Rotation and wobbling motion in triaxially deformed nuclei

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A quantum mechanical method of rotation and wobbling motion in triaxially deformed nuclei is represented within the framework of time-dependent Hartree-Fock theory. For such systems, the intrinsic frame is defined by imposing constraints of principal-axis frame. With aid of the canonical formulation of the constrained system, the Dirac quantization of the classical system is performed. It is shown that the commutation relations of angular momentum in the intrinsic frame then exactly satisfy the body-fixed frame. Furthermore, a method of describing large amplitude collective motion in the constrained system is proposed by extending the self-consistent collective-coordinate method.

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I. INTRODUCTION

Recent experimental data of the energy and γ-ray spectrum at high spin provide much detailed information about the high-spin states deexciting toward the yrast line. A model describing such states is the triaxial rotor model of Bohr and Mottelson [1]. In this model, the level above the yrast line corresponds to a new collective motion (wobbling motion) which is the dynamical fluctuation of angular velocity vector. This, however, is a phenomenological model. One needs a microscopic description as a more realistic calculation. The self-consistent cranking (SCC) plus random-phase approximation (RPA) [2,3] is useful for describing the wobbling motion above the yrast line. However, this is restricted to small oscillations. The purpose of this paper is to discuss general rotation and wobbling motion in triaxially deformed nuclei.

If a many-body system displays a triaxial stable deformation, it is simpler to describe it from a moving frame (intrinsic frame) of reference. How is the intrinsic frame microscopically defined? This is considered as follows. In the intrinsic frame, the deformation breaks rotational invariance or dynamical rotational symmetry. The rotational invariance corresponds to the rotational SO(3) symmetry. This is analogous to gauge theory [4]. Then the conditions determining the intrinsic frame (intrinsic frame conditions) correspond to the gauge-fixing conditions in the gauge theory. With this meaning, we cannot uniquely determine the intrinsic frame. In this paper, we will choose the principal-axis (PA) frame condition as the intrinsic frame conditions. Thus, we have to treat the constrained systems to defined the intrinsic frame. Since the nucleus is an isolated system whose Hamiltonian should be manifestly rotational invariant in the space-fixed frame, the symmetry breaking has to be restored. The intrinsic states corresponding to different orientations should yield degenerate energies. This gives rise to three zero-frequency modes corresponding to the components of angular momentum. One represents rotation (Goldstone mode) [5], and the others are spurious modes associated with the redundancy of the choice of the intrinsic frame. Furthermore, there is the difficulty of the divergent norm associated with the zero-frequency modes. When we go beyond the RPA to higher orders, naive perturbation theory fails due to the presence of these zero-frequency modes. Therefore, the perturbation method needs to be modified in the presence of such zero-frequency modes. To tackle this, Bes *et al.* [6] have recently proposed a microscopic theory using the collective coordinate method [7] in the path-integral formulation. Furthermore, they investigated a canonical formulation [8] using the Becchi-Rouet-Stora-Tyutin (BRST) transformation [9] in the gauge theory. On the other hand, Kerman and Onishi [10] have studied the nuclear rotation including precession and wobbling motion within the semiclassical method used on the time-dependent mean-field theory. It is, however, not clear whether the zero-frequency modes are exactly treated in their theory.

In this paper, we propose the quantum mechanical treatment of the rotation and the wobbling motion in the triaxially deformed nuclei within the framework of the time-dependent Hartree-Fock (TDHF) theory. The approach in the present paper differs from that of Bes *et al.* [7,8]. This is done by the canonical formulation in the TDHF theory with the constraints (the intrinsic frame conditions) which determine the choice of the intrinsic frame. The TDHF theory is considered to be a powerful tool for describing the dynamics of nonlinear systems such as the nucleus. With use of the canonical variables, the TDHF equations have the same form as the canonical equations of motion in classical mechanics [11]. In fact, the boson expansion theory is reduced to the TDHF theory under the replacement of the boson operators with the classical canonical variables. With this meaning, to obtain the spectra of the bound states it is necessary to quantize the above-mentioned classical constrained system. The quantization is then carried out by the Dirac quantization.

As is well known, the frequency of the small fluctuation around a static Hartree-Fock field is the same as that given by the RPA. The RPA gives a way of how to determine the collective subspace, although it is a linear approximation. The RPA is a quantal theory and leads to the excitation energy of the first excited state. Marumori *et al.* recently proposed a microscopic theory beyond the RPA to highly nonlinear system including the large amplitude collective motion within the framework of the TDHF theory, which is called self-consistent collective-coordinate method [12]. They gave a scheme for choosing the collective degree of freedom. We will extend the self-consistent collective-coordinate method to the case of the constrained system.

In Sec. II, starting from the many-fermion system, we present the canonical formulation of the nuclear rotation and the wobbling motion within the framework of the TDHF theory, and perform the canonical quantization using Dirac bracket. Then it is shown that the angular-momentum algebra in the intrinsic frame obeys exactly the minus-sign rules of the usual commutation relations which are well known as the commutation rules with respect to the rotating body-fixed frame. Our method in Sec. II is applied to the triaxial rotor at high spin in Sec. III. Choosing the PA frame conditions as the gauge-fixing conditions, we discuss the static Hartree-Bogoliubov (HB) approximation plus RPA (HB+RPA). It is shown that the result obtained is identical to the SCC+RPA equations of Marshalek. In Sec. IV, the self-consistent collective-coordinate method is extended to the case of the constrained system. A short summary and some concluding remarks are given in Sec. V.

II. TDHF METHOD IN INTRINSIC FRAME AND CANONICAL QUANTIZATION

We first start from the Hamiltonian operator with single-particle states of a spherical mean field:

$$\hat{H}=\sum_{\alpha\beta} T_{\alpha\beta}\hat{c}_\alpha^\dagger\hat{c}_\beta+\tfrac{1}{2}\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}\hat{c}_\alpha^\dagger\hat{c}_\beta^\dagger\hat{c}_\delta\hat{c}_\gamma \;, \tag{2.1}$$

where the operator $\hat{c}_\alpha$ ($\hat{c}_\alpha^\dagger$) denotes the fermion annihilation (creation) operator in the single-particle state α. In terms of the particle-creation operator $\hat{a}_\lambda^\dagger$ and the hole-creation operator $\hat{b}_i^\dagger$, the above Hamiltonian operator (2.1) can be written as

$$\hat{H}=\sum_i T_{ii}+\sum_{ij} V_{ijij}+\sum_\lambda \epsilon_\lambda \hat{a}_\lambda^\dagger\hat{a}_\lambda-\sum_i \hat{b}_i^\dagger\hat{b}_i +\tfrac{1}{2}\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}:\hat{c}_\alpha^\dagger\hat{c}_\beta^\dagger\hat{c}_\delta\hat{c}_\gamma: \;, \tag{2.2}$$

