Closed form theory of elastic breakup and applications to astrophysically relevant heavy ion reactions

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(Received 31 January 1992)

We investigate the breakup process in light heavy ion reactions. With the help of a separable approximation proposed in a previous paper and using closed forms for radial integrals, we obtain simple expressions for the breakup cross section. The theory is applied to the reaction $^{16}O+^{28}Si\rightarrow\alpha+^{12}C+^{28}Si$, and the results are shown to agree with the experimental data.

PACS number(s): 25.70.Mn, 95.30.Cq

I. INTRODUCTION

Breakup reactions constitute an important piece of the total reaction cross section of intermediate energy heavy ion collisions. It is now quite common to separate the breakup process into two physically distinct processes: elastic breakup, and inclusive inelastic breakup (also called breakup fusion). Whereas semi-analytical treatments of the latter process have been extensively developed [1], only numerical recipes based on either post or prior distorted wave Born approximation (DWBA) description are available in the literature. Elastic breakup reactions are of great importance since it represents an appreciable part of the total break-up cross section. Furthermore, it can be used as a vehicle through which fusion cross section of light heavy ions at extremely low energies, of great relevance to astrophysical research, can be extracted through detailed balance arguments [2,3]. The above fact clearly calls for more theoretical analysis of the elastic breakup process in order to render the numerics simpler.

The aim of the present paper is to develop a semi-analytical treatment of the elastic breakup cross section. In this endeavor, we rely heavily on the work of Frahn [4], who considered heavy ion inelastic and transfer reactions at low energies. The approximations he employed are, in fact, more valid at the higher energies we consider here since they are, based to some extent, on the eikonal method. We apply our theory to some recent data on the reaction $^{16}O+^{28}Si\rightarrow\alpha+^{12}C+^{28}Si$ at $4A$ MeV and find reasonable agreement.

The paper is organized as follows. In Sec. II the DWBA amplitude for the elastic breakup process is fully analyzed and its multipolarity content is studied. In Sec. III closed form expressions for the nuclear and Coulomb parts of the amplitude are derived. In Sec. IV the formalism is applied to the reaction $^{16}O+^{28}Si\rightarrow\alpha+^{12}C+^{28}Si$ at $4A$ MeV and the resulting breakup cross section is compared to the recent data of Carlin *et al.* [5]. Finally, in Sec. V several concluding remarks are made.

II. THEORY OF DIRECT BREAKUP

Let us consider the breakup of a projectile a in a collision with a target A. Assuming that the projectile is formed by fragments b and x, this process can be represented as

$$a+A\rightarrow b+x+A\ . \tag{1}$$

Following Ref. [7], we adopt the prior representation and use the DWBA. The corresponding T matrix can then be written

$$T_{f\leftarrow i}=\langle\chi_f^{(-)}(\mathbf{R})\phi_f(\mathbf{r})|\Delta V(\mathbf{r},\mathbf{R})|\chi_i^{(+)}(\mathbf{R})\phi_i(\mathbf{r})\rangle\ , \tag{2}$$

where $\mathbf{r}$ is the vector between the fragments b and x and $\mathbf{R}$ is the projectile-target relative coordinate. The coupling interaction is [7]

$$\Delta V(\mathbf{r},\mathbf{R})=U_{xA}(\mathbf{r}_{xA})+U_{bA}(\mathbf{r}_{bA})-U_{aA}(\mathbf{R})\ . \tag{3}$$

Above, $U_{aA}(\mathbf{R})$ is the projectile-target optical potential and $U_{xA}(\mathbf{r}_{xA})$ and $U_{bA}(\mathbf{r}_{bA})$ are fragment-target optical potentials. The coordinates $\mathbf{r}_{xA}$ and $\mathbf{r}_{bA}$ are given by

$$\mathbf{r}_{xA}=\mathbf{R}-\frac{m_b}{m_a}\mathbf{r},\quad \mathbf{r}_{bA}=\mathbf{R}+\frac{m_x}{m_a}\mathbf{r}\ , \tag{4}$$

where m_i stands for the mass of particle i. In Eq. (2), $\chi_i^{(+)}$ ($\chi_f^{(-)}$) is the distorted wave with outgoing (ingoing) boundary condition, $\phi_i(\mathbf{r})$ is the bound ground state of the two fragments in the projectile, and $\phi_f(\mathbf{r})$ is the final state with the fragments in the continuum.

