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Empirical model for three-quasiparticle states

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An empirical model is proposed for three-quasiparticle states (3*qp*) which is based on the experimental data of one- and two-quasiparticle states in neighboring nuclei. Calculations have been performed for 3*qp* states in ^{163}Er, ^{175}Lu, and ^{177}Lu to check the validity of the model. The ordering of the levels in a given 3*qp* quadruplet is correctly reproduced in all nuclei. Two strong rules are proposed for (*nnp*) or (*ppn*) configurations according to which the highest-lying member of a given quadruplet always has a spin combination in which the spins of like particles are parallel while those of unlike particles are antiparallel and the state having all three spins in the same direction cannot lie lowest in energy. For (*nnn*) or (*ppp*) configurations, however, the state having all three spins in the same direction will be the highest in energy.

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At excitation energies ≥ 1 MeV, which is approximately the energy gap 2Δ in the rare-earth region, a proton or a neutron pair can break up and form a three-quasiparticle state (3qp) in an odd-A nucleus. Two kinds of 3qp states are possible: those having all the three particles of the same kind (*nnn* and *ppp*) and others having a combination of two kinds of particles (*npp* and *nnp*). In a deformed nucleus, coupling of the three quasiparticles in Nilsson states having K values, say, K_1, K_2, and K_3, leads to four possible intrinsic states with resultant $K=|K_1 \pm K_2 \pm K_3|$. These four intrinsic states split up due to the residual interaction between the three neutrons and/or protons. A careful study of this problem should therefore lead to a better understanding of the residual interaction between neutrons and protons. Since the 3qp states occur at rather high excitation energies, considerable admixture with vibrational states is possible. Experimental identification of 3qp states therefore often conflicts with a vibrational assignment or even a single-particle assignment. Reliable data on 3qp states are therefore scarce. On the basis of theoretical calculations, Soloviev [1] had suggested that specific spin-flip beta transitions may feed 3qp states of the (*nnp*) and (*ppn*) type. Experimental data on 3qp states have recently been presented by Jain *et al.* [2] and, in specific context of fast beta decays, by Sood and Sheline [3]. It may be noted that a complete quadruplet of 3qp states is yet to be observed in any nucleus. It therefore remains an open challenge for experimentalists to reliably identify 3qp multiplets in odd-mass nuclei and, in particular, to observe examples of all the four couplings of the same 3qp configuration.

Earlier theoretical calculations [4,5] based on the residual interaction were only partially successful in explaining the splittings and orderings of these states. In the present paper, we propose an empirical model to describe the 3qp multiplets. This model is basically an extension of the model used by Hoff *et al.* [6] for 2qp states in odd-odd nuclei. The model is based on the following assumptions.

(i) The excitation energy of a given configuration is the sum of each of the odd-nucleon excitations and can be taken from the experimental data for neighboring odd-mass nuclei, i.e.,

$$E_I = E_{\rm qp} + (\hbar^2/2\mathcal{J})[I(I+1) - K^2 + \delta_{K,1/2}a(-1)^{(I+1/2)}(I+\tfrac{1}{2})] . \quad (1)$$

(ii) The effective moment of inertia for three-quasiparticle states can be expressed as

$$\mathcal{J}_{\rm 3qp} = \mathcal{J}_{\text{even-even}} + \sum_{i=1}^{3} \Delta\mathcal{J}_{(i)}$$

and

$$\Delta\mathcal{J} = \mathcal{J} - \mathcal{J}_{\text{even}-\text{even}} , \quad (2)$$

where $\mathcal{J}$ is the moment of inertia for the relevant rotational band in the same nucleus, and $\Delta\mathcal{J}$ is the increment to even-even core moment of inertia (i.e., $\mathcal{J}_{\text{even}-\text{even}}$) due to unpaired nucleon. Therefore, the effective moment of inertia for 3qp states becomes

$$\mathcal{J}_{\rm 3qp} = \mathcal{J}_{(1)} + \mathcal{J}_{(2)} + \mathcal{J}_{(3)} - 2\mathcal{J}_{\text{even}-\text{even}} . \quad (3)$$

(iii) The contribution of the effective residual interaction can be taken as a sum of the neutron-proton/proton-proton/neutron-neutron interaction energies from 2qp states in the neighboring nuclei. The interaction energy gives the splitting between the triplet and the singlet members of a 2qp doublet. Therefore, it can be expressed in terms of splitting energy and the Newby shift.