where : : denotes the normal ordered product with respect to the particle and hole. The TDHF equations are given as

$$\delta\langle\phi|(i\,d/dt-\hat{H})|\phi\rangle=0 \;. \tag{2.3}$$

Here, δ means variation and $|\phi\rangle$ is a general time-dependent Slater determinant

$$|\phi\rangle=\exp\left[\sum_{i\lambda}(C_{\lambda i}\hat{a}_\lambda^\dagger\hat{b}_i^\dagger-C_{\lambda i}^*\hat{b}_i\hat{a}_\lambda)\right]|0\rangle \;, \tag{2.4}$$

where the vacuum $|0\rangle$ of particles and holes satisfies the relation

$$\hat{a}_\lambda|0\rangle=\hat{b}_i|0\rangle=0 \;, \tag{2.5}$$

and $C_{\lambda i}$ are complex variables. Let us now introduce the new variables (β_μ,β_μ^*) connected by

$$\beta=\frac{\sin\sqrt{CC^\dagger}}{\sqrt{CC^\dagger}}C \;, \quad \beta^*=C^\dagger\frac{\sin\sqrt{CC^\dagger}}{\sqrt{CC^\dagger}} \;, \tag{2.6}$$

which satisfy the relations

$$\left\langle\phi\left|\frac{\partial}{\partial\beta_\mu}\right|\phi\right\rangle=\tfrac{1}{2}\beta_\mu^* \;, \quad \left\langle\phi\left|\frac{\partial}{\partial\beta_\mu^*}\right|\phi\right\rangle=-\tfrac{1}{2}\beta_\mu \;. \tag{2.7}$$

Here, μ means the set $\{i\lambda\}$ of the particle λ and the hole i. Then the TDHF equations (2.3) are expressed by the canonical form

$$i\dot{\beta}_\mu=[\beta_\mu,H]_P \;, \quad i\dot{\beta}_\mu^*=[\beta_\mu^*,H]_P \;, \tag{2.8}$$

where the dot denotes the time derivative. The density matrix corresponds to the c-number version of the generalized Holstein-Primakoff representation. The H is the classical version $H=<\phi|\hat{H}|\phi>$ of the original Hamiltonian operator $\hat{H}$. Hereafter, we put F as the expectation value $<\phi|\hat{F}|\phi>$ for arbitrary operator $\hat{F}$. The Poisson bracket $[F,G]_P$ is defined as

$$[F,G]_P=\sum_\mu\left[\frac{\partial F}{\partial\beta_\mu}\frac{\partial G}{\partial\beta_\mu^*}-\frac{\partial G}{\partial\beta_\mu}\frac{\partial F}{\partial\beta_\mu^*}\right] . \tag{2.9}$$

Now let us consider the rotating triaxial nuclei. Since the TDHF equations (2.1) refer to the space-fixed frame, it is convenient to go to the rotating frame from the space-fixed frame. Then both coordinate systems are connected by the Eulerian angles $(\theta_1,\theta_2,\theta_3)$, which are the dynamical variables depending on time. The Hamiltonian in such an arbitrary rotating frame is written as

$$\tilde{H}=H-\sum_k \Omega_k J_k \;, \tag{2.10}$$

where J_k are the classical versions of the angular-momentum operator $\hat{J}_k=\sum_{mm'}(J_k)_{mm'}\hat{c}_m^\dagger\hat{c}_{m'}$ with respect to the space-fixed frame, and Ω_k are the angular velocities with respect to the rotating frame. The angular velocities Ω_k are expressed by the Eulerian angles θ_i as follows:

$$\Omega_k=\sum_i V_{ki}\dot{\theta}_i \;, \tag{2.11}$$

where the transformation matrix V is given as

$$V=\begin{bmatrix} -\sin\theta_2\cos\theta_3 & \sin\theta_3 & 0 \\ \sin\theta_2\sin\theta_3 & \cos\theta_3 & 0 \\ \cos\theta_2 & 0 & 1 \end{bmatrix} . \tag{2.12}$$

Since the Hamiltonian is rotationally invariant, the physical results do not depend on the choice of the rotating frame. This implies the gauge invariance corresponding to the SO(3) symmetry. Thus, we need the gauge-fixing conditions that determine the intrinsic frame. We impose the constraints:

$$\alpha_k\approx 0 \;, \quad (k=1,2,3) \;, \tag{2.13}$$

satisfying the conditions

$$\mathrm{Det}([J_k,\alpha_l]_P)\neq 0 \ , \tag{2.14}$$

$$[\alpha_k,\alpha_l]_P=0 \ . \tag{2.15}$$

Since the Poisson bracket must be worked out before we make use of the constraint equations, we use a different equality sign $\approx$ from the usual $=$. With this meaning, we call Eqs. (2.13) weak equations. We cannot uniquely determine the α_k satisfying the conditions (2.14) and (2.15). The consistency conditions for arbitrary time then are

$$i\dot{\alpha}_k=[\alpha_k,\tilde{H}]_P=[\alpha_k,H]_P-\sum_l \Omega_l[\alpha_k,J_l]_P=0 \ . \tag{2.16}$$

From these conditions, the angular velocities Ω_k are

$$\Omega_k=-\sum_l [H,\alpha_l]_P\Phi_{lk}^{-1} \ , \tag{2.17}$$

where Φ_{lk}^{-1} are the inverse matrix elements of $[\alpha_k,J_l]_P$ given by

$$\sum_l [\alpha_k,J_l]_P\Phi_{lk'}^{-1}=\delta_{kk'} \ . \tag{2.18}$$

Inserting (2.11) into (2.17), we obtain the relationship

$$\sum_i V_{ki}\dot{\theta}_i=-\sum_l [H,\alpha_l]_P\Phi_{lk}^{-1} \ . \tag{2.19}$$

These differential equations give the connection between the Eulerian angles θ_i and the mean-field variables (β_μ,β_μ^*). Upon solving the differential equations (2.19), one finds the Eulerian angles θ_i are expressed by the mean-field variables (β_μ,β_μ^*). The Hamiltonian $\tilde{H}$ of Eq. (2.10) then satisfies the consistency conditions. However, Eq. (2.16) admits solutions for which $\alpha_k\neq 0$. Such solutions involve the admixture of spurious mode. In order to eliminate the spurious mode, for an arbitrary physical quantity F we define $\tilde{F}$ as follows:

$$\tilde{F}=F+\sum_{kl}[F,\alpha_l]_P\Phi_{lk}^{-1}\chi_k+\sum_{kl}[F,\chi_l]_P\Psi_{lk}^{-1}\alpha_k \ , \tag{2.20a}$$

$$\chi_k=J_k-I_k \ , \tag{2.20b}$$

where I_k is the collective version of the angular momentum referred to the intrinsic frame, and Φ_{lk}^{-1} are the inverse matrix elements of $[J_k,\alpha_l]_P$ given by

$$\sum_l [J_k,\alpha_l]_P\Phi_{lk'}^{-1}=\delta_{kk'} \ , \tag{2.21}$$

and $\tilde{F}$ is the invariant part satisfying the relationship

$$[\tilde{F},\alpha_k]_P=0 \ . \tag{2.22}$$

Putting $F=\alpha_k$, $\tilde{\alpha}_k$ satisfy $\tilde{\alpha}_k=0$ as the strong equality. α_k and χ_k are second class. It is now convenient to introduce the Dirac bracket defined as

$$[F,G]_D=[F,G]_P+\sum_{kl}[F,\alpha_k]_P\Phi_{kl}^{-1}[J_l,G]_P+\sum_{kl}[F,J_k]_P\Psi_{kl}^{-1}[\alpha_l,G]_P \ . \tag{2.23}$$

Then the Dirac brackets of the mean-field variables (β_μ,β_μ^*) become

$$[\beta_\mu,\beta_\nu^*]_D=\delta_{\mu\nu}+\sum_{kl}[\beta_\mu,\alpha_k]_P\Phi_{kl}^{-1}[J_l,\beta_\nu^*]_P+\sum_{kl}[\beta_\mu,J_k]_P\Psi_{kl}^{-1}[\alpha_l,\beta_\nu^*]_P \ , \tag{2.24a}$$

$$[\beta_\mu,\beta_\nu]_D=\sum_{kl}[\beta_\mu,\alpha_k]_P\Phi_{kl}^{-1}[J_l,\beta_\nu]_P+\sum_{kl}[\beta_\mu,J_k]_P\Psi_{kl}^{-1}[\alpha_l,\beta_\nu]_P \ . \tag{2.24b}$$

The Dirac brackets of the angular momentum are

$$[J_k,J_l]_D=-i\sum_m \epsilon_{klm}J_m \ . \tag{2.25}$$

The TDHF equations in the intrinsic frame are

$$i\dot{\beta}_\mu=[\beta_\mu,H]_D \ , \quad i\dot{\beta}_\mu^*=[\beta_\mu^*,H]_D \ . \tag{2.26}$$

Let us next perform the canonical quantization with constraints. Following the procedure of the Dirac quantization [13], the quantization is carried out by the replacement

$$[\ ,\ ]_D\to[\ ,\ ] \ , \quad \beta_\mu\to\hat{\beta}_\mu \ , \tag{2.27a}$$

$$J_k\to\hat{J}_k \ , \quad \alpha_k\to\hat{\alpha}_k \ . \tag{2.27b}$$

Then Eqs. (2.23)–(2.26) become

$$[\hat{F},\hat{G}]=[\hat{F},\hat{G}]_P+\sum_{kl}[\hat{F},\hat{\alpha}_k]_P\hat{\Phi}_{kl}^{-1}[\hat{J}_l,\hat{G}]_P+\sum_{kl}[\hat{F},\hat{J}_k]_P\hat{\Psi}_{kl}^{-1}[\hat{\alpha}_l,\hat{G}]_P \ , \tag{2.28}$$

$$[\hat{\beta}_\mu,\hat{\beta}_\nu^*]=\delta_{\mu\nu}+\sum_{kl}[\hat{\beta}_\mu,\hat{\alpha}_k]_P\hat{\Phi}_{kl}^{-1}[\hat{J}_l,\hat{\beta}_\nu^*]_P+\sum_{kl}[\hat{\beta}_\mu,\hat{J}_k]_P\hat{\Psi}_{kl}^{-1}[\hat{\alpha}_l,\hat{\beta}_\nu^*]_P \ , \tag{2.29a}$$

$$[\hat{\beta}_\mu,\hat{\beta}_\nu]=\sum_{kl}[\hat{\beta}_\mu,\hat{\alpha}_k]_P\hat{\Phi}_{kl}^{-1}[\hat{J}_l,\hat{\beta}_\nu]_P+\sum_{kl}[\hat{\beta}_\mu,\hat{J}_k]_P\hat{\Psi}_{kl}^{-1}[\hat{\alpha}_1,\hat{\beta}_\nu]_P \ , \tag{2.29b}$$

$$[\hat{J}_k,\hat{J}_l]=-i\sum_m \epsilon_{klm}\hat{J}_m \ , \tag{2.30}$$

$$i\dot{\hat{\beta}}_\mu=[\hat{\beta}_\mu,\hat{H}] \ , \quad i\dot{\hat{\beta}}_\mu^*=[\hat{\beta}_\mu^*\hat{H}] \ , \tag{2.31}$$

where $[\hat{F},\hat{G}]_P$ means the operator that is obtained by the replacement (2.27) after working out the Poisson bracket. From Eqs. (2.29a) and (2.29b), it is clear that $\hat{\beta}_\mu$ and $\hat{\beta}_\mu^*$ are not bosons. The commutation relations (2.29a) and (2.29b) contain the deviations from the boson rules. Putting $\hat{F}=\hat{\beta}_\mu$ and $\hat{G}=\hat{\alpha}_k$ in Eq. (2.28), it is easily found that $[\hat{\beta}_\mu,\hat{\alpha}_k]=0$. This means that the $\hat{\alpha}_k$ play the role of constants of motion due to the constraints (2.13). The angular-momentum algebra (2.30) obeys exactly the minus-sign rules of the usual commutation relations which are well known as the commutation rules with respect to the rotating body-fixed frame. This is due to the nonbosonic commutation relations (2.29a) and (2.29b).

III. WOBBLING MOTION AT HIGH SPIN

In this section, we will apply our method in the previous section to triaxial rotor at high spin. The powerful tool for describing the yrast states of axially symmetric rotor is the self-consistent cranking method. In the triaxial rotor, the levels above the yrast states are described by quantized wobbling modes. We now give the general formulation of the wobbling motion at high spin. Let us first consider the pairing plus quadrupole-quadrupole Hamiltonian operator

$$\hat{H}=\hat{H}_{\text{s.p.}}-\tfrac{1}{4}G\hat{P}^{\dagger}\hat{P}-\tfrac{1}{2}\kappa\sum_{M}\hat{Q}^{\dagger}_{2M}\hat{Q}_{2M}\ , \tag{3.1a}$$

$$\hat{H}_{\text{s.p.}}=\sum_{m}\epsilon_m\hat{c}^{\dagger}_m\hat{c}_m\ , \tag{3.1b}$$

$$\hat{P}=\sum_{m}s_m\hat{c}_m\hat{c}_{\bar{m}}\ ,\quad s_m=(-1)^{j-m}\ , \tag{3.1c}$$

$$\hat{Q}_{2M}=\sum_{mm'}\langle m|r^2Y_{2M}|m'\rangle\hat{c}^{\dagger}_m\hat{c}_{m'}\ , \tag{3.1d}$$