It is convenient to separate nuclear and Coulomb parts in the interactions and approximate the nuclear part of the optical potentials as [6,9]

$$U_{jA}(\mathbf{r}_{jA})=U_j^N e^{-|\mathbf{r}_{jA}|^2/\alpha_j^2} . \tag{5}$$

It is well known that Gaussian forms fail in giving a good overall description of heavy ion optical potentials. However, it can accurately reproduce the tail region, which dominates the breakup process. Assuming that the optical potentials have the same α_j, i.e., $\alpha_b=\alpha_x=\alpha_a\equiv\alpha$, the nuclear part of the coupling interaction takes the form

$$\Delta V^N(\mathbf{r},\mathbf{R})=U_{aA}^N(\mathbf{R})\left\{u_b \exp\left[-\left[\frac{r^2 m_b^2}{m_a^2\alpha^2}+\frac{2m_b\mathbf{r}\cdot\mathbf{R}}{m_a\alpha^2}\right]\right]+u_x \exp\left[-\left[\frac{r^2 m_x^2}{m_a^2\alpha^2}+\frac{2m_x\mathbf{r}\cdot\mathbf{R}}{m_a\alpha^2}\right]\right]-1\right\} , \tag{6}$$

where $u_b=U_{bA}^N/U_{aA}^N$ and $u_x=U_{xA}^N/U_{aA}^N$. Carrying out partial wave expansions in Eq. (6), the nuclear part of the T matrix [Eq. (2)] takes the factorized form

$$T_{f\leftarrow i}^N=\sum_{LM}(T_{\rm el}^N)_{LM}(T_{\rm exc}^N)_{LM} , \tag{7}$$

where the elastic part is

$$(T_{\rm el}^N)_{LM}=4\pi\langle\chi_f^{(-)}(\mathbf{R})|Y_{LM}^*(\hat{R})U_{aA}^N(R)|\chi_i^{(+)}(\mathbf{R})\rangle , \tag{8}$$

and the excitation part is

$$(T_{\rm exc}^N)_{LM}=\sum_{j=b,x}\langle\phi_f(\mathbf{r})|Y_{LM}(\hat{r})v_{jL}^N(r)|\phi_i(\mathbf{r})\rangle . \tag{9}$$

We have introduced the quantity

$$v_{jL}^N(r)=(-1)^{n_j}e^{-(m_j r/m_a\alpha)^2}j_L\left[i\frac{2m_j r R_T}{m_a\alpha^2}\right] , \tag{10}$$

where $n_j=L$ for $j=x$; $n_j=0$ for $j=b$, and j_L are spherical Bessel functions. In the imaginary argument of j_L, we made the approximation of replacing the coordinate R by the sum $R_T=R_a+R_A$. This approximation greatly simplifies the calculation of form factors and it is reasonable because the breakup dominantly occurs at closest approach in grazing collisions.

For heavy collision partners, the Coulomb contribution to the breakup process is also important. In this case the multipole expansion is straight-forward and we obtain [7]

$$\Delta V^C(r,\mathbf{R})=4\pi Z_A e^2\sum_{L\neq 0,M}\left\{\left[Z_b\left[-\frac{m_x}{m_a}\right]^L+Z_x\left[\frac{m_b}{m_a}\right]^L\right]\times\frac{r^L Y_{LM}^*(\hat{R})Y_{LM}(\hat{r})}{(2L+1)R^{L+1}}\right\} . \tag{11}$$

Using this expression in Eq. (2), we obtain a separable form for T^C, analogous to Eq. (7), with the elastic factor

$$(T_{\rm el}^C)_{LM}=\frac{4\pi Z_A e^2}{2L+1}\langle\chi_f^{(-)}(\mathbf{R})|\frac{Y_{LM}^*(\hat{R})}{R^{L+1}}|\chi_i^{(+)}(\mathbf{R})\rangle , \tag{12}$$

and the excitation factor

$$(T_{\rm exc}^C)_{LM}=\left[Z_b\left[-\frac{m_x}{m_a}\right]^L+Z_x\left[\frac{m_b}{m_a}\right]^L\right]\times\langle\phi_f(\mathbf{r})|r^L Y_{LM}(\hat{r})|\phi_i(\mathbf{r})\rangle . \tag{13}$$

