Nuclear structure anomalies arising from the use of Bonn interactions and possible resolutions

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Although the tensor interaction in the first order does not affect the single-particle splitting for closed *LS* shell nuclei, e.g., ^{4}He, ^{16}O, it causes the $j=l-\frac{1}{2}$ member of a spin-orbit pair to come towards or even below the $j=l+\frac{1}{2}$ member for an open shell. With the bare Bonn *A* interaction, the splitting $\epsilon_{p_{1/2}}-\epsilon_{p_{3/2}}$ is 4.2 MeV for an ^{16}O core but is -3.0 MeV for a closed $p_{3/2}$ core of ^{12}C. In large space shell-model calculations, this single-particle energy inversion leads to too high an occupancy of the $p_{1/2}$ orbit and pushes the wave functions of the low-lying states too much towards the *LS* limit. This manifests itself in too low a magnetic dipole transition rate from the ground state (0_1^+) to the 1_1^+, $T=1$ state. Various mechanisms are investigated in attempts to cure this problem including the relativistic effects of the Dirac-Brueckner-Hartree-Fock approach, also the effects of core polarization on the effective interaction and finally the effects of changing the meson masses in the nuclear medium as parametrized in the Hosaka-Toki interaction. The Dirac effects tend to increase the spin-orbit interaction while the change of the meson masses yields a weaker effective tensor force. Both effects improve the results of the nuclear structure calculations. The core polarization graphs involving phonon exchange also improve the results but other graphs, e.g., particle-particle ladders and hole-hole diagrams work in the opposite direction.

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I. INTRODUCTION

It is definitely one of the main aims of theoretical nuclear physics to develop a many-body theory to evaluate the properties of nuclear many-body systems starting from a realistic model of the nucleon-nucleon (*NN*) interaction which has been fitted to *NN* scattering and the data of the deuteron. A lot of effort has been made in particular to describe bulk properties of nuclei like binding energy and radius and the saturation point of nuclear matter. Such attempts turned out to be successful only after the relativistic effects of the so-called Dirac-Brueckner-Hartree-Fock (DBHF) were taken into account [1–3]. In order to verify the importance of these relativistic effects for nuclear structure, it is crucial to search for other observables which are sensitive to these effects. Investigations have been made on the residual interaction between valence nucleons in the 1*s*-0*d* shell [4]. It turned out, however, that the nuclear density relevant for two valence nucleons in the 1*s*-0*d* shell is too small to exhibit strong Dirac effects.

In this work we continue to pursue the above objective. The emphasis in the above work [4] was the good agreement for the spectra in a valence space when a relativistic formulation with realistic interactions is used and the fact that almost identical results are obtained with and without inclusion of the relativistic effects. In this work the emphasis is on things that *disagree* with experiment and whether the combined effects of modern *G* matrices which fit *NN* data much better than the old Reid [5] or Hamada-Johnston [6] interactions did, and the ideas of relativity, medium modifications, etc., can cure this disagreement.

An important point, which was emphasized in previous works [7,8] is that in order to really test an interaction, one should use it not only to calculate the interactions between valence nucleons but also to calculate the single-particle energies. Most calculations up to now have been hybrid in the sense that although the residual interaction is calculated from fundamentals, the single-particle energies are taken from experiment. Such a procedure usually leads to better agreement with experiment but it avoids a more stringent test on the quality of the interaction. Since the single-particle energies are due to the interaction with all nucleons, their evaluation also probes the effective interaction at higher densities than the residual interaction between valence nucleons at the surface.

Indeed we will soon find that there is some pressing disagreement with experiment using modern realistic bare *G* matrices and that the disagreement can be traced to discrepancies in the calculation of single-particle energies. For example, in performing a calculation of the energies of the well studied 1^+ states in ^{12}C (the $T=0$ state at 12.71 MeV and the $T=1$ state at 15.11 MeV), we find that the magnetic dipole transition rate from the ground state to the 1_1^+ $T=1$ state is much too small when a bare

