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Equivalent local potentials for energy dependent nonlocal interactions

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Various equivalent local potentials and their Perey factors are discussed. The Wronskian and inversion-type equivalent local potentials for the energy dependent nonlocal interaction induced by the coupling of the elastic to the nonelastic channels are investigated numerically. Their Perey factors are found to be closer to unity than those associated with exchange-type nonlocal potentials.

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I. INTRODUCTION

Equivalent local potentials (ELP) have been introduced to provide a simple and practical *local* description of the elastic scattering in systems for which the original interaction is nonlocal. This nonlocal interaction may be an energy independent potential like the Perey-Buck potential [1] or the microscopic potential of the resonating group method (RGM) [2]. The nonlocality of the latter arises basically from exchange effects due to the action of the Pauli principle, and is often phenomenologically represented by a Perey-Buck potential.

Another type of nonlocal interaction is that arising from the dynamical coupling of the elastic to nonelastic channels. Here the elastic channel wave function can, in principle, also be calculated from a single-channel Schrödinger equation containing the energy dependent, nonlocal generalized optical potential [3,4]. This may then again be regarded as a nonlocal potential for which an ELP can be constructed. In this case, however, it is more convenient to relate the ELP directly to the system of coupled-channel equations itself, rather than to the corresponding generalized optical potential, for which explicit calculations have, in fact, only rarely been made [5-7].

In the region of interaction the original, "nonlocal," and the equivalent, "local," elastic wave functions usually differ. Their ratio, the Perey factor [8,9] (more precisely, its deviation from unity), is in a sense a measure of the nonlocality of the original interaction. Perey factors also provide a measure of the off-shell effects due to the difference between the phase equivalent local and the nonlocal interactions. They show up, e.g., in distorted-wave calculations for heavy-ion reactions [10] and in bremsstrahlung [11].

In the present paper we point out that there is a significant difference between the Perey factor for a nonlocal potential arising from Pauli exchange effects, and the Perey factor for an interaction whose nonlocality is a consequence of the coupling of the elastic to the nonelastic channels: The original Perey effect [12] for the nonlocal Perey-Buck potential refers to the first case, and results in a Perey factor considerably smaller than unity. On the other hand, our present calculations for the coupled-channel case yield a Perey factor which may also be larger than unity; more importantly, it is generally much closer to unity than in the case of the exchange-type Perey-Buck potentials.

Clearly, different types of nonlocal interactions lead, in general, to different types of Perey factors. Even for the same nonlocal interaction, different ELP's may be defined, and the corresponding Perey factors will differ from each other. We deem it worthwhile to make a few comments on the properties of some of the different ELP's. The coupled-channel nonlocal interaction considered in the present paper is actually used to calculate two different ELP's: the Wronskian-type ELP [13] and the ELP obtained by inversion of the nonlocal elastic-scattering phase shifts [14–17]. Although different in detail, the corresponding Perey factors turn out to exhibit qualitatively the same behavior, viz., near equality to unity, as pointed out above.

The result that the coupled-channel nonlocal interaction gives rise to Perey factors with values near unity is of considerable interest. It implies that the corresponding ELP's produce nearly the same elastic wave functions as the original coupled-channel interaction; these ELP's are therefore not only on-shell (phase shift) equivalent, but also nearly off-shell (wave function) equivalent.

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One consequence of these results is that optical potentials obtained by fitting the elastic-scattering data (or better by inversion of the phase shifts, if they are available) are apparently better suited to calculate the elastic channel wave function in the coupled-channel case than one might have expected from the known situation for exchange-type nonlocalities.

The coupled-channel interaction used in the present paper is only a schematic one. However, this is unlikely to affect the conclusions, which should be generally valid. These conclusions help to justify the widespread use of fitted local optical potentials for on-shell as well as off-shell elastic channel calculations in situations where nonelastic channels are coupled in significantly.

The paper is organized as follows: In Sec. II, we shall discuss some theoretical aspects of the ELP, commenting on some of the current types found in the literature. Particular attention is given to the Wronskian and inversion-type ELP's. These are determined numerically in Sec. III for the same energy dependent nonlocal interaction arising from a schematic system of six coupled channels modeled on n-^{58}Ni scattering at an incident energy of 60 MeV. Section IV contains our conclusions.

II. THEORETICAL CONSIDERATIONS

In this section we make some remarks on various kinds of ELP's in current use.

(i) Trivially equivalent local potential. This potential, the simplest one, in a way, is obtained from the nonlocal wave function $\Psi_{\rm NL}(\mathbf{r})$ by the simple prescription [4, 9, 13, 18, 19]

$$V(\mathbf{r})=\left[\left[E+\frac{1}{2m}\hbar^2\nabla^2\right]\Psi_{\rm NL}(\mathbf{r})\right]\Big/\Psi_{\rm NL}(\mathbf{r}) \ . \tag{1}$$

In terms of the density $\rho(\mathbf{r})$ and current $\mathbf{j}(\mathbf{r})$ calculated from the nonlocal wave function,

$$\rho(\mathbf{r})=|\Psi_{\rm NL}(\mathbf{r})|^2, \quad \mathbf{j}(\mathbf{r})=\frac{\hbar}{m}\mathrm{Im}\Psi^*_{\rm NL}\nabla\Psi_{\rm NL} \ , \tag{2}$$

the trivially equivalent local potential can also be written as

$$V(\mathbf{r})=E-\frac{\hbar^2}{4m}[\nabla^2\rho/\rho-(\nabla\rho)^2/2\rho]-\mathbf{j}^2/\rho^2+i\hbar\nabla\cdot\mathbf{j}/\rho \ . \tag{3}$$

Clearly, the potential (1) yields a local wave function Ψ_L which is identical to the original nonlocal wave function $\Psi_{\rm NL}$. Therefore, the Perey factor,

$$F(\mathbf{r})\equiv|\Psi_{\rm NL}(\mathbf{r})/\Psi_L(\mathbf{r})| \ , \tag{4}$$

is here equal to unity. In this exceptional case, the Perey factor is, of course, not a measure of the nonlocality or of anything.

While the trivially equivalent local potential fulfills the function of a potential when substituted in the appropriate place in the Schrödinger equation, it depends, of course, on as many parameters (energy, angular momentum, angle, etc.) as the wave function itself. It fluctuates greatly (e.g., it becomes infinite at the nodes of the wave function) and produces only that wave function from which it is calculated in the first place. It is less a potential than a functional of the wave function: its real part is a simple functional of the density and the current, and its imaginary part is the divergence of the current per unit density. Both are quite interesting quantities but they are less fundamental than a potential, which is expected to have a more universal application. Nevertheless, the trivially equivalent local potential, appropriately smoothed, has sometimes proven itself to be useful [19, 20].

(ii) Equivalent local potential for a nonlocal potential in WKB approximation. This potential has been defined for cases where the original nonlocal potential is real and independent of the energy, i.e., of the exchange type (Hartree-Fock or RGM potential). It has been obtained in three-dimensional space [1,2] and for partial waves [13,21]. The expression for the equivalent local potential need not be quoted here; the Perey factor $F(\mathbf{r})$ is simply given in terms of the ELP $V(E)$ by the formula [2, 22, 23]

$$F(\mathbf{r})=|1-\partial V(E)/\partial E|^{1/2} \ . \tag{5}$$

For exchange-type nonlocal potentials, $F(r)\leq 1$ ("Perey effect" [12]).

In its three-dimensional form, the "RGM + WKB" [21] leads to potentials which are energy dependent but only weakly angular-momentum dependent. In the partial-wave form of the WKB approximation, however, we obtain ELP's which are both energy dependent and strongly angular-momentum dependent at short distances. The same applies to the corresponding Perey factors.

It must be emphasized, however, that relation (5) is not valid when the original nonlocal potential is itself energy dependent, as is the case for all generalized optical potentials which take account of dynamical effects like the coupling to inelastic channels [3,4]. The numerical investigation in the next section will provide evidence for this.

(iii) Wronskian equivalent local potential. An exact quantal equivalent local potential can be derived for a particular partial wave l such that the nodes of the nonlocal and local partial wave functions coincide [13]. This potential is given by [24]

$$V_l(r)=k^2-l(l+1)/r^2-W_l''(r)/2W_l(r) + 3[W_l'(r)/2W_l(r)]^2-W_l^1(r)/W_l(r) \ , \tag{6}$$

with $W_l(r)\equiv W(u_l(r),v_l(r))$ and $W_l^1(r)\equiv W(u_l'(r),v_l'(r))$, where $W(x,y)=xy'-x'y$ is the Wronskian, and $u_l(r)$ and $v_l(r)$ are the regular and irregular nonlocal partial wave functions normalized such that asymptotically $W_l(\infty)=1$. Expression (6) is easily derived from Eqs. (2.2) and (2.8) of Ref. [9]. The Perey factor is given by

$$F_l(r)=|W_l(r)|^{1/2} \ . \tag{7}$$

Relations (6) and (7) are also valid for energy dependent nonlocal interactions. Clearly the equivalent local potential (6) is angular-momentum dependent. It has been shown [9] that the "RGM + WKB" potential in its partial-wave form is an approximation to the fully quantal Wronskian ELP. The deviation of the Wronskian $W_l(r)$ from unity is a measure of the nonlocality of the interaction, in accordance with the fact that two linearly independent solutions u_l, v_l of a local potential would, of course, yield $W_l(r)=1$. Equation (7) therefore provides a direct connection between the nonlocality of the original interaction and the ratio of the nonlocal over the local wave function, i.e., the Perey factor. For nonlocal interactions arising from the coupling to nonelastic channels, the nonlocal wave functions $u_l(r)$ and $v_l(r)$ are, of course, simply the corresponding wave functions in the elastic channel. The nonlocality then does not refer to a potential but to the dynamics of the coupled-channel scattering system itself. If one calculates a generalized optical potential for this system, this must have nonlocalities consistent with the Perey factor (7). Such a potential has been obtained in Refs. [5–7].

(iv) Equivalent local potential obtained by inversion. The last method we wish to discuss here for introducing an ELP is that by inversion [14–17]. Here one takes elastic-scattering phase shifts from a nonlocal potential or from a coupled-channel calculation, and finds the equivalent local potential by solving the associated inverse scattering problem. The Perey factor is then calculated as the ratio of the true elastic channel wave function, i.e., the nonlocal wave function, over the wave solution of the local potential from inversion. This last step is conveniently carried out for each partial wave separately.

In the calculations of the next section we shall use an inversion scheme for fixed energy [25, 26]. The ELP's resulting from this procedure are independent of angular momentum, but do depend on energy. One may also consider an inversion scheme at fixed angular momentum, which would lead to an energy independent, but angular-momentum-dependent ELP. We remark that this has been done for the nucleon-nucleon interaction, e.g., the Paris and Bonn potentials [27], and for n-d scattering [28]. It is found there that the difference in off-shell effects for the two potentials is rather small, corresponding to a Perey factor near unity. We further note that the ELP's obtained by inversion cover the two limiting cases of ELP's, i.e., those for which the ELP is either purely energy dependent or purely angular-momentum dependent (Marchenko inversion), while the "WKB + RGM" and Wronskian ELP's are both energy and angular-momentum dependent.

The preceding considerations show that one and the same nonlocal interaction can be associated with different ELP's, and, therefore, different Perey factors. On the other hand, we remark that a case is known where two different potentials, a nonlocal and an equivalent quasilocal potential containing a gradient term [29, 30], have the same Perey factor.

III. NUMERICAL INVESTIGATIONS

We have considered a schematic example of six coupled channels modeled on n-^{58}Ni scattering at an incident energy of 60 MeV. Forerunners of this work are Refs. [7, 29]. The diagonal potentials in this coupled-channel system are all given by the same expression (r in fm)

$$V_n(r)=V_0 \exp(-r^2/a^2) \tag{8}$$

with $V_0=-50$ MeV and $a=5$ fm. The coupling potentials are also identical between all channels, with the value $(n,n'=0,\ldots,5)$

$$V_{nn'}(r)=V_c \frac{d}{dr}[1+\exp\{(r-r_0)/b\}]^{-1} \tag{9}$$

with $V_c=-16$ MeV fm, $r_0=5$ fm, and $b=0.5$ fm. The coupling is peaked in the surface region $r\simeq 5$ fm, where it has the value of 8 MeV. The excitation energies (in MeV) in the five inelastic channels are $\varepsilon_1=2.92$, $\varepsilon_2=4.89$, $\varepsilon_3=8.01$, $\varepsilon_4=12.24$, and $\varepsilon_5=17.59$.

(i) Wronskian ELP. Solving for the regular and irregular wave functions of the six-channel system the Wronskian ELP (6) and the Wronskian Perey factor (7) have been calculated for the partial waves $l=0$, 1, 7, and 10 (cf., Figs. 1 and 2). The ELP shows strong oscillatory effects of the coupling in the surface region $r\simeq 5$ fm. The deviation of the Perey factor from unity is also concentrated in the surface region. The Perey factor increases with angular momentum; moreover, it changes from values larger than unity to values smaller than unity as the turning point for the partial wave moves across the region of coupling. In addition to the Perey factor, we give a direct comparison between the nonlocal and local wave functions in Fig. 3.

(ii) Inversion-type ELP. The calculations for the inversion-type ELP make use of the elastic-scattering function $S_l=\exp(2i\delta_l)$ calculated from the six-channel system. This function is shown in the Argand plot of Fig. 4. It has been "inverted" into a local angular-momentum-independent potential using the method of Refs. [25, 26]. The potential is shown in Fig. 5. The

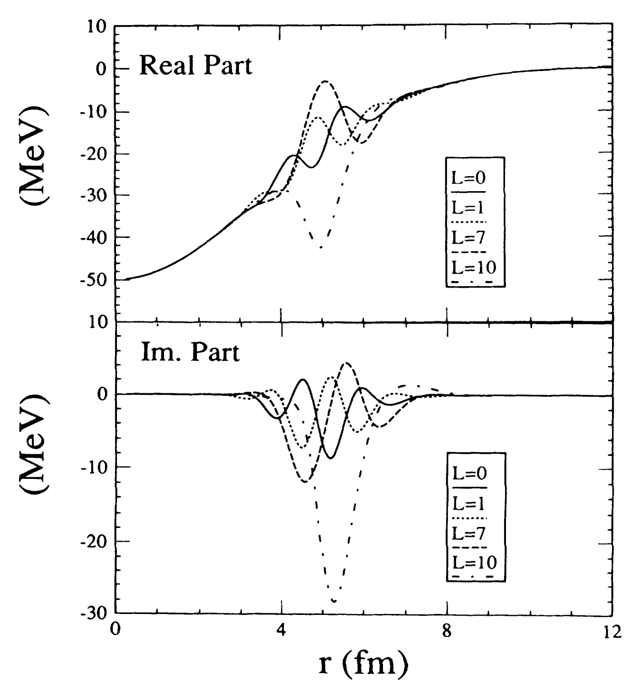

FIG. 1. Wronskian-type equivalent local potential $V_l(r)$ for $l=0$, 1, 7, and 10.

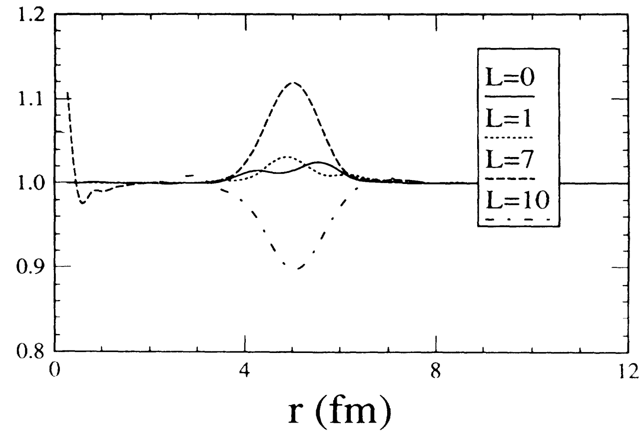

FIG. 2. Wronskian-type Perey factor $|W_l(r)|^{1/2}$ for $l=0$, 1, 7, and 10.

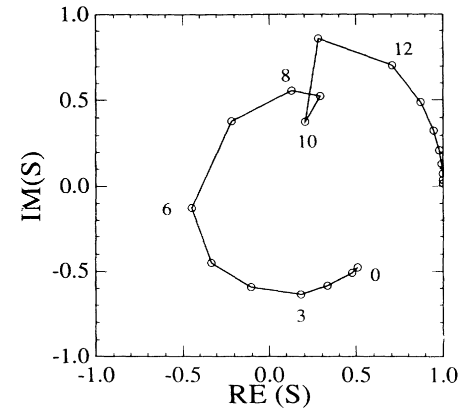

FIG. 4. Argand plot of the elastic-scattering function S_l.

imaginary part is partly positive, i.e., emissive. The fact that $\mathrm{Im}\,V>0$ in some regions inside the nucleus must not be seen simply as an artifact of the definition of the equivalent local potential. It can indeed be shown that the elastic wave function of the coupled-channel problem does lead to local emissive current sources in the nucleus, $\nabla\cdot\mathbf{j}>0$ (cf., for example, the imaginary part of the "Ψ potential" of Ref. [18] which is proportional to $\nabla\cdot\mathbf{j}$). These are regions where more flux is transferred from the inelastic channels to the elastic one than vice versa.

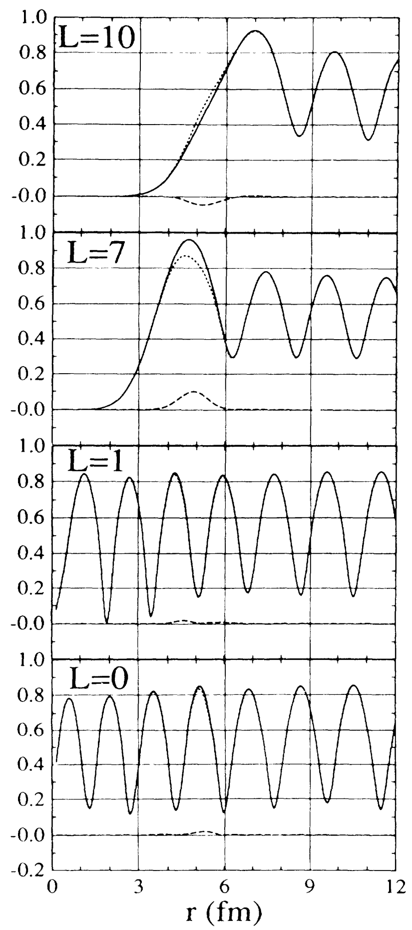

FIG. 3. Local wave function $|\Psi_L|$ (dotted curves) for the Wronskian-type equivalent local potential in comparison with the nonlocal wave function $|\Psi_{\mathrm{NL}}|$ (solid curves). The dashed curves represent the difference $|\Psi_{\mathrm{NL}}|-|\Psi_L|$.

We now study the partial wave function Ψ_L of this local potential in comparison with the elastic channel wave function obtained in the coupled-channel calculation [7] (the nonlocal wave function Ψ_{NL}). These two wave functions are shown in Fig. 6 for $l=0$, 1, 7, and 10. From this figure one could, of course, extract the Perey factor. We have chosen not to do so explicitly owing to numerical inaccuracies in the inversion, which would completely distort the Perey factor at points where Ψ_L is small. It can nevertheless be seen that the Perey factor for the dynamical six-channel system is sometimes smaller and sometimes larger ("anti-Perey effect" [17]) than unity.

We see from the results for the two types of ELP's that the coupled-channel nonlocality has a rather different effect on the Perey factor than the exchange-type nonlocality (a fact which is sometimes ignored in actual applications, cf., Ref. [31] in this connection). When "averaged" over the region of interaction and over the partial waves, the coupled-channel Perey factor is much closer to unity than the Perey-Buck one (which is ~ 0.85).

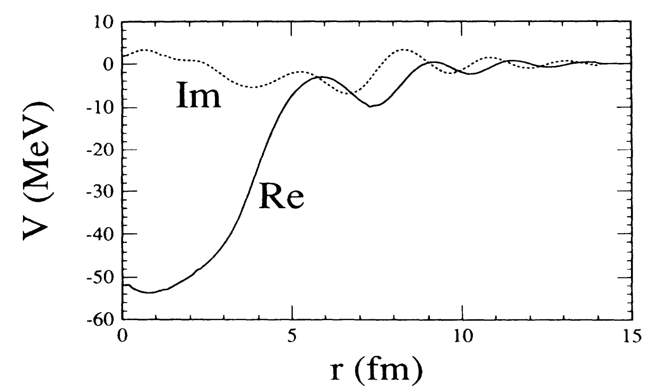

FIG. 5. Equivalent local potential obtained by inversion of the elastic scattering function S_l of Fig. 4.

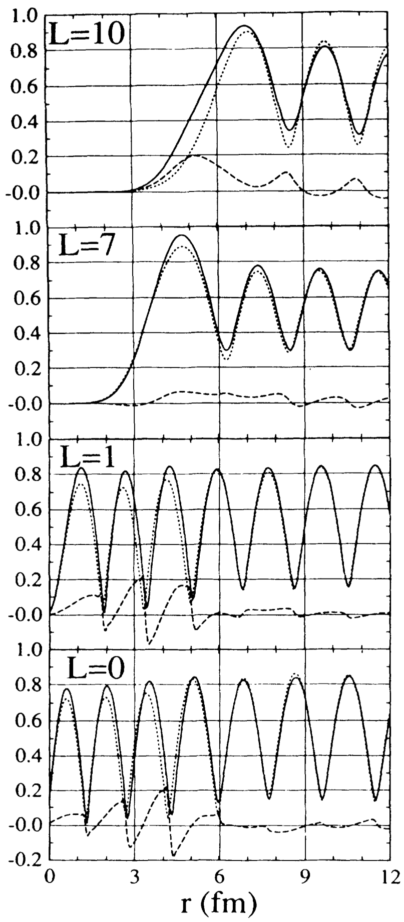

FIG. 6. Local wave function $|\Psi_L|$ for the inversion-type potential in comparison with the nonlocal wave function $|\Psi_{NL}|$. The labeling of the curves is the same as in Fig. 3.

Comparison of Fig. 6 with Fig. 3 shows, moreover, that local and nonlocal wave functions are closer to one another for the Wronskian ELP than for the inversion-type ELP.

By considering various incident energies in the neighborhood of 60 MeV, we have been able to compute the energy derivative of the inversion-type equivalent local potential. Substituting this quantity in Eq. (5), we can find the value of the WKB-type Perey factor; however, we know it to be invalid, since the nonlocal interaction is not energy independent in the present case. And, indeed, we have found that the Perey factor (5) differs more strongly from unity than the inversion-type Perey factor derived from Fig. 6. Apparently, the intrinsic energy dependence of the generalized optical potential, if correctly taken into account by a modification of Eq. (5), would compensate to some degree for the effects of its nonlocality. It is interesting to note that it has also been observed "empirically" in the analysis of the calculations of Ref. [31] that the large exchange-type Perey effect is almost completely canceled in a case where channel coupling plays a role.

We have discussed two different ELP's for the energy dependent nonlocal interaction induced by coupling to nonelastic channels. The Wronskian ELP depends on the angular momentum; it gives rise to Perey factors which differ from unity only in the surface region of the potential. The inversion-type ELP, on the other hand, is independent of angular momentum. For the determination of the Wronskian ELP one needs to know the full "nonlocal" wave function, whereas the input for calculating the inversion-type ELP are simply the "nonlocal" phase shifts.

From the point of view of minimizing the off-shell deviations arising from the difference of the nonlocal and local wave functions, the Wronskian ELP appears to be preferable: the difference $|\Psi_{NL}|-|\Psi_L|$ is smaller and it is concentrated in the surface region, in accordance with physical intuition. The suppression of the angular-momentum dependence in the inversion-type ELP apparently increases the difference $|\Psi_{NL}|-|\Psi_L|$ and extends its range beyond the surface region into the interior, where the partial wave functions of the ELP appear to display a phase shift as compared to those of the nonlocal interaction. Generally, it depends on the context in which the ELP is to be used, whether one or the other type of ELP is to be preferred.

IV. CONCLUSIONS

In the present paper it has been our aim to elucidate various aspects of the concept and of the explicit forms of the equivalent local potential. In particular, we have made clear the difference between an ELP describing the elastic scattering in a system with exchange effects but no coupling to nonelastic channels, and an ELP for a system with nonelastic couplings. In the former case the ELP is equivalent to an energy independent nonlocal interaction while in the latter case the ELP is equivalent to a nonlocal interaction which has an intrinsic dynamical energy dependence. Correspondingly, the relation between the nonlocal and local wave function (the Perey factor) is different in the two cases.

Two types of equivalent local potential for a coupled system of elastic and inelastic channels have been studied numerically. In the particular schematic example used in the investigation, it turns out that the local and nonlocal wave functions are closer to one another (the Perey factor is closer to unity) than for the better known exchange-type Perey-Buck nonlocalities. One consequence of this result is that optical potentials obtained by fitting the elastic-scattering data are apparently better suited to calculate the elastic channel wave function in the coupled-channel case than one might have expected from our experience with Perey-Buck nonlocalities. This may help to justify the use of fitted local optical potentials for on-shell as well as off-shell elastic channel calculations in situations where nonelastic channels are coupled in significantly. It would be desirable to check the general validity of our findings for a wide range of realistic coupled systems, but at present we are not in a position to do so.

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