where $\hat{H}_{\text{s.p.}}$ is a spherially symmetric single-particle Hamiltonian, $\hat{P}$ is the $J=0$ pair operator, G is the strength of the pairing interaction, $\hat{Q}_{2M}$ are components of the mass quadrupole tensor, and κ is the strength of the quadrupole-quadrupole interaction. To obtain the SCC basis, we must vary the expectation value of the Hamiltonian operator (3.1a) with the subsidiary conditions that the angular-momentum operator $\hat{J}_x$ and the total number operator $\hat{N}=\sum_m\hat{c}^{\dagger}_m\hat{c}_m$ have certain expectations. This gives the equation of the variational principle

$$\delta\langle\Omega|\hat{H}'|\Omega\rangle=0\ , \tag{3.2a}$$

$$\hat{H}'=\hat{H}-\Omega\hat{J}_x-\lambda\hat{N}\ . \tag{3.2b}$$

The Lagrange multipliers Ω and λ are self-consistently determined by

$$\langle\Omega|\hat{J}_x|\Omega\rangle=\sqrt{I(I+1)}\ , \tag{3.3a}$$

$$\langle\Omega|\hat{N}|\Omega\rangle=N\ , \tag{3.3b}$$

where I and N are the spin of the rotor and the total particle number, respectively. Here, Ω is interpreted as the angular velocity about the x-axis, λ is the chemical potential, and $|\Omega\rangle$ is the quasiparticle vacuum of the angular velocity Ω defined by

$$\hat{a}_i|\Omega\rangle=0\ . \tag{3.4}$$

The quasiparticle operators $\hat{a}_i$ are defined by the generalized Bogolyubov transformation

$$\hat{a}_i=\sum_{m}(A^0_{mi}\hat{c}_m+B^0_{mi}\hat{c}^{\dagger}_m)\ , \tag{3.5a}$$

$$\hat{a}^{\dagger}_i=\sum_{m}(A^0_{mi}\hat{c}^{\dagger}_m+B^0_{mi}\hat{c}_m)\ , \tag{3.5b}$$

where A^0_{mi} and B^0_{mi} are the coefficients of the particle part and the hole part, respectively. The matrices A^0 and B^0 are real values, and satisfy the following relations expressing the unitary property:

$$\sum_{m}(A^0_{mi}A^0_{mj}+B^0_{mi}B^0_{mj})=\delta_{ij}\ , \tag{3.6a}$$

$$\sum_{m}(A^0_{mi}B^0_{mj}+B^0_{mi}A^0_{mj})=0\ , \tag{3.6b}$$

$$\sum_{i}(A^0_{mi}A^0_{ni}+B^0_{mi}B^0_{ni})=\delta_{mn}\ , \tag{3.6c}$$

$$\sum_{i}(A^0_{mi}B^0_{ni}+B^0_{mi}A^0_{ni})=0\ , \tag{3.6d}$$

and the inverse relations

$$\hat{c}_m=\sum_{i}(A^0_{mi}\hat{a}_i+B^0_{mi}\hat{a}^{\dagger}_i)\ , \tag{3.7a}$$

$$\hat{c}^{\dagger}_m=\sum_{i}(A^0_{mi}\hat{a}^{\dagger}_i+B^0_{mi}\hat{a}_i)\ . \tag{3.7b}$$

While the interaction $-\Omega\hat{J}_x$ in the Hamiltonian (3.2b) violates time-reversal symmetry, the symmetry and antisymmetric linear combinations of time-reversal conjugate states are the eigenstates of $\hat{J}_z$. It is then convenient to introduce two "signature" classes. In this paper, however, we do not distinguish such states for simplicity. Then the Hamiltonian operator $\hat{H}$ is expressed in terms of the quasiparticle operators $(\hat{a}_i,\hat{a}^{\dagger}_i)$:

$$\hat{H}=E_0(I)+\sum_{i}E_i\hat{a}^{\dagger}_i\hat{a}_i-\tfrac{1}{4}G{:}\hat{P}^{\dagger}\hat{P}{:}$$
$$-\tfrac{1}{2}\kappa\sum_{M}{:}\hat{Q}^{\dagger}_{2M}\hat{Q}_{2M}{:}+\Omega\hat{J}_x\ , \tag{3.8}$$

where the "dangerous terms" $\hat{a}^{\dagger}_i\hat{a}^{\dagger}_j$ and $\hat{a}_j\hat{a}_i$ are eliminated. Here, $E_0(I)$ is the yrast-state energy (SCC energy) which is evaluated using the Hartree-Bogolyubov factorization:

$$E_0(I)=\langle\hat{H}_{\text{s.p.}}\rangle-\tfrac{1}{4}G\langle\hat{P}^{\dagger}\rangle\langle\hat{P}\rangle$$
$$-\tfrac{1}{2}\kappa\sum_{M}\langle\hat{Q}^{\dagger}_{2M}\rangle\langle\hat{Q}_{2M}\rangle\ , \tag{3.9}$$

where E_i are the quasiparticle energies, and : : denotes the normal ordered product with respect to the quasiparticle. Here, the expectation values $\langle\ \rangle$ are with respect to the vacuum state $|\Omega\rangle$. We must choose the coefficients A^0_{mi} and B^0_{mi} in such a way that Eq. (3.2a) is minimized; it is equivalent to the procedure of making the dangerous terms vanish:

$$E_iA^0_{mi}=(\epsilon_m-\lambda)A^0_{mi}+\sum_{m'}\Gamma_{mm'}A^0_{m'i}$$
$$-\Omega\sum_{m'}(J_x)_{mm'}A^0_{m'i}-\Delta s_mB^0_{-mi}\ , \tag{3.10a}$$

$$E_iB^0_{mi}=-(\epsilon_m-\lambda)B^0_{mi}-\sum_{m'}\Gamma_{mm'}B^0_{m'i}$$
$$+\Omega\sum_{m'}(J_x)_{mm'}B^0_{m'i}+\Delta s_mA^0_{-mi}\ , \tag{3.10b}$$

$$\Gamma_{mm'}=-\sum_{\nu}(-)^{\nu}D^0_{2-\nu}\langle m|Q_{2\nu}|m'\rangle\ , \tag{3.11a}$$

$$\Delta=G\sum_{im>0}s_mB^0_{mi}A^0_{-mi}\ , \tag{3.11b}$$

$$D^0_{2\nu}=\kappa\sum_{imm'}\langle m|r^2Y_{2\nu}|m'\rangle B^0_{mi}B^0_{m'i}\ , \tag{3.11c}$$

where $\Gamma_{mm'}$, Δ, and $D^0_{2\nu}$ are the Hartree-Fock potential, the pairing potential, and the quadrupole deformation, respectively. The time-dependent Hartree-Bogoliubov (TDHB) equations in the uniformly rotating frame are given as

$$\delta\langle\phi_\Omega|(i\,d/dt-\hat{H}-\Omega\hat{J}_x)|\phi_\Omega\rangle=0\ . \tag{3.12}$$

A time-dependent Slater determinant $|\phi_\Omega\rangle$ with respect to the quasiparticles is defined by

$$|\phi_\Omega\rangle=\exp\left[\sum_{ij}(\tilde{C}_{ij}\hat{a}_i^\dagger\hat{a}_j^\dagger-\tilde{C}_{ij}^*\hat{a}_j\hat{a}_k)\right]|\Omega\rangle\ , \tag{3.13}$$

where $\tilde{C}_{ij}$ are complex variables. Introducing the new variables $(\tilde{\beta}_\mu,\tilde{\beta}_\mu^*)$ defined by

$$\tilde{\beta}=\frac{\sin\sqrt{\tilde{C}\tilde{C}^\dagger}}{\sqrt{\tilde{C}\tilde{C}^\dagger}}\tilde{C}\ ,\quad \tilde{\beta}^*=\tilde{C}^\dagger\frac{\sin\sqrt{\tilde{C}\tilde{C}^\dagger}}{\sqrt{\tilde{C}\tilde{C}^\dagger}}\ , \tag{3.14}$$

which satisfy the relations

$$\left\langle\phi_\Omega\left|\frac{\partial}{\partial\tilde{\beta}_\mu}\right|\phi_\Omega\right\rangle=\tfrac{1}{2}\tilde{\beta}_\mu^*\ ,\qquad \left\langle\phi_\Omega\left|\frac{\partial}{\partial\tilde{\beta}_\mu^*}\right|\phi_\Omega\right\rangle=-\tfrac{1}{2}\tilde{\beta}_\mu\ , \tag{3.15}$$

and following the same procedure as the previous section, we get

$$[\tilde{\beta}_\mu,\tilde{\beta}_\nu^*]_D=\delta_{\mu\nu}+\sum_{kl}[\tilde{\beta}_\mu,\alpha_k]_P\Phi_{kl}^{-1}[J_l,\tilde{\beta}_\nu^*]_P+\sum_{kl}[\tilde{\beta}_\mu,J_k]_P\Psi_{kl}^{-1}[\alpha_l,\tilde{\beta}_\nu^*]_P\ , \tag{3.16a}$$

$$[\tilde{\beta}_\mu,\tilde{\beta}_\nu]_D=\sum_{kl}[\tilde{\beta}_\mu,\alpha_k]_P\Phi_{kl}^{-1}[J_l,\tilde{\beta}_\nu]_P+\sum_{kl}[\tilde{\beta}_\mu,J_k]_P\Psi_{kl}^{-1}[\alpha_l,\tilde{\beta}_\nu]_P\ , \tag{3.16b}$$

$$[J_k,J_l]_D=-i\sum_m\epsilon_{klm}J_m\ , \tag{3.16c}$$

and the TDHB equations in the intrinsic frame

$$i\dot{\tilde{\beta}}_\mu=[\tilde{\beta}_\mu,H]_D\ ,\quad i\dot{\tilde{\beta}}_\mu^*=[\tilde{\beta}_\mu^*,H]_D\ . \tag{3.17}$$

Here, μ means the set $\{ij\}$ of the quasiparticle states i and j. The Hamiltonian H is expressed by

$$\begin{aligned}H&=\langle\phi_\Omega|\hat{H}|\phi_\Omega\rangle\\&=E_0(I)+\sum_\mu\mathcal{E}_\mu\tilde{\beta}_\mu^*\tilde{\beta}_\mu-\tfrac{1}{4}GP^\dagger P\\&\quad-\tfrac{1}{2}k\sum_\mu Q_{2M}^\dagger Q_{2M}+\Omega J_x\ ,\end{aligned} \tag{3.18a}$$

$$P=\langle\phi_\Omega|\hat{P}|\phi_\Omega\rangle\ ,\quad Q_{2M}=\langle\phi_\Omega|\hat{Q}_{2M}|\phi_\Omega\rangle\ , \tag{3.18b}$$

where $\mathcal{E}_\mu=E_i+E_j$ is the energy of a quasiparticle pair. As gauge-fixing conditions which determine the intrinsic frame, we choose conditions in the PA frame as follows:

$$\alpha_x=Q_{22}-Q_{2-2}\approx0\ , \tag{3.19a}$$

$$\alpha_y=Q_{21}+Q_{2-1}\approx0\ , \tag{3.19b}$$

$$\alpha_z=Q_{21}-Q_{2-1}\approx0\ . \tag{3.19c}$$

We call these conditions the PA frame conditions. Then the Poisson brackets $[J_k,\alpha_l]_P$ and $[\alpha_k,\alpha_l]_P$ are

$$[J_k,\alpha_l]_P=\begin{bmatrix}2Q_{22}^{(+)} & \alpha_z & \alpha_y\\ \alpha_z & Q_{22}^{(+)}+\sqrt{3/2}Q_{20}^{(+)} & \alpha_x\\ -i\alpha_y & -i\alpha_x & -iQ_{22}^{(+)}+i\sqrt{3/2}Q_{20}^{(+)}\end{bmatrix}, \tag{3.20a}$$

$$[\alpha_k,\alpha_l]_P=0\ , \tag{3.20b}$$

where $Q_{22}^{(+)}$ and $Q_{20}^{(+)}$ are defined by

$$Q_{22}^{(+)}=Q_{22}+Q_{2-2}\ ,\quad Q_{20}^{(+)}=2Q_{20}\ . \tag{3.21}$$

Under the PA frame conditions (3.19a)–(3.19c), $[J_k,\alpha_l]_P$ becomes diagonal and the determinant is written as

$$\mathrm{Det}([J_k,\alpha_l]_P)\approx-2iq_{22}^{(+)}(Q_{22}^{(+)}+\sqrt{3/2}Q_{20}^{(+)})\times(Q_{22}^{(+)}-\sqrt{3/2}Q_{20}^{(+)})\ . \tag{3.22}$$

Thus, the PA frame conditions satisfy the conditions (2.14) and (2.15).

Let us next apply the random-phase approximation of the TDHB equations (3.17) in the intrinsic frame. The angular momentum (J_y,J_z) and (α_y,α_z) in the RPA order are given as

$$J_y=i\sum_\mu J_\mu^y(\tilde{\beta}_\mu^*-\tilde{\beta}_\mu)\ ,\quad J_z=\sum_\mu J_\mu^z(\tilde{\beta}_\mu^*+\tilde{\beta}_\mu)\ , \tag{3.23a}$$

$$\begin{aligned}\alpha_y&=i\sum_\mu Q_\mu^{(1)}(+)(\tilde{\beta}_\mu^*+\tilde{\beta}_\mu)\ ,\\ \alpha_z&=i\sum_\mu Q_\mu^{(1)}(-)(\tilde{\beta}_\mu^*-\tilde{\beta}_\mu)\ .\end{aligned} \tag{3.23b}$$

The linearized equations of motion for the complex variables $\tilde{\beta}_\mu$ and $\tilde{\beta}_\mu^*$ are

$$\begin{aligned}i\dot{\tilde{\beta}}_\mu&=\mathcal{E}_\mu\tilde{\beta}_\mu+\frac{\kappa}{2\gamma_+^2}J_\mu^y\sum_\nu Q_\nu^{(1)}(+)\mathcal{E}_\nu(\tilde{\beta}_\nu^*-\tilde{\beta}_\nu)\\&\quad-\frac{\kappa}{2\gamma_-^2}J_\mu^z\sum_\nu Q_\nu^{(1)}(-)(\tilde{\beta}_\nu^*+\tilde{\beta}_\nu)\ ,\end{aligned} \tag{3.24a}$$

$$i\dot{\tilde{\beta}}^*_\mu = -\mathcal{E}_\mu \tilde{\beta}_{\mu*} + \frac{\kappa}{2\gamma_+^2} J^y_\mu \sum_\nu Q^{(1)}_\nu(+)\mathcal{E}_\nu(\tilde{\beta}^*_\nu - \tilde{\beta}_\nu) + \frac{\kappa}{2\gamma_-^2} J^z_\mu \sum_\nu Q^{(1)}_\nu(-)(\tilde{\beta}^*_\nu + \tilde{\beta}_\nu) , \quad (3.24b)$$

where γ_+ and γ_- are the deformation parameters defined by

$$\gamma_+ = \frac{\kappa}{2}\langle [J_y, \alpha_y]_P \rangle = \frac{\kappa}{2}(\langle Q^{(+)}_{22}\rangle + \sqrt{3/2}\langle Q^{(+)}_{20}\rangle) , \quad (3.25a)$$

$$\gamma_- = -\frac{i\kappa}{2}\langle [J_z, \alpha_z]_P \rangle = -\frac{\kappa}{2}(\langle Q^{(+)}_{22}\rangle - \sqrt{3/2}\langle Q^{(+)}_{20}\rangle) . \quad (3.25b)$$

We now assume that the time dependence of the complex variables $(\tilde{\beta}_\mu, \tilde{\beta}^*_\mu)$ are written as

$$\tilde{\beta}_\mu = \mathcal{A}_\mu e^{i\omega t} , \quad \tilde{\beta}^*_\mu = \mathcal{A}_\mu e^{-i\omega t} . \quad (3.26)$$

Substituting the above equations into the RPA equations (3.24a)–(3.24b), the frequencies are given by solutions of the following equations:

$$\omega^2 = \Omega^2 \frac{[\mathcal{J}_x - \mathcal{J}_y(\omega) - \omega^2\Omega^{-1}S(\omega)][\mathcal{J}_x - \mathcal{J}_z(\omega) - \omega^2\Omega^{-1}S(\omega)]}{[\mathcal{J}_y(\omega) + \Omega S(\omega)][\mathcal{J}_z(\omega) + \Omega S(\omega)]} , \quad (3.27a)$$

$$\mathcal{J}_x = \frac{I}{\Omega} , \quad \mathcal{J}_y(\omega) = 2\sum_\mu \frac{|J^y_\mu|^2 \mathcal{E}_\mu}{\mathcal{E}^2_\mu - \omega^2} , \quad (3.27b)$$

$$\mathcal{J}_z(\omega) = 2\sum_\mu \frac{|J^z_\mu|^2 \mathcal{E}_\mu}{\mathcal{E}^2_\mu - \omega^2} , \quad S(\omega) = 2\sum_\mu \frac{J^y_\mu J^z_\mu}{\mathcal{E}^2_\mu - \omega^2} . \quad (3.27c)$$

This is just same result as that derived by Marshalek [2]. Then the RPA Hamiltonian of the wobbling motion is expressed as

$$\hat{H}_{\rm RPA} = E_0(I) + \sum_n \omega_n \hat{O}^\dagger_n \hat{O}_n , \quad (3.28)$$

where $\hat{O}_n$ and $\hat{O}^\dagger_n$ are the normal modes of the RPA equations.

IV. SELF-CONSISTENT COLLECTIVE-COORDINATE METHOD WITH CONSTRAINTS

In the rotating triaxially deformed systems, the body-fixed frame was defined by imposing some constraints. These constraints were nothing but the gauge-fixing conditions, and it was possible to choose conditions (2.13). As such a possible choice, we imposed the PA frame conditions (3.19a)–(3.19c) and (2.20b). Then, α_k and χ_k were second class. In the previous section, we examined the wobbling motion at high spin in the framework of the TDHB method with the second-class constraints. In the TDHB theory, the frequency of the small fluctuation around a static Hartree-Bogoliubov field (SCC) is the same as that given by the RPA theory. As is well known, the RPA is a method for the collective motion in many-fermion systems, and is based on the linearization of equations of motion for the quasiparticle pairs. It was then shown that the SCC+RPA equations of Marshalek [2] can be derived.

For the description of the system in which the nonlinear effect becomes very important, Marumori *et al.* [11] recently have proposed the self-consistent collective-coordinate method. This method was investigated by several authors, and further applied to realistic nuclei. In the self-consistent collective-coordinate method, the basic equations on the collective submanifold are written in terms of the collective canonical variables $\{\eta_i, \eta^*_i;\ i=1,2,\ldots,M\}$ as follows:

$$\left\langle \phi \left| \frac{\partial}{\partial \eta_i} \right| \phi \right\rangle = \tfrac{1}{2}\eta^*_i , \quad \left\langle \phi \left| \frac{\partial}{\partial \eta^*_i} \right| \phi \right\rangle = -\tfrac{1}{2}\eta_i , \quad (4.1a)$$

$$[\beta_\mu, H]_P = [\beta_\mu, H]^{(\eta,\eta^*)}_P , \quad (4.1b)$$

$$[\beta^*_\mu, H]_P = [\beta^*_\mu, H]^{(\eta,\eta^*)}_P , \quad (4.1c)$$

where $[F,G]_P$ is given by Eq. (2.9) and $[F,G]^{(\eta,\eta^*)}_P$ is defined as

$$[F,G]^{(\eta,\eta^*)}_P = \sum_i \left[\frac{\partial F}{\partial \eta_i}\frac{\partial G}{\partial \eta^*_i} - \frac{\partial G}{\partial \eta_i}\frac{\partial F}{\partial \eta^*_i} \right] . \quad (4.2)$$

In this section, we will give the extension of the self-consistent collective-coordinate method to the case of the constrained system.

The TDHF equations with the second-class constraints are given as

$$\delta \left\langle \phi \left| \left[id/dt - \hat{H} - \sum_l \lambda_l \hat{\Phi}_l \right] \right| \phi \right\rangle = 0 , \quad (4.3a)$$

with constraints

$$\Phi_l = \langle \phi | \hat{\Phi}_l | \phi \rangle \approx 0 , \quad (4.3b)$$

where λ_l are the Lagrange multipliers with respect to the constraints (4.3b). Let us now assume that the collective submanifold is specified by only a few collective canonical variables $\{\eta_i,\eta_i^*;\ i=1,2,\ldots,M\}$. This means that it may be possible to separate the full TDHF $2N$ degrees of freedom into two types of degrees of freedom, which are collective variables $\{\eta_i,\eta_i^*;\ i=1,2,\ldots,M\}$ and noncollective variables $\{\xi_\alpha,\xi_\alpha^*;\ \alpha=1,2,\ldots,(N-M)\}$:

$$\beta_\mu=\beta_\mu(\eta_i,\eta_i^*;\xi_\alpha,\xi_\alpha^*)\ ,\quad \beta_\mu^*=\beta_\mu^*(\eta_i,\eta_i^*;\xi_\alpha,\xi_\alpha^*)\ . \tag{4.4}$$

Then, Eq. (4.3a) is written as

$$\delta\left\langle\phi\left|\left[i\sum_i\left[\dot\eta_i\frac{\partial}{\partial\eta_i}+\dot\eta_i^*\frac{\partial}{\partial\eta_i^*}\right]+i\sum_\alpha\left[\dot\xi_\alpha\frac{\partial}{\partial\xi_\alpha}+\dot\xi_\alpha^*\frac{\partial}{\partial\xi_\alpha^*}\right]-\hat H-\sum_l\lambda_l\hat\Phi_l\right]\right|\phi\right\rangle=0\ . \tag{4.5}$$

To get the canonical formulation, let us impose the following conditions:

$$\left\langle\phi\left|\frac{\partial}{\partial\eta_i}\right|\phi\right\rangle=\tfrac{1}{2}\eta_i^*\ ,\quad \left\langle\phi\left|\frac{\partial}{\partial\eta_i^*}\right|\phi\right\rangle=-\tfrac{1}{2}\eta_i\ , \tag{4.6a}$$

$$\left\langle\phi\left|\frac{\partial}{\partial\xi_\alpha}\right|\phi\right\rangle=\tfrac{1}{2}\xi_\alpha^*\ ,\quad \left\langle\phi\left|\frac{\partial}{\partial\xi_\alpha^*}\right|\phi\right\rangle=-\tfrac{1}{2}\xi_\alpha\ . \tag{4.6b}$$

With the use of these conditions, Eq. (4.5) can be written as

$$\delta\left[\frac{i}{2}\sum_i(\dot\eta_i\eta_i^*-\dot\eta_i^*\eta_i)+\frac{i}{2}\sum_\alpha(\dot\xi_\alpha\xi_\alpha^*-\dot\xi_\alpha^*\xi_\alpha)-H-\sum_l\lambda_l\Phi_l\right]=0\ , \tag{4.7}$$

where the classical Hamiltonian H is defined as

$$H=\langle\phi|\hat H|\phi\rangle\ . \tag{4.8}$$

Therefore, the variational principle of Eq. (4.7) leads to the Hamilton equations of motion

$$i\dot\eta_i=\frac{\partial H}{\partial\eta_i^*}+\sum_l\lambda_l\frac{\partial\Phi_l}{\partial\eta_i^*}\ ,\quad i\dot\eta_i^*=-\frac{\partial H}{\partial\eta_i}-\sum_l\lambda_l\frac{\partial\Phi_l}{\partial\eta_i}\ , \tag{4.9a}$$

$$i\dot\xi_\alpha=\frac{\partial H}{\partial\xi_\alpha^*}+\sum_l\lambda_l\frac{\partial\Phi_l}{\partial\xi_\alpha^*}\ ,\quad i\dot\xi_\alpha^*=-\frac{\partial H}{\partial\xi_\alpha}-\sum_l\lambda_l\frac{\partial\Phi_l}{\partial\xi_\alpha}\ . \tag{4.9b}$$

When the constraints (4.3b) are assumed to be of second class, they should satisfy the following consistency conditions:

$$i\dot\Phi_l=\left[\Phi_l,H+\sum_m\lambda_m\Phi_m\right]_P=0\ . \tag{4.10}$$

Then, the Lagrange multipliers λ_l are obtained as

$$\lambda_l=-(A^{-1})_{lm}[\Phi_m,H]_P\ , \tag{4.11}$$

where $(A^{-1})_{lm}$ is the inverse matrix of $[\Phi_l,\Phi_m]_P$. It is now convenient to introduce the Dirac bracket defined as

$$[F,G]_D=[F,G]_P-\sum_{lm}[F,\Phi_l]_P(A^{-1})_{lm}[\Phi_m,G]_P\ . \tag{4.12}$$

Then, the Dirac bracket is separated into two types of the Dirac bracket, which are the collective and noncollective parts:

$$[F,G]_D=[F,G]_D^{(\eta,\eta^*)}+[F,G]_D^{(\xi,\xi^*)}\ . \tag{4.13}$$

Here, the Dirac brackets $[F,G]_D^{(\eta,\eta^*)}$ and $[F,G]_D^{(\xi,\xi^*)}$ are defined as

$$[F,G]_D^{(\eta,\eta^*)}=[F,G]_P^{(\eta,\eta^*)}-\sum_{lm}[F,\Phi_l]_P^{(\eta,\eta^*)}(A^{-1})_{lm}[\Phi_m,G]_P^{(\eta,\eta^*)}\ , \tag{4.14a}$$

$$[F,G]_D^{(\xi,\xi^*)}=[F,G]_P^{(\xi,\xi^*)}-\sum_{lm}[F,\Phi_l]_P^{(\xi,\xi^*)}(A^{-1})_{lm}[\Phi_m,G]_P^{(\xi,\xi^*)}\ , \tag{4.14b}$$

where the Poisson bracket $[F,G]_P^{(\xi,\xi^*)}$ is given by

$$[F,G]_P^{(\xi,\xi^*)}=\sum_\alpha\left[\frac{\partial F}{\partial\xi_\alpha}\frac{\partial G}{\partial\xi_\alpha^*}-\frac{\partial G}{\partial\xi_\alpha}\frac{\partial F}{\partial\xi_\alpha^*}\right]. \tag{4.15}$$

The Hamilton equations of motion (4.9a) and (4.9b) can be given in terms of the collective and noncollective equations of motion

$$i\dot\eta_i=[\eta_i,H]_D^{(\eta,\eta^*)}\ ,\quad i\dot\eta_i^*=[\eta_i^*,H]_D^{(\eta,\eta^*)}\ , \tag{4.16a}$$

$$i\dot\xi_\alpha=[\xi_\alpha,H]_D^{(\xi,\xi^*)}\ ,\quad i\dot\xi_\alpha^*=[\xi_\alpha^*,H]_D^{(\xi,\xi^*)}\ . \tag{4.16b}$$

On the other hand, the equations of motion of the original variables (β_μ,β_μ^*) are expressed as

$$i\dot\beta_\mu=[\beta_\mu,H]_D=[\beta_\mu,H]_D^{(\eta,\eta^*)}+[\beta_\mu,H]_D^{(\xi,\xi^*)}\ , \tag{4.17a}$$

$$i\dot\beta_\mu^*=[\beta_\mu^*,H]_D=[\beta_\mu^*,H]_d^{(\eta,\eta^*)}+[\beta_\mu^*,H]_D^{(\xi,\xi^*)}\ , \tag{4.17b}$$

where the Dirac bracket $[F,G]_D$ is defined by Eq. (4.12). Following the same procedure as Yamamura and Kuriyama [14], let us now investigate how the collective submanifold can be specified in the case of the constrained system. The noncollective degrees of freedom should be frozen on the collective submanifold; $\dot{\xi}_\alpha=\dot{\xi}^*_\alpha=0$. From Eq. (4.16b), this leads us to the following relations:

$$\|[\xi_\alpha,H]_D^{(\xi,\xi^*)}\|=0\ ,\quad \|[\xi^*_\alpha,H]_D^{(\xi,\xi^*)}\|=0\ , \tag{4.18}$$

where the notation $\|F(\eta_i,\eta^*_i;\xi_\alpha,\xi^*_\alpha)\|$ denotes the value on the collective submanifold

$$\|F(\eta_i,\eta^*_i;\xi_\alpha,\xi^*_\alpha)\|=F(\eta_i,\eta^*_i;\xi_\alpha=0,\xi^*_\alpha=0)\ . \tag{4.19}$$

Conditions (4.18) are the so-called equations of the collective submanifold. On the collective submanifold, using the relations (4.18), we can rewrite Eqs. (4.17a) and (4.17b) in the following form:

$$\|i\dot{\beta}_\mu\|=\|[\beta_\mu,H]_D\|=\|[\beta_\mu,H]_D^{(\eta,\eta^*)}\|\ , \tag{4.20a}$$

$$\|i\dot{\beta}^*_\mu\|=\|[\beta^*_\mu,H]_D\|=\|[\beta^*_\mu,H]_D^{(\eta,\eta^*)}\|\ . \tag{4.20b}$$

Therefore, instead of Eq. (4.18) we can obtain the equations of the collective submanifold as follows:

$$\|[\beta_\mu,H]_D\|=\|[\beta_\mu,H]_D^{(\eta,\eta^*)}\|\ , \tag{4.21a}$$

$$\|[\beta^*_\mu,H]_D\|=\|[\beta^*_\mu,H]_D^{(\eta,\eta^*)}\|\ . \tag{4.21b}$$

Conditions (4.6a) and (4.6b) guarantee the collective and noncollective variables to be canonical, and specify the type of canonical variables. As emphasized by Yamamura and Kuriyama [14], the Hamilton equations of motion are invariant under arbitrary canonical transformation. In this case, we fix the canonical coordinate system within conditions (4.6a) and (4.6b). On the collective submanifold, conditions (4.6a) then become

$$\left\|\left\langle\phi\left|\frac{\partial}{\partial\eta_i}\right|\phi\right\rangle\right\|=\tfrac{1}{2}\eta^*_i\ ,\qquad \left\|\left\langle\phi\left|\frac{\partial}{\partial\eta^*_i}\right|\phi\right\rangle\right\|=-\tfrac{1}{2}\eta_i\ . \tag{4.22}$$

Finally, we can obtain the basic equations which specify the collective submanifold:

$$\left\|\left\langle\phi|\frac{\partial}{\partial\eta_i}|\phi\right\rangle\right\|=\tfrac{1}{2}\eta^*_i\ ,\qquad \left\|\left\langle\phi|\frac{\partial}{\partial\eta^*_i}|\phi\right\rangle\right\|=-\tfrac{1}{2}\eta_i\ , \tag{4.23a}$$

$$\|[\beta_\mu,H]_D\|=\|[\beta_\mu,H]_D^{(\eta,\eta^*)}\|\ , \tag{4.23b}$$

$$\|[\beta^*_\mu,H]_D\|=\|[\beta^*_\mu,H]_D^{(\eta,\eta^*)}\|\ . \tag{4.23c}$$

These are the equations which are replaced the Poisson bracket in Eqs. (4.1b) and (4.1c) with the Dirac bracket. The basic equations (4.23a)–(4.23c) are solved by an iterative method with the collective variables (η_i,η^*_i) expansion. The collective Hamiltonian $H(\eta_i,\eta^*_i)$ obtained in such a way must be quantized in order to calculate the excitation spectra. As is well known, there is the ambiguity of operator ordering. Here, we adopt the quantization procedure of Matsuo and Matsuyanagi [15]. The collective coordinates (η_i,η^*_i) in the collective Hamiltonian and the Dirac bracket $[\ ,\ ]_D$ are replaced by the collective operators $(\hat{\eta},\hat{\eta}^*)$ and the commutation relation $[\ ,\ ]$, respectively, and the normal ordering is taken:

$$\eta_i\to\hat{\eta}_i\ ,\quad \eta^*_i\to\hat{\eta}^*_i\ ,\quad [\ ,\ ]_D^{(\eta,\eta^*)}\to[\ ,\ ]\ . \tag{4.24a}$$

$$H(\eta_i,\eta^*_i)\to :H(\hat{\eta}_i,\hat{\eta}^*_i):\ . \tag{4.24b}$$

V. CONCLUSION

A quantum mechanical method for the general rotation and wobbling motion has been derived from the viewpoint of the gauge theory within the framework of the mean-field theory. The conditions determining the intrinsic frame correspond to the gauge-fixing conditions in the gauge theory. It is feasible to describe the general rotation and wobbling motion beyond the leading order. The intrinsic frame was determined by imposing constraints which correspond to the gauge-fixing conditions in the gauge theory. In this paper, we chose the PA frame conditions as such constraints. There are other kinds of constraints. In fact, we have discussed spin-orientation frame conditions in a previous paper [16].

Furthermore, we have extended the self-consistent collective-coordinate method to the case of the constrained system. The basic equations of the self-consistent collective-coordinate method with the constraints are just the equations obtained by replacing the Poisson bracket in the conventional equations (4.1b) and (4.1c) with the Dirac bracket, and can be solved consistently by the iterative procedure. The microscopic triaxial rotor has been discussed in this paper. It is interesting to study particle-rotor model microscopically. This may be done by applying the extended TDHF theory [14] to our method. This investigation is now in progress.

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