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Bonn *A* interaction, as defined in Table A.2 of Ref. [3], is used. We trace the problem to the fact that the wave functions are too close to the *LS* coupling limit. At this limit the spin part of the magnetic dipole transition vanishes, leaving only the orbital part. The reason we are too close to the *LS* limit is due to the fact, as noted many years ago by Wong [11] and also by Scheerbaum [12], that the $p_{1/2}$ level in ^{12}C comes close to or even below the $p_{3/2}$ level. This is the opposite of what happens for closed *LS* cores like ^{4}He and ^{16}O. This inversion is due in part to the central interaction but is mainly due to the tensor interaction contained in Bonn *A*. That the tensor interaction is causing problems with the Bonn *A* interaction is at first surprising because the *D*-state probability in the deuteron with this interaction is only 4.4%, much less than what the old interactions gave (e.g., the Reid interaction yielded about 7% *D*-state admixture). Evidently the *D*-state admixture in the deuteron is not the most relevant quantity to gauge the effects of the tensor interaction in nuclei.

We discuss mechanisms which might cure this problem by making the tensor interaction weaker in the nucleus and/or the spin-orbit interaction stronger. It would appear that rather severe modifications of the nucleon in the nuclear medium are needed to cure this problem. We emphasize again that this problem would not have been evident if we did not use the same interaction to calculate single-particle energies as is used as a residual interaction between valence nucleons.

II. RESULTS FOR BARE *G* MATRICES

The results for calculations with the *bare G* matrix calculated for various modern one-boson-exchange interactions are given in Table I. All *G*-matrix elements were calculated in a basis of oscillator functions (oscillator parameter $b=1.72$ fm) assuming a Pauli operator which forbids scattering into the intermediate states with one particle in the 0*s* or 0*p* shell or both particles in 1*s*-0*d* shell and a constant starting energy of -5 MeV [9]. In Table I we give results for five versions of the Bonn potential. One of them (denoted by "BA1") is defined in Table A.1 of Ref. [3]. It is obtained by a fit to the *NN* interaction using the Blankenbecler-Sugar equation for the evaluation of phase shifts, data of the deuteron and in solving the Bethe-Goldstone equation, and assuming a pseudoscalar coupling for π and η mesons. This approach is typically used in nonrelativistic Brueckner-

TABLE I. Selected properties for the carbon isotopes in shell-model calculations with bare two-body matrix elements of Bonn *A* and Bonn *C* interactions. Energies are in units of MeV, magnetic moments are in μ_N, and $B(M1)$ in μ_N^2. The single-particle splitting $\Delta\epsilon_p$ between $0p_{1/2}$ and $0p_{3/2}$ orbits and the splitting $\Delta\epsilon_d$ between $0d_{3/2}$ and $0d_{5/2}$ orbits are also calculated with the same interaction relative to different cores.

Nucleus	Quantity	BA1[a]	BA2[a]	BA3[a]	BA4[a]	BC4[a]	CKII[b]	Expt.
^{4}He	$\Delta\epsilon_p$	2.749	2.595	3.448	4.060	4.025		
^{12}C	$\Delta\epsilon_p$	-3.013	-2.710	-1.264	-0.246	-0.431		
^{16}O	$\Delta\epsilon_p$	4.161	3.949	5.238	6.159	6.109		
	$\Delta\epsilon_d$	6.154	5.828	7.518	8.705	8.628		
^{12}C	$E_{1_1^+0}$	11.93	10.60	10.36	10.22	9.677	12.43	12.71
	$E_{1_1^+1}$	13.17	12.45	12.18	12.00	11.37	15.23	15.11
	$B(M1)_{1_1^+0}$	0.0013	0.0015	0.0035	0.0060	0.0061	0.0148	0.0435
	$B(M1)_{1_1^+1}$	0.639	0.693	1.139	1.565	1.703	2.510	2.85
	$N_{p_{1/2}}(0_1^+)$[c]	2.200	2.177	1.970	1.798	1.758	1.470	
	$N_{p_{1/2}}(1_1^+0)$[c]	1.673	1.677	1.521	1.435	1.437	1.559	
	$N_{p_{1/2}}(1_1^+1)$[c]	1.838	1.826	1.654	1.550	1.545	1.492	
^{13}C	$\mu_{1/2_1^-}$	0.994	0.976	0.887	0.814	0.775	0.701	0.7024
	$\mu_{3/2_1^-}$	-0.970	-0.969	-0.887	-0.824	-0.822	-0.934	
	$E_{3/2_1^-}$	0.305	0.350	0.898	1.456	1.548	3.673	3.684
^{14}C	$B(\mathrm{GT})_{0_1^+1\to 1_1^+0}$[d]	4.795	3.980	0.036	0.060	0.008	0.024	~ 0