Based on these assumptions, the excitation energies of the members of a 3qp quadruplet can be expressed as the sum of the energy required to break a nucleon pair plus the sum of the three odd-nucleon excitations plus a term for the rotational energy and the residual interaction between the unpaired nucleon, i.e.,

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$$
\begin{aligned}
E(I)=&E_{\text{pairing}}+E_{\text{qp}}+E_{\text{rot}}+E_{\text{int}}\\
=&2\Delta+E^{(1)}_{\text{qp}}+E^{(2)}_{\text{qp}}+E^{(3)}_{\text{qp}}+(\hbar^2/2\mathcal{J}_{3\text{qp}})[I(I+1)-K^2]\\
&-\{\sigma_1[\tfrac{1}{2}-\delta_{\Sigma_{(1,2)},0}]E_{\text{split}_{(1,2)}}+\sigma_2[\tfrac{1}{2}-\delta_{\Sigma_{(2,3)},0}]E_{\text{split}_{(2,3)}}+\sigma_3[\tfrac{1}{2}-\delta_{\Sigma_{(1,3)},0}]E_{\text{split}_{(1,3)}}\}\\
&-\{\sigma_1[\delta_{K_{(1,2)},0}(-1)^I E_{N_{(1,2)}}\Pi_{(1,2)}]+\sigma_2[\delta_{K_{(2,3)},0}(-1)^I E_{N_{(2,3)}}\Pi_{(2,3)}]+\sigma_3[\delta_{K_{(1,3)},0}(-1)^I E_{N_{(1,3)}}\Pi_{(1,3)}]\}\ ,
\end{aligned}
\tag{4}
$$

where Δ is the pairing gap energy, E_{qp} the quasiparticle energy of the odd nucleon, K the band quantum number, Σ the intrinsic spin, and $\sigma_n=(-1)$ for like particles and $(+1)$ for unlike particles. E_{split} is the splitting energy between the singlet and triplet states, E_N is the Newby shift, and Π is the parity for the 2qp configuration as specified in expression (4). The input to this expression is the data from neighboring odd-A, odd-odd, and even-even nuclei. It is a well-known fact that the triplet state lies lower in energy in the doubly odd 2qp doublet and there is no exception to this rule. Similarly, the singlet state lies lower in energy in a doubly even 2qp doublet and the experimental data available so far support this fact. If we further take into account the empirical fact that the splitting energy in a doubly odd 2qp doublet is of the order of 100 keV and that in a doubly even 2qp doublet is nearly 400–500 keV, the energy expression (4) implies with near certainty that the highest-lying spin combination in a 3qp quadruplet will be the one where the spins of like particles are parallel and unlike particles are antiparallel. Also, the state whose spin combination has all three particles aligned in the same direction (↑↑↑) will never be the lowest in energy in a 3qp quadruplet. However, if all three qp's are of the same kind (*nnn* or *ppp*), the state having a spin combination (↑↑↑) will always be the highest in energy in a 3qp quadruplet.

Validity of this model has been tested for three nuclei, i.e., ^{163}Er, ^{175}Lu, and ^{177}Lu, where sufficient experimental data on 2qp states in the neighboring nuclei are available.

^{163}Er. The only known 3qp configuration in this nucleus is $\{\frac{1}{2}^+[411]_p\otimes\frac{7}{2}^-[523]_p\otimes\frac{5}{2}^-[523]_n\}$. This configuration gives rise to four levels with K values of $\frac{13}{2}$, $\frac{11}{2}$, $\frac{3}{2}$, and $\frac{1}{2}$ all having positive parity. Out of these, only two levels ($\frac{3}{2}^+$ and $\frac{1}{2}^+$) have been observed by Gnatovich *et al.* [5] in the β decay of ^{163}Tm. The $\frac{3}{2}^+$ and $\frac{1}{2}^+$ states are observed at 1538.4 and 1802.0 keV, respectively, with low logft values (≈ 5). These low logft values rule out the assignment of a one-quasiparticle configuration to these levels.