A. Calculation of $(T_{\rm el})_{LM}$

To evaluate the nuclear and the Coulomb parts of $(T_{\rm el})_{LM}$, we expand $\chi_i^{(+)}(\mathbf{R})$ and $\chi_f^{(-)}(\mathbf{R})$ in partial waves. We obtain

$$(T_{\rm el}^N)_{LM}=\frac{(4\pi)^2}{k_i k_f}\sum_{\lambda,l,m}i^{\lambda-l}\sqrt{(2\lambda+1)(2l+1)}\langle l\lambda 00|l\lambda 00\rangle\times\langle l\lambda m0|l\lambda LM\rangle Y_{LM}^*(\hat{\mathbf{k}}_f)\times I_{l\lambda L}^N(k_f,k_i) , \tag{14}$$

where $\mathbf{k}_i$ ($\mathbf{k}_f$) is the initial (final) wave vector of the projectile-target relative motion and $I_{l\lambda L}^N(k_f,k_i)$ are the radial integrals

$$I_{l\lambda L}^N(k_f,k_i)=\int_0^\infty u_\lambda^*(k_f R)U_{aA}^N(R)u_l(k_i R)dR . \tag{15}$$

Above, we have introduced the radial wave functions $u(kR)$, which corresponds to those of Ref. [8] divided by kR.

The Coulomb part of $T_{\rm el}$ is given by Eq. (14), with the replacement of the nuclear radial integral by its Coulomb counterpart

$$I_{l\lambda L}^C(k_f,k_i)=Z_A e^2\int_0^\infty\frac{u_\lambda^*(k_f R)u_l(k_i R)}{(2L+1)R^{L+1}}dR . \tag{16}$$

In Ref. [7], the radial integrals were calculated with the eikonal approximation. In the present work we will not follow this procedure so that the validity of our results can be extended to low energies and large angles.

B. Calculation of $(T_{\rm exc})_{LM}$

To simplify the calculation we approximate the initial and the final internal states of the projectile as

$$\phi_i(\mathbf{r})\equiv\phi_i(r)=N_i e^{-r^2/t^2} \tag{17}$$

and

$$\phi_f(\mathbf{r})=e^{-i\mathbf{q}_f\cdot\mathbf{r}}-\sqrt{8}\exp\left[-\frac{q_f^2t^2}{4}+\frac{r^2}{t^2}\right]. \qquad (18)$$

In Eq. (17), t is a parameter chosen as to give the right projectile size and N_i is the appropriate normalization constant. The second term on the right-hand side (RHS) of Eq. (18) has been included in order that ϕ_f be orthogonal to ϕ_i. If one expands ϕ_f in partial waves and uses the expansion in Eqs. (9) and (13), we obtain

$$(T^N_{\rm exc})_{LM}=N_i\sum_{j=b,x}\left[(\pi/\gamma_j)^{3/2}(-1)^{n_j+L}Y_{LM}(\hat{q}_f)j_L\left[\frac{m_jR_T}{m_a\gamma_j\alpha^2}q\right]\times\exp\left[\frac{4m_j^2R_T^2-m_a^2\alpha^2q^2}{4\gamma_jm_a^2\alpha^2}\right]-\sqrt{2}\delta_{L,0}(\pi/\Gamma_j)^{3/2}\exp\left[\frac{4m_j^2R_T^2-m_a^2\Gamma_j\alpha^4t^2q^2}{4m_a^2\Gamma_j\alpha^4}\right]\right], \qquad (19)$$

with

$$\gamma_j=\frac{m_j^2}{m_a^2\alpha^2}+\frac{1}{t^2},\quad \Gamma_j=\gamma_j+\frac{1}{t^2}, \qquad (20)$$

and the excitation part becomes

$$(T^C_{\rm exc})_{LM}=4\sqrt{2\pi}i^L\left[Z_b\left[-\frac{m_x}{m_a}\right]^L+Z_x\left[\frac{m_b}{m_a}\right]^L\right]\times\left[\frac{t^2}{2}\right]^{3/2}e^{-t^2q_f^2/4}(q_f)^LY_{LM}(\hat{q}_f). \qquad (21)$$

C. The dominant terms in the multipole expansion

One expects that the nuclear part of the T matrix will be dominated by the monopole term. To check this point we have studied the contribution of the lowest multipoles in the breakup reaction $^{16}O+^{28}Si\to{}^{12}C+\alpha+^{28}Si$. For this purpose we use the orientation average or $(T^C_{\rm exc})_{LM}$, obtained through the replacement of $Y_{LM}(\hat{q}_f)$ by $1/\sqrt{4\pi}$.

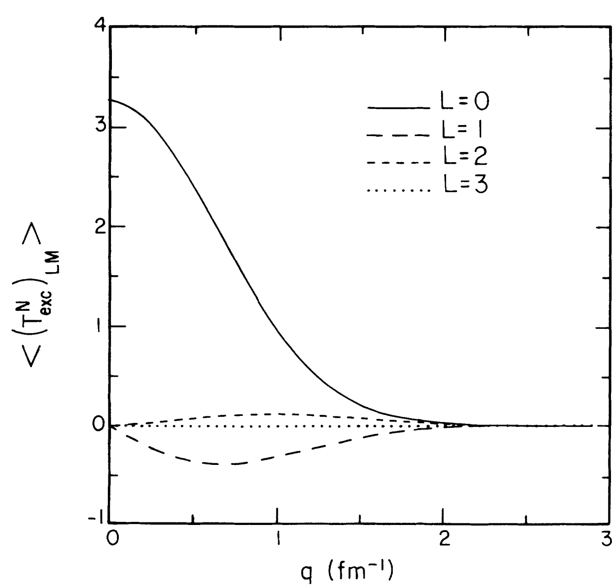

FIG. 1. The excitation T matrix, $T^N_{\rm exc}(q)$, Eq. (9) for $L=0$, $L=1$, $L=2$, and $L=3$ for the system $^{16}O+^{28}Si\to\alpha+^{12}C+^{28}Si$, at $4A$ MeV. The optical potential used is a typical strong absorption potential (see Ref. [1]).

We followed this procedure for the multipolarities $L=0$, 1, 2, and 3 and the results (assuming $M=0$) are presented in Fig. 1, as a function of q_f. As the monopole term clearly dominates, we will henceforth neglect the contributions of higher multipoles.

The Coulomb part should be dominated by the dipole term. However, according to Eq. (21), the odd multipolarity contributions vanish when the two fragments have the same charge to mass ratio. As this frequently occurs, and it will be the case in the breakup reaction considered in Sec. IV, we will restrict the sum over multipolarities to the quadrupole term.

III. CLOSED FORM FOR THE BREAKUP T MATRIX

In this section we approximate the radial integrals of Eqs. (15) and (16) by closed expressions. This way one eliminates the need of optical model codes in the calculation of the breakup cross section.

A. The radial integrals of the nuclear potential

Taking for the nuclear part of the elastic S matrix a typical strong absorption parametrization [10]

$$S_l^N(k)=\frac{1}{1+\exp[(l-\bar{l})/\bar{\Delta}+i\bar{\alpha}]} \qquad (22)$$

where $\bar{l}$ and $\bar{\Delta}$ are the parameters

$$\bar{l}=l_0\sqrt{E-E_0},\quad \bar{\Delta}=\Delta_0\left[\frac{1-E/E_0}{1-E/2E_0}\right], \qquad (23)$$

and E is the collision energy in the c.m. frame. The constant phase $\bar{\alpha}$ and the parameters E_0, Δ_0, and l_0 depend on the collision partners. For the $^{16}O+^{28}Si$ system, which we study in Sec. IV, these parameters are [11]

$$E_0=17.5\ \text{MeV},\quad l_0=6.033,\quad \Delta_0=0.054,\quad\text{and}\quad \bar{\alpha}=1.5. \qquad (24)$$

A straightforward generalization of the closed form of Frahn [4] leads to the approximation

$$I_l^N(k_i,k_f)=\frac{d}{dl}[S_l^N(k_i)S_l^N(k_f)]^{1/2}, \qquad (25)$$

which can be easily calculated from Eq. (22).

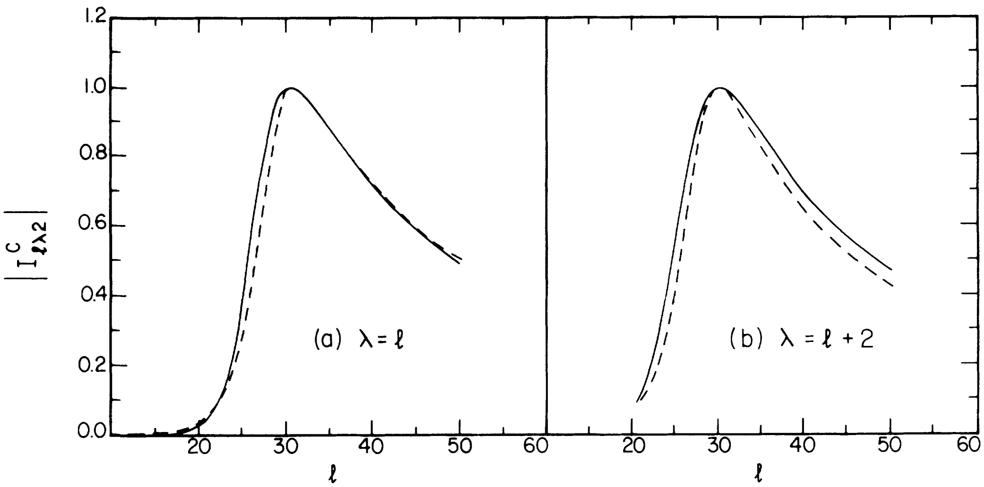

FIG. 2. The Coulomb radial integral for $L=2$ vs l. Solid curve is the exact result obtained from the radial wave functions, Eq. (16). The dashed curve represents Eq. (26). See text for details.

B. The radial integrals of the Coulomb potential

To evaluate the radial integrals of the quadrupole term, which dominates the Coulomb contribution to the T matrix, we use the approximation [12]

$$I^C_{l\lambda L}(k_i,k_f)=C_0[S_l^N(k_i)S_\lambda^N(k_f)]^{1/2}\mathcal{J}_{l\lambda L}(k_i,k_f)\ , \tag{26}$$

where $\mathcal{J}$ is the pure Coulomb radial integral, defined as in Eq. (16) with the replacement of the optical radial wave functions u_l and u_λ by the corresponding regular Coulomb functions F_l and F_λ, as defined in Ref. [13]. C_0 is an undetermined multiplication factor. If we neglect the variation of the collision energy, taking $k_i=k_f\equiv k$, the quadrupole radial integrals can be calculated analytically [13]. The result is

$$\mathcal{J}_{l\lambda 2}=\frac{2\eta^2}{3|(l+1+i\eta)(l+2+i\eta)|}\ , \tag{27}$$

for $l=\lambda\pm 2$, and

$$\mathcal{J}_{l\lambda 2}=\frac{2\eta^2}{l(l+1)(l+2)}\left[\eta\pi[\coth(\eta\pi)-1]+2(l+1)-2\eta^2\sum_{n=0}^{\infty}\frac{1}{n^2+\eta^2}(2l+1)\right]\ , \tag{28}$$

for $l=\lambda$. In Eqs. (27) and (28), η is the Sommerfeld parameter.

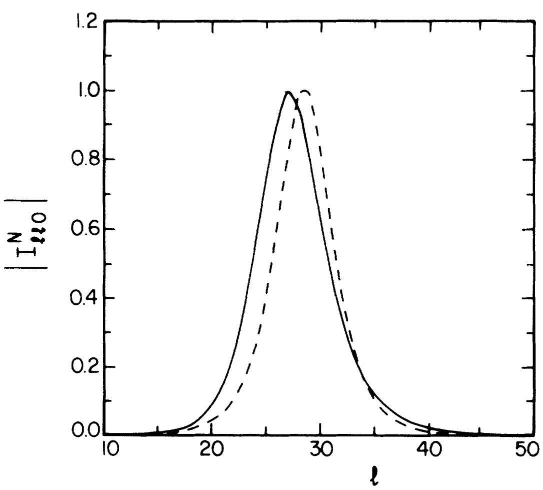

FIG. 3. The nuclear radial integral vs l. The solid curve represents Eq. (15), whereas the dashed curve represents Eq. (25). An overall normalization is involved. See text for details.

IV. APPLICATION: THE $^{16}O+^{28}Si\rightarrow\alpha+^{12}C+^{28}Si$ REACTION

In this section we apply the formalism described in the previous sections to the elastic breakup of ^{16}O in the field of ^{28}Si, at $4A$ MeV laboratory energy, which has recently been measured by Carlin *et al.* [5]. Before presenting the comparison of our theory and the data, we first access the validity of the approximations employed in deriving our closed form expressions. In Fig. 2 we show the Coulomb

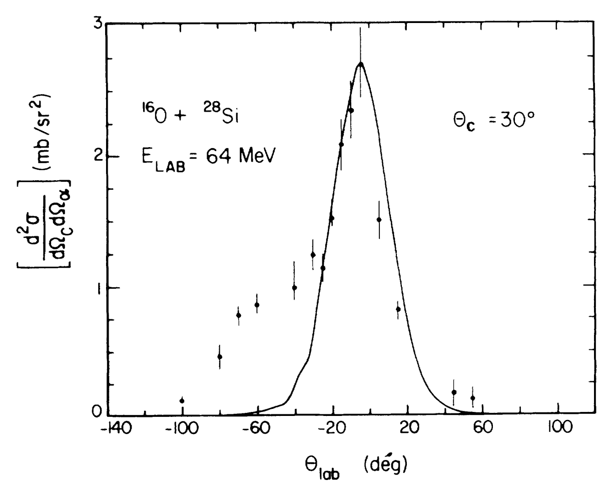

FIG. 4. Comparison between our theory (solid curve) and the experimental data of Ref. [5].

radial integrals defined in Eq. (16) (solid line) and the corresponding closed form approximations (dashed line) described in Eqs. (26)–(28). Both the cases $\lambda=l$ [Fig. 2(a)] and $\lambda=l+2$ [Fig. 2(b)] are considered. It is clear that the closed form expressions for $I^{C}_{l\lambda 2}$ accounts very well for the shape of the exact expression. It misses the width by at most 5%. We should remind the reader that in the reaction considered here the Coulomb contribution is negligible. However, we present the above comparison for completeness.

Next, we consider the nuclear radial integrals. In Fig. 3 we show the comparison between the exact one (solid line), defined in Eq. (15), and the corresponding closed form of Eq. (25) (dashed line). Again the agreement is reasonable. The position of the peak is slightly shifted (by $\Delta l \simeq 2$).

In Fig. 4 we show the result of the calculation with the full closed amplitude. The data points are taken from Ref. [5]. An overall normalization factor was employed. The agreement with the major peak is quite reasonable. The large tail at negative angles corresponds to a different process, namely incomplete fusion [14].

V. CONCLUSIONS

In this paper, closed form expressions for the elastic breakup amplitude of heavy ions are derived and analyzed. These expressions were found to represent the exact amplitudes very well. Comparison of the theory with the data for the reaction the $^{16}O+^{28}Si \rightarrow \alpha+^{12}C+^{28}Si$, at $4A$ MeV shows reasonable agreement. Application to other systems as well as the extraction of fusion cross section from the inverse reaction will be made in the future.

[1] M. S. Hussein and K. McVoy, Nucl. Phys. **A445**, 124 (1985).

[2] G. Baur, C. A. Bertulani, and H. Rebel, Nucl. Phys. **A459**, 188 (1986); in *Proceedings of the International Symposium on Weak and Electromagnetic Interactions in Nuclei,* Heidelberg, 1986, edited by H. V. Klapdor (Springer-Verlag, Berlin, 1986), p. 980.

[3] T. Motobayashi *et al.,* Phys. Lett. B **264**, 259 (1991); J. Kiener *et al.,* Phys. Rev. C **44**, 2195 (1991); J. Hesselbarth and K. T. Knöpfle, Phys. Rev. Lett. **67**, 2773 (1991); S. B. Gazes *et al., ibid.* **68**, 150 (1992).

[4] W. E. Frahn, Phys. Rev. C **21**, 1870 (1980).

[5] N. Carlin *et al.,* Phys. Rev. C **40**, 91 (1989).

[6] P. J. Karol, Phys. Rev. C **11**, 1203 (1975).

[7] C. A. Bertulani and M. S. Hussein, Nucl. Phys. **A524**, 306 (1991).

[8] D. F. Jackson, *Nuclear Reactions* (Methuen, London, 1970).

[9] J. O. Rasmussen, L. F. Canto, and X. J. Qiu, Phys. Rev. C **33**, 2033 (1986).

[10] J. A. MacIntyre, S. D. Baker, and T. L. Watts, Phys. Rev. **116**, 1212 (1959).

[11] W. E. Frahn, M. S. Hussein, L. F. Canto, and R. Donangelo, Nucl. Phys. **A369**, 122 (1981).

[12] M. J. Sopkovitch, Nuovo Cimento **26**, 186 (1962).

[13] K. Alder and A. Winther, *Coulomb Excitation* (Academic, New York, 1966).

[14] N. Added *et al.,* Nucl. Phys. A (to be published).