[a]BA1, nonrelativistic $m^*=m=938.9$ MeC/c^2; BA2, Dirac spinors $m^*=m$; BA3, Dirac spinors $m^*=729.1$ MeV/c^2; BA4, Dirac spinors $m^*=630.0$ MeV/c^2; BC4, Dirac spinors $m^*=630.0$ MeV/c^2, higher *D* state probability (5.5%) for the deuteron.
[b]Cohen-Kurath "(8-16)TBME" interaction, see Ref. [14].
[c]Averaged number of nucleons in the $0p_{1/2}$ orbit.
[d]$B(\mathrm{GT})$ is defined so as to include the factor $(1.251)^2$.

Hartree-Fock calculations. The interaction BA1 is furthermore characterized by a weak tensor force (calculated D-state probability for the deuteron of 4.4%).

Such a weak tensor component is also a characteristic for the interactions denoted by "BA2," "BA3," and "BA4". They are derived from the potential A defined in Table A.2 of Ref. [3] using the Thompson approximation for the three-dimensional reduction of the Bethe-Salpeter scattering equation with pseudovector coupling for π and η mesons. This approach can be used in DBHF calculations. In this approach one takes into account that the self-energy of a nucleon in the nuclear medium consists of a large attractive scalar component (A) and a repulsive, time-like vector component (B)

$$\hat{U} = A + B\gamma^0 \; . \tag{2.1}$$

In nonrelativistic calculations the attractive and repulsive components cancel each other to a large extent leading to single-particle energies which are small as compared to the rest mass of the nucleon. If, however, such a self-energy is inserted into a Dirac equation for the nucleons in the nuclear medium

$$(\not{k} - m - \hat{U})\tilde{u}(p,s) = 0 \; , \tag{2.2}$$

and assume for simplfication that the components of the self-energy are independent of the momentum of the nucleon k, one obtains a solution for this Dirac equation

$$\tilde{u}(k,s) = \left[\frac{E^* + m^*}{2m^*}\right]^{1/2} \begin{bmatrix} 1 \\ \dfrac{\sigma\cdot\mathbf{k}}{E^*+m^*} \end{bmatrix} \chi_\sigma \; , \tag{2.3}$$

with $E^* = (k^2 + m^{*2})^{1/2}$. This solution is very similar to the one of the free Dirac equations except that the ratio of small to large component in the Dirac spinor is characterized by a Dirac mass

$$m^* = m + A \; , \tag{2.4}$$

which in the nuclear medium can be much smaller than the free mass of the nucleon m. In order to investigate this medium dependence of the Dirac spinors we have considered three choices: For BA2, $m^* = m = 938.9$ MeV/c^2 (the value for the vacuum), for BA3, $m^* = 729.1$ MeV/c^2 (this is a value typical for the density at the surface of a light nucleus [10] like ^{16}O), and for BA4, $m^* = 630.0$ MeV/c^2 (a value more typical for the global density of a light nucleus). Note that for BA3 and BA4, we have already included the nuclear medium effects through the use of an Dirac mass less than the free space value, $m^* < m$. In order to investigate the sensitivity of the results with respect to the strength of the tensor component we furthermore consider potential C defined in Table A.2 of Ref. [3] which exhibits a D-state probability of 5.5%. We shall also use the relativistic version with $m^* = 630.0$ MeV/c^2 and therefore it is denoted by "BC4" in analogy to the Bonn A G matrix BA4.

In the first row of Table I the single-particle splitting $\Delta\epsilon_p = \epsilon_{p_{1/2}} - \epsilon_{p_{3/2}}$ with respect to a ^{4}He core [i.e., $(0s_{1/2})^4$] calculated with the bare G matrices is given. This splitting is entirely due to the two-body spin-orbit interaction (which contracts into a one-body spin-orbit interaction). The values for BA1 and BA2 are 2.749 and 2.595 MeV, respectively. Note that the tensor interaction in the first order does not contribute at all to the single-particle splitting when the core is a closed LS shell like ^{4}He or ^{16}O.

