Phenomenological approach to high-spin quantum fields based on the Weinberg formalism

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(Received 9 January 1992)

We investigate the possibility of constructing a covariant phenomenology of high-spin particles based on the work of Weinberg. We put forth a simple algebraic method for investigating the dispersion relations $E=E(\mathbf{p},m)$ satisfied by the solutions to the free-particle Weinberg equations. We find that these equations support solutions which do not satisfy $E^2=\pm\sqrt{\mathbf{p}^2+m^2}$. This motivates a proposal to build the phenomenology without explicit reference to a wave equation. We demonstrate the practicality of this approach by providing techniques for constructing the covariant spinors, free-particle field operators, and propagators which could then be used to construct a covariant perturbation theory.

PACS number(s): 11.10.Qr, 11.30.Cp

At new medium-energy accelerators, such as CEBAF, PILAC at LAMPF, and KAON, it becomes possible to study the excited states of the hadron in great detail. An interesting possibility would be to produce an excited hadron in the nuclear medium via a quasielastic scattering and then to use the final-state interaction to study the interaction of the hadron with the nuclear medium. Since these hadrons can have a high spin, this reaction and others require a consistent Lorentz covariant phenomenology of high-spin particles. At the moment, there does not exist a totally satisfactory formalism within which to build this phenomenology. While, in principle, the Bargmann and Wigner formalism [1] is general enough to treat any spin, its calculational usefulness does not easily extend beyond the spin-$\frac{3}{2}$ formalism of Rarita and Schwinger [2,3]. Even for spin $\frac{3}{2}$ there seem to be several misconceptions [4] and fundamental difficulties [5,6] associated with this formalism.

An alternate approach to the quantum field theory of high-spin particles was proposed by Weinberg [7]. The equations that he proposed have found some use in phenomenologies [8–10]. We here investigate the Weinberg equations at the free-particle level and find that they support solutions for which $E\neq\pm\sqrt{\mathbf{p}^2+m^2}$, a situation we call kinematic acausality. This makes it difficult to utilize these equations in any phenomenology. One would have to somehow ensure that the couplings did not mix in any of the unwanted solutions. However, the logic put forth by Weinberg [7,11] suggests an alternate approach. A phenomenology can be constructed from the ingredients of a covariant perturbation theory. These ingredients, covariant spinors, field operators, propagators, and model interactions, can all be constructed without reference to any wave equation. Elsewhere [12] we develop this approach in detail, providing explicit expressions for the spinors up through $j=2$ and presenting some possible model interactions.

The first point to be made here is that the free-particle Weinberg equations support acausal solutions. We here put forward a very simple and totally algebraic way of demonstrating this. We begin with the general form of the spin-j Weinberg [11] equations,

$$(\gamma_{\{\mu\}}p^{[\mu]}-m^{2j}I)\psi(\mathbf{p})=0\ , \tag{1}$$

where $\{\mu\}$ is a set of $2j$ Lorentz indices and $p^{[\mu]}$ is a product of $2j$ contravariant energy-momentum vectors. For $j=\frac{1}{2}$, Eq. (1) is the usual Dirac equation for spin-$\frac{1}{2}$ particles. For one-time- and three-space-dimensional spacetime there are $[4(4+1)\cdots(4+2j-1)]/(2j)!$ γ matrices of dimension $2(2j+1)\times 2(2j+1)$ that are symmetric in the $2j$ Lorentz indices $\{\mu\}$. To make this more concrete, we provide [12] the explicit expression for the case $j=2$. For this example case, the operator in Eq. (1) is, in the chiral representation,

$$[\gamma^{\rm ch}_{\{\mu\}}p^{[\mu]}]_{j=2}=\gamma^{\rm ch}_{\mu\nu\lambda\rho}p^\mu p^\nu p^\lambda p^\rho$$
$$=\begin{pmatrix} 0 & \begin{array}{c}\{\eta_{\mu\nu}p^\mu p^\nu\eta_{\lambda\rho}p^\lambda p^\rho \\ +2(\mathbf{J}\cdot\mathbf{p})[(\mathbf{J}\cdot\mathbf{p})+p^0]\eta_{\mu\nu}p^\mu p^\nu \\ +\frac{2}{3}(\mathbf{J}\cdot\mathbf{p})[(\mathbf{J}\cdot\mathbf{p})^2-\mathbf{p}^2][(\mathbf{J}\cdot\mathbf{p})+2p^0]\}\end{array} \\ \begin{array}{c}\{\eta_{\mu\nu}p^\mu p^\nu\eta_{\lambda\rho}p^\lambda p^\rho \\ +2(\mathbf{J}\cdot\mathbf{p})[(\mathbf{J}\cdot\mathbf{p})-p^0]\eta_{\mu\nu}p^\mu p^\nu \\ +\frac{2}{3}(\mathbf{J}\cdot\mathbf{p})[(\mathbf{J}\cdot\mathbf{p})^2-\mathbf{p}^2][(\mathbf{J}\cdot\mathbf{p})-2p^0]\}\end{array} & 0 \end{pmatrix}. \tag{2}$$

To study the solutions to these equations, we note that Eq. (1) can be viewed as a set of $2(2j+1)$ homogenous coupled equations. Consequently, the existence of solutions to the equation requires

$$\det(\gamma_{\{\mu\}}p^{[\mu]}-m^{2j}I)=0 \ . \tag{3}$$

This equation is a simple algebraic method for obtaining the dispersion relation for the energy $E=E(\mathbf{p},m)$ that the solutions to the equation must satisfy. For a given j this equation is a $2j[2(2j+1)]$th order equation in E, (40th order in E for $j=2$), which we then factor in order to obtain the dispersion relation for E. We have done this for $j=\frac{1}{2}$, 1, $\frac{3}{2}$, and 2, and the results are given in Table I for the spin-2 Weinberg equation. We find that of the $2j[2(2j+1)]$ solutions there are $N_C(j)=2(2j+1)$ solutions that satisfy $E=\pm\sqrt{\mathbf{p}^2+m^2}$, i.e., the "kinematically causal" solutions. We also find $N_A(j)=2j[2(2j+1)]-2(2j+1)=2(2j-1)(2j+1)$ solutions that do *not* satisfy the correct dispersion relation, the "kinematically acausal" solutions. The solutions given in Table I are consistent with the observation made in Ref. [13] that this type of equation is an operator expression of $(p_\mu p^\mu)^{2j}=m^{4j}$. Some of the solutions imply propagation faster than the speed of light; we label these solutions "tachyonic." Only for $j=\frac{1}{2}$, the Dirac equation, are there no acausal solutions.

The work of Weinberg, however, does not crucially depend on the existence of a wave equation. As he noted, the covariant spinors can be constructed directly from the precepts of quantum mechanics and Poincaré covariance. We note [12] that a generalization to arbitrary spin of the approach used in [14] for spin $\frac{1}{2}$ provides a practical way of constructing the needed spinors. The particle/antiparticle spinors, $u_{+j}(\mathbf{p}),\ldots,u_{-j}(\mathbf{p})$; $v_{+j}(\mathbf{p}),\ldots,v_{-j}(\mathbf{p})$, are obtained by the action of the boost operator (given here in the generalized canonical representation)

$$M_{\rm ca}(\mathbf{p})=\begin{bmatrix}\cosh(\mathbf{J}\cdot\boldsymbol{\varphi}) & \sinh(\mathbf{J}\cdot\boldsymbol{\varphi})\\ \sinh(\mathbf{J}\cdot\boldsymbol{\varphi}) & \cosh(\mathbf{J}\cdot\boldsymbol{\varphi})\end{bmatrix} , \tag{4}$$

on the $2(2j+1)$ rest spinors in the form of the $2(2j+1)$-dimensional columns

$$u_{+j}(\mathbf{0})=\begin{bmatrix}N(j)\\0\\0\\\vdots\\0\end{bmatrix},\ldots,\ v_{-j}(\mathbf{0})=\begin{bmatrix}0\\0\\\vdots\\0\\N(j)\end{bmatrix} . \tag{5}$$

Here $N(j)$ is a normalization factor which can be chosen conveniently so that in the $m\to 0$ limit (i) the "rest spinors" vanish and (ii) the $m\to 0$ covariant spinors have a nonsingular norm. The simplest choice that satisfies these criteria is $N(j)=m^j$. In Eq. (4) the parameter $\boldsymbol{\varphi}$ is defined by $\cosh\varphi=E/m$, $\sinh\varphi=|\mathbf{p}|/m$, $\hat{\boldsymbol{\varphi}}=\mathbf{p}/|\mathbf{p}|$.

TABLE I. Dispersion relations $E=E(\mathbf{p},m)$ associated with the spin-2 Weinberg equation.

Solutions	Multiplicity	Interpretation
$E=+\sqrt{\mathbf{p}^2+m^2}$	5	Causal
$E=-\sqrt{\mathbf{p}^2+m^2}$	5	Causal
$E=+\sqrt{\mathbf{p}^2-m^2}$	5	Acausal, tachyonic
$E=-\sqrt{\mathbf{p}^2-m^2}$	5	Acausal, tachyonic
$E=+\sqrt{\mathbf{p}^2+im^2}$	5	Acausal
$E=-\sqrt{\mathbf{p}^2+im^2}$	5	Acausal
$E=+\sqrt{\mathbf{p}^2-im^2}$	5	Acausal
$E=-\sqrt{\mathbf{p}^2-im^2}$	5	Acausal

Here $\mathbf{p}$ is the three-momentum of the particle, and $\mathbf{J}$ are the usual $(2j+1)\times(2j+1)$ angular momentum operators with J_z diagonal.

From the covariant spinors we can construct the field operators $\Psi^j(x)$. The arguments that are used for the spin-$\frac{1}{2}$ case apply equally well here and give

$$\Psi^j(x)=\sum_{\sigma=-j}^{+j}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2\omega_{\mathbf{p}}}\times[u_\sigma(\mathbf{p})a(\mathbf{p},\sigma)\exp(-ipx)+v_\sigma(\mathbf{p})b^\dagger(\mathbf{p},\sigma)\exp(+ipx)] , \tag{6}$$

with $\omega_{\mathbf{p}}=\sqrt{m^2+\mathbf{p}^2}$ and $\overline{\Psi}^j(x)\equiv\Psi^{j\dagger}(x)\gamma_{00\cdots 0}$. The construction of the Feynman-Dyson propagator from the $(j,0)\oplus(0,j)$ matter fields is conceptually straightforward. The propagator *cannot* be identified with the Green function

$$[(i)^{2j}\gamma_{\{\mu\}}\partial^{[\mu]}-m^{2j}I]G^{(j,0)\oplus(0,j)}(x-x')=\delta^4(x-x') \ . \tag{7}$$

This operator would propagate the acausal as well as the causal solutions. However, the propagator needed to construct a perturbation theory is properly defined as the vacuum expectation value of time-ordered field operators,

$$\langle x|\mathcal{S}^j_{\rm FD}|y\rangle\equiv\langle\ |T[\Psi^j(x)\overline{\Psi}^j(y)]|\ \rangle \ . \tag{8}$$

Of all the relativistic wave equations of the Weinberg form, Eq. (1), it is only for the Dirac $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ spinors that the Green function, Eq. (7), and the vacuum expectation value of time-ordered field operators, Eq. (8), are the same operator. Using $\{a_\sigma(\mathbf{p}),a^\dagger(\mathbf{p}')\}=(2\pi)^3 2\omega_{\mathbf{p}}\delta_{\sigma\sigma'}\delta(\mathbf{p}-\mathbf{p}')$ for fermions and a similar relation for bosons (with anticommutator replaced by commutator), we obtain the configuration space Feynman-Dyson propagator for arbitrary spin:

$$\langle x|\mathcal{S}^j_{\rm FD}|y\rangle=\sum_{\sigma=-j}^{+j}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2\omega_{\mathbf{p}}}[u_\sigma(\mathbf{p})\overline{u}_\sigma(\mathbf{p})e^{-ip(x-y)}\theta(x^0-y^0)+\epsilon v_\sigma(\mathbf{p})\overline{v}_\sigma(\mathbf{p})e^{+ip(x-y)}\theta(y^0-x^0)] , \tag{9}$$

with $\epsilon=+1$ for bosons and -1 for fermions. The Feynman-Dyson propagator in momentum space is

$$\langle k'|S^j_{\mathrm{FD}}|k\rangle=\int\frac{d^4x}{(2\pi)^3}\frac{d^4y}{(2\pi)^3}e^{ik'x}e^{-iky}\langle x|\mathcal{S}^j_{\mathrm{FD}}|y\rangle$$
$$=-\frac{i\delta^{(4)}(k'-k)}{(2\pi)^2 2\omega_{\mathbf{k}}}\sum_{\sigma=-j}^{+j}\left[\frac{u_\sigma(\mathbf{k})\overline{u}_\sigma(\mathbf{k})}{k_0+i\eta-E(\mathbf{k})}-\epsilon\frac{v_\sigma(-\mathbf{k})\overline{v}_\sigma(-\mathbf{k})}{k_0-i\eta+E(\mathbf{k})}\right]. \tag{10}$$

Note that only the physically acceptable causal solutions of the Weinberg equations enter these propagators. Thus, the explicit construction of the covariant spinors by using the boost operator given in Eq. (4) provides the field operators, Eq. (6), and the propagators, Eqs. (9) and (10).

We have demonstrated that the Weinberg equations for free particles of arbitrary spin support solutions with incorrect dispersion relations. A phenomenology that utilizes these equations would have to somehow avoid any admixtures of these unphysical solutions. However, as was pointed out by Weinberg [11], the wave equation plays a secondary role in this approach to high-spin fields. We are thus led to the proposition that the phenomenology be defined by the perturbation theory which can be constructed from the free-particle fields, the Feynman-Dyson propagator, and model interactions. Here, we provide a method of constructing the spinors, fields, and propagators that are required in this approach.

This work was supported, in part, by the National Science Foundation.

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