Calculations were made by using expression (4). The intrinsic excitation energies for neutron and protons were taken from the experimental data for ^{163}Er and an average of the experimental values for ^{161}Ho and ^{165}Tm, respectively [2]. The splitting energies were estimated from the experimental data for ^{168}Tm (Ref. [7]), ^{164}Ho (Ref. [8]), and ^{166}Er (Ref. [9]). The results of our calculation are shown in Fig. 1. All energies are normalized to the $\frac{3}{2}^+$ band in order to study their relative position. The thin and the thick arrows indicate the spin directions of the like and unlike particles in the figure.

Besides reproducing the orderings of the known levels, we find that the highest-lying energy state has a spin combination in which the spins of the like particles are in the same direction (a triplet) but that of the unlike particle is in the opposite direction as suggested earlier. The theoretical calculations [5] based on the model developed by Pyatov and Chernyshev [4] suggest the ordering of the levels to be $\frac{3}{2}^+$ (lowest), $\frac{13}{2}^+$, $\frac{1}{2}^+$, and $\frac{11}{2}^+$ (highest), which is identical with the ordering obtained from our empirical model.

^{175}Lu. Four three-quasiparticle states associated with

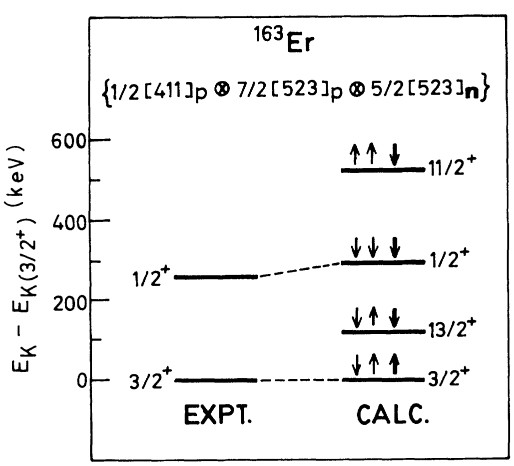

FIG. 1. Energies of the members of the three-quasiparticle configuration observed in ^{163}Er. Experimental energies are plotted on the left and the calculated energies are on the right. Thin arrows show the spin of like particles and the thick arrows the unlike particle.

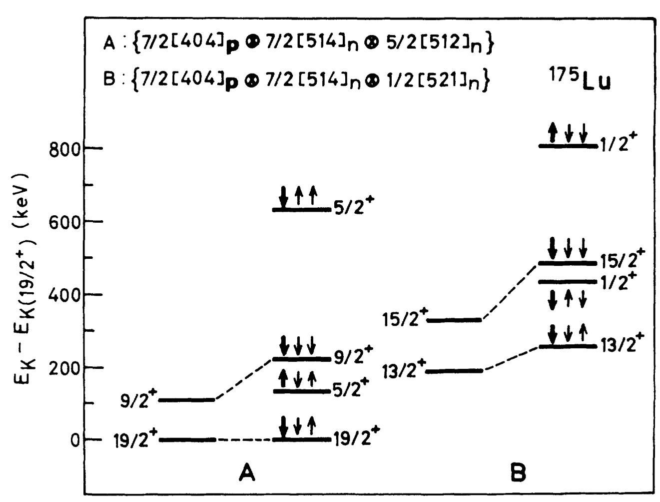

FIG. 2. Similar to Fig. 1 but for ^{175}Lu.

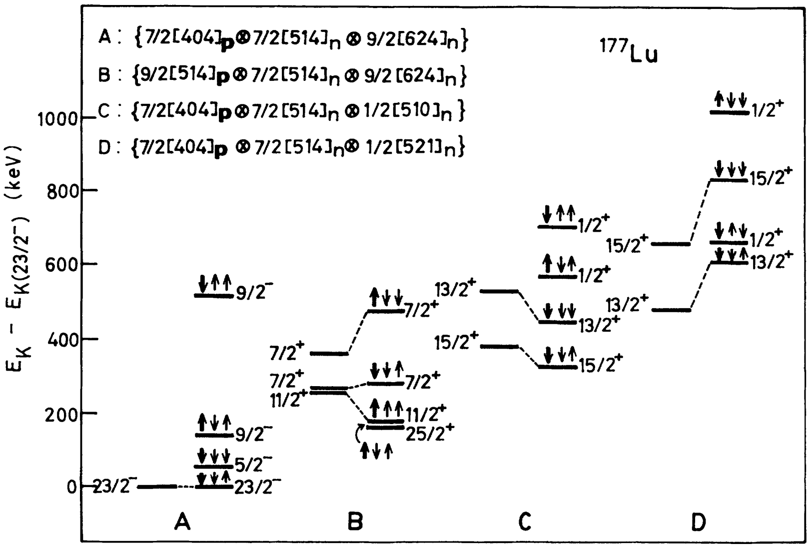

FIG. 3. Similar to Fig. 1 but for ^{177}Lu.

the $\{\frac{7}{2}^+[404]_p \otimes \frac{7}{2}^-[514]_p \otimes \frac{5}{2}^-[512]_n\}$ and $\{\frac{7}{2}^+[404]_p \otimes \frac{7}{2}^-[514]_p \otimes \frac{1}{2}^-[521]_n\}$ configurations were strongly populated in the (d,t) reaction [10]. Two levels each are known in both the configurations with tentative assignments.

Calculations from our model were performed for these configurations. The input data on intrinsic excitation energies for neutrons and protons were taken from the experimental data for ^{175}Lu, ^{173}Yb, and ^{177}Hf (Ref. [2]). The values of splitting energies were estimated from the experimental data for ^{174,176}Lu (Refs. [7,11]) and ^{176}Hf (Ref. [9]). The Newby shift value was taken to be -68.0 keV in the case of the $\{\frac{7}{2}[404]_p \otimes \frac{7}{2}[514]_n\}$ two-quasiparticle configuration as deduced from the data of ^{176}Lu.

The results are shown in Fig. 2 along with the experimental data. All values are normalized to the $\frac{19}{2}^+$ bandhead of the $\{\frac{7}{2}^+[404]_p \otimes \frac{7}{2}^-[514]_p \otimes \frac{5}{2}^-[512]_n\}$ configuration. Again, the highest-lying level is seen to possess the same spin combination as in the case of ^{163}Er. Also, the ordering of the levels are correctly reproduced for all the four known members of the two 3qp configurations.

^{177}Lu. We have considered all four configurations, i.e., A: $\{\frac{7}{2}[404]_p \otimes \frac{7}{2}[514]_n \otimes \frac{9}{2}[624]_n\}$; B: $\{\frac{9}{2}[514]_p \otimes \frac{7}{2}[514]_n \otimes \frac{9}{2}[624]_n\}$; C: $\{\frac{7}{2}[404]_p \otimes \frac{7}{2}[514]_n \otimes \frac{1}{2}[510]_n\}$; and D: $\{\frac{7}{2}[404]_p \otimes \frac{7}{2}[514]_n \otimes \frac{1}{2}[521]_n\}$. Configuration B has three of the four members experimentally known. In addition, configurations A, C, and D have one, two, and two members experimentally known, respectively. In all, eight members of the four 3qp configurations have been observed. Therefore, this nucleus is a good example to test the validity of our model.

The data on intrinsic 1qp states for protons and neutrons have been taken from the nucleus ^{177}Lu and average values for the nuclei ^{175}Yb and ^{179}Hf, respectively [2]. Also, the splitting energies ($E_{\rm split}$) and Newby shifts (E_N) have been taken from ^{176}Lu (Ref. [7]), ^{180}Ta (Ref. [12]), and ^{176,178}Hf (Ref. [9]). The results obtained from our calculations are shown in Fig. 3. All energies are normalized to the energy of $\frac{23}{2}^-$ member of the configuration A. We find that order of the levels for different multiplets is correctly reproduced. Also, the highest-energy state in each of the multiplets has the same spin combination as that in ^{163}Er and ^{175}Lu.

We find that our model is able to correctly reproduce the orderings and the splittings of three-quasiparticle states for different quadruplets. We also propose two strong rules on the basis of our analysis: Firstly, the highest-energy state always has a spin combination in which the spins of like particles are parallel and that of unlike particles are antiparallel ($\uparrow\uparrow\downarrow$) and, secondly, the state having all three spins in the same direction ($\uparrow\uparrow\uparrow$) cannot lie lowest in energy in a 3qp quadruplet. As a result of these two rules, the lowest-lying state must have a spin combination which is either ($\uparrow\downarrow\uparrow$) or ($\downarrow\uparrow\uparrow$). However, the only strong rule applicable when all three particles are of the same kind puts the spin combination ($\uparrow\uparrow\uparrow$) highest in a 3qp quadruplet. The model which is presented here may prove very useful to the experimentalists in their search for new 3qp states and the associated rotational bands.

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