Let us go down the column corresponding to BA1 and consider various points of interest. For an ^{16}O core the splitting $\Delta\epsilon_p = \epsilon_{p_{1/2}} - \epsilon_{p_{3/2}}$ increases to 4.16 MeV, again entirely due to the two-body spin-orbit interaction. The fact that $\Delta\epsilon_p$ is larger in ^{16}O than in ^{4}He is in accord with experiment. The main point of interest in this work is reached, when we consider $\Delta\epsilon_p$ in ^{12}C with the assumption of a closed $p_{3/2}$ core. We find that $\Delta\epsilon_p$ for ^{12}C is negative: -3.013 MeV. That is to say, the $p_{1/2}$ level comes below the $p_{3/2}$ level. This is due in part to the central interaction but is mainly due, as noted by Wong [11] and Scheerbaum [12] (see also Ref. [7]), to the tensor interaction, which, in contrast to the spin-orbit interaction, gives a large and negative contribution to the single-particle splitting for an open shell core. The single-particle splitting $\Delta\epsilon_p$ for different cores is also shown schematically in Fig. 1.

The fact that $p_{1/2}$ comes so much below $p_{3/2}$ would present no problem were it not for the fact, as we will demonstrate soon, that it is not supported by experiment. It is interesting to note that although the Bonn A interaction yields a relatively smaller percentage of the D-state admixture in the deuteron than the other G matrices, the tensor interaction of this G matrix still presents us with the old problems. At this point it would appear that things can be made better by either decreasing the tensor interaction and/or increasing the spin-orbit interaction.

Before proceeding down the BA1 column of Table I, we remind the reader that in previous works [7,8], we noted that a naive one-particle–one-hole (1p-1h) calculation of the 1_1^+0 state (we use the notation J_1^+T to represent the lowest positive-parity state with angular momentum J and isospin T) in ^{12}C with a closed spherical $p_{3/2}$ core as a ground state yields a collapse. That is

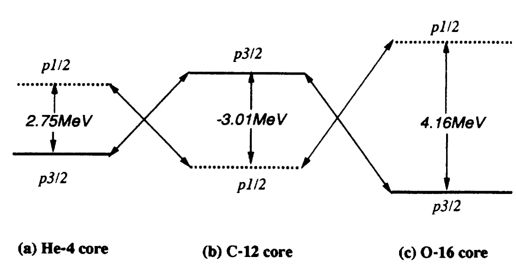

FIG. 1. The calculated single-particle splitting $\Delta\epsilon_p$ between $0p_{1/2}$ and $0p_{3/2}$ orbits with respect to (a) ^{4}He core [i.e., $(0s_{1/2})^4$], (b) ^{12}C core [i.e., $(0s_{1/2})^4(0p_{3/2})^8$], and (c) ^{16}O core [i.e., $(0s_{1/2})^4(0p_{3/2})^8(0p_{1/2})^4$]. The nonrelativistic Bonn A (BA1) interaction is used.

to say, with the BA1 interaction, the 1_1^+0 state of the configuration $(p_{1/2}p_{3/2}^{-1})^{1^+,0}$ came below the ground state ($E=-1.38$ MeV). Likewise the 1_1^+1 state energy was very close to zero ($E=1.41$ MeV). However, experimentally the 1^+ states are at a rather high excitation energy: 12.71 MeV for $T=0$ and 15.11 MeV for $T=1$. This "collapse" of the 1^+ state in the 1p-1h calculation is not too surprising since our initial assumption of a closed $p_{3/2}$ core is inconsistent with the $p_{1/2}$ level coming below the $p_{3/2}$ level.

We now perform a complete p-shell calculation of the 0^+ (ground) state and $1_1^+ T=0,1$ excited states in ^{12}C using the OXBASH code provided by Brown [13]. Since all configurations within the major shell are allowed, there is no problem about the inconsistent ground state.

The results for the 1^+ excitation energies in the full p space calculation, although not in perfect agreement with experiment, are greatly improved over the 1p-1h calculation: $E_{1_1^+0}=11.93$ MeV and $E_{1_1^+1}=13.17$ MeV. The 1^+ states are now at a respectably high energy.

