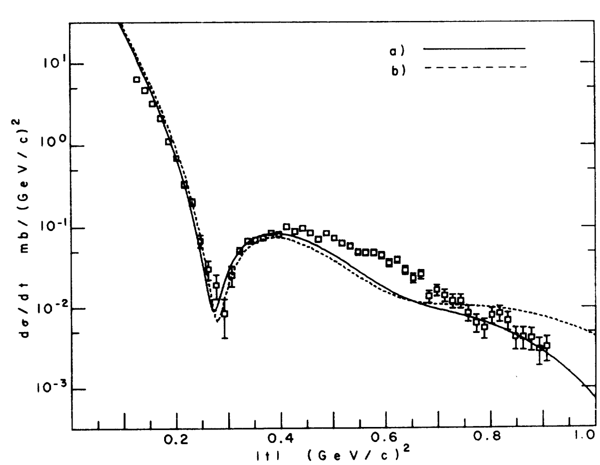


FIG. 3. Effect of inclusion of extended neutron. KH80 phase representation of two-body amplitude and modified Gaussian nuclear form factor. (a) point neutron; (b) extended neutron.

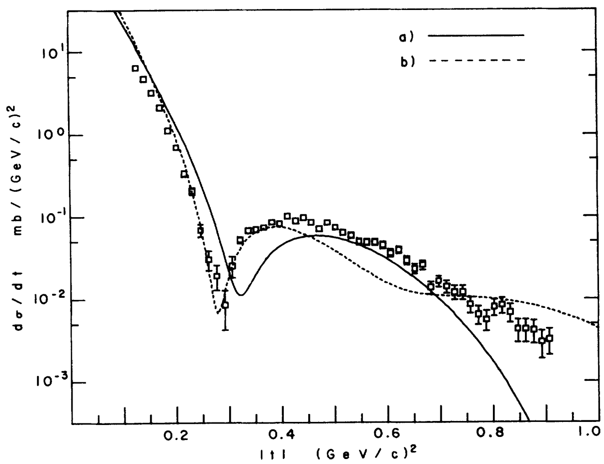


FIG. 4. Effect of varying phase-parameter representation of two-body amplitude. Modified Gaussian nuclear form factor, dipole nucleon form factor, and extended neutron. (a) KRLH phase parameters, (b) KA84 phase parameters.

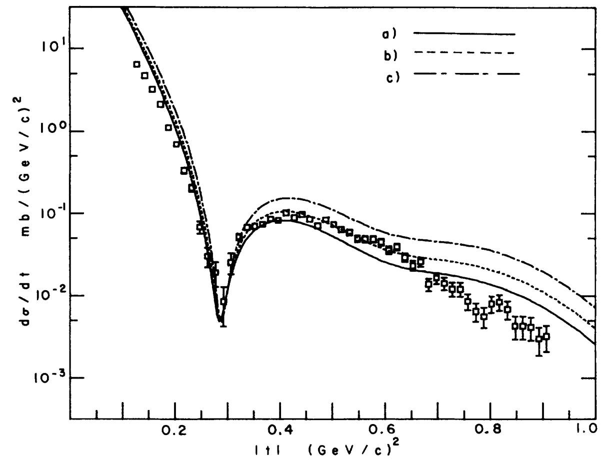


FIG. 5. Effect of increasing nucleon radius. KA84 phase parameters, modified Gaussian nuclear form factor, dipole nucleon form factor, extended neutron. Increase nucleon radius by (a) 10%, (b) 20%, (c) 30%.

this case, the KH80 phase-parameters set as tabulated in the SAID [2] program. The nuclear density is represented by the modified Gaussian shape, and the nucleon shape is given by (a) Gaussian and (b) dipole form factors. As may be seen, the dipole shape for the nucleon gives a fit to the data which is superior to that of the Gaussian at large momentum transfer. It is interesting to note that the calculated cross section shows an inflection at the location at which one would expect the second diffraction minimum, while the data do not. The calculated cross section obtained with the two-body amplitude given in terms of the partial-wave amplitudes matches the data better than one obtained from the exponential amplitude when each is combined with acceptable nuclear and nucleon form factors.

Figure 3 shows the effect of including the form factor of the neutron in obtaining the density of nuclear material. As expected, the effect does not become significant until large momentum transfers but beyond 0.7 (GeV/$c$), the effect is quite noticeable.

In Fig. 4, we compare additional phase-parameter sets from the SAID [2] program; (a) KRLH and (b) KA84. Both calculations are performed with the modified Gaussian nuclear density and the dipole nucleon form factor. Both calculations also contain the effects of the neutron charge density. Comparison of these plots together with that shown in Fig. 3(b) shows that the scattering obtained with the KA84 set is significantly better than that obtained with the KRLH set and is somewhat superior to that of KH80.

Finally, in Fig. 5, we examine the effect of arbitrarily increasing the nucleon rms radius in the dipole form factor by 10%, 20%, and 30%. This would simulate the effect of nuclear deconfinement, proposed [8] as a possible explanation of the European Muon Collaboration effect [9]. All calculations are carried out with the KA84 phase-parameter set and the modified Gaussian nuclear density, and include the effect of the extended neutron. As may be seen, the effect of even a 10% change in the nucleon rms radius is easily seen in these calculations.

## CONCLUSIONS

We find that when the nuclear and nucleon shapes are realistically represented, by the modified Gaussian and dipole form factors, respectively, an acceptable fit to the data is obtained when using a phase-shift representation for the pion-nucleon amplitude. Our best fit is obtained with the KA84 phase-parameter set. The exponential amplitude yields a satisfactory fit to the data only when used in conjunction with a Gaussian nucleon form factor, which is a poor representation of the proton shape. The effect of the extended size of neutron is significant at the higher end of the experimental momentum transfers.

The fact that the scattering observables computed from a potential obtained with realistic two-body information and reliable nuclear and nucleon shapes are in the best agreement with the data, leads to confidence in the calculational procedure. The very noticeable effect of increasing the nucleon radius suggests that analysis of pion scattering from light nuclei could yield information about the effects of nucleon substructure on pion nuclear observables. The suggestion of a second diffraction minimum in the observables calculated with partial-wave amplitudes, while the data show no such minimum, suggests that higher-order pion-nucleon scatterings need to be investigated.

[1] B. Bedeček, M. Chevallier, G. Fäldt, and A. Hallgren, Phys. Scr. **29**, 207 (1984).
[2] Scattering Analysis Interactive Dial-in, R. A. Arndt, J. M. Ford, and L. Roper, Phys. Rev. D **32**, 1085 (1985).
[3] J. S. McCarthy, I. Sick, and R. R. Whitney, Phys. Rev. C **15**, 1396 (1977).
[4] R. Hofstadter, F. Bumiller, and M. R. Yearian, Rev. Mod. Phys. **30**, 482 (1958).
[5] L. Koester *et al.*, Phys. Rev. Lett. **36**, 1021 (1976).
[6] J. S. Chalmers and A. M. Saperstein, Phys. Rev. **156**, 1099 (1967).
[7] L. C. Liu and C. M. Shakin, Phys. Rev. C **14**, 1885 (1976).
[8] R. L. Jaffe, Phys. Rev. Lett. **50**, 228 (1983); F. E. Close, R. G. Roberts, and G. G. Ross, Phys. Lett. **129B**, 341 (1983).
[9] European Muon Collaboration, J. J. Aubert *et al.*, Phys. Lett. **123B**, 275 (1983).