However, by looking at the $M1$ transition $0_1^+0 \rightarrow 1_1^+1$, we see that there is a problem. The value of $B(M1)$ is much smaller than experiment—the calculated value is $0.64\mu_N^2$ but experiment gives $2.85\mu_N^2$.

We claim that the reason for the calculated value of $B(M1)$ being too small is due to the fact that the $p_{1/2}$ single-particle energy comes down too low relative to $p_{3/2}$. This can best be discussed by considering the value of $B(M1)$ in the jj and LS coupling limits. For the jj coupling, the configuration of the 1^+ state is, as mentioned before, $(p_{1/2}p_{3/2}^{-1})$, and the value of $B(M1)_{0_1^+\rightarrow 1^+1}$ is $11.26\mu_N^2$. In the LS limit, however, $B(M1)$ is negligibly small. The reason is that in the LS limit the *spin* part of $B(M1)$ vanishes and one gets contributions only from the orbital part and the ratio $(g_l/g_s)^2_{\text{isovector}}$ is very small: $(0.5/4.706)^2$. Now moving the single-particle level $p_{1/2}$ towards $p_{3/2}$ tends to take us towards the LS limit (indeed the LS coupling would be much more relevant in nuclear physics if there were no spin-orbit splitting).

To further show that the effective $p_{1/2}$ level is too low, we compare the Bonn A results for $B(M1)$ and for the *occupancy* of the $p_{1/2}$ orbit with the old Cohen-Kurath calculation [14] (CKII in Table I). The Cohen-Kurath phenomenological interaction included a tensor part and had a remarkably good fit to all the known data in the p shell of which we only show a small portion. The excitation energies of the 1_1^+0 and 1_1^+1 states with CKII are 12.43 and 15.23 MeV and the value of $B(M1)$ is $2.51\mu_N^2$ (recall that the experimental numbers are 12.71 MeV, 15.11 MeV, and $2.85\mu_N^2$, respectively).

Before making a comparison of the occupancy in the $p_{1/2}$ level, $N_{p_{1/2}}$, we note that in the naive spherical model for ^{12}C the occupancy of the $p_{1/2}$ orbit in the ground state would be $N_{p_{1/2}}=0$ and in the 1_1^+ $T=0,1$ states, it would be $N_{p_{1/2}}=1$. The results for the full p-space calculation with BA1 are qualitatively different. We find that the 1_1^+1 state has *less* $p_{1/2}$ occupancy than the ground state: $N_{p_{1/2}}(1_1^+1)=1.838$, $N_{p_{1/2}}(0_1^+0)=2.200$. However, one must remember that ^{12}C is deformed in the ground state so the comparison with Cohen-Kurath is more relevant. For CKII the occupancy of the $p_{1/2}$ level is slightly large for the 1^+ states than that for the ground state but only barely so. The relevant numbers are $N_{p_{1/2}}(1_1^+1)=1.470$ and $N_{p_{1/2}}(0_1^+0)=1.492$. Compared to CKII we find that with BA1 we have too much $p_{1/2}$ occupancy in the ground state (2.20 vs 1.47). This is consistent with what we were pointing out before: The $p_{1/2}$ level is coming down too low.

To show that the behavior in ^{12}C is not an isolated case of where there is too much configuration mixing due to the tensor interaction being too strong and/or the spin-orbit interaction too weak, we consider the neighboring nucleus ^{13}C. We consider the difference in energy of the $J=\frac{3}{2}^-$ first excited state and the $J=\frac{1}{2}^-$ ground state, and also the magnetic moment of the ground state.

The zeroth-order picture of ^{13}C is that of a closed $p_{3/2}$ shell and a valence $p_{1/2}$ neutron. The $\frac{3}{2}{}_1^-$ state would consist of an excitation of a $p_{3/2}$ nucleon to the $p_{1/2}$ shell. The ground-state Schmidt moment (for a $j=l-\frac{1}{2}$ neutron) is $-j/(j+1)\mu$, which is $0.64\mu_N$. Configuration mixing makes the magnetic moment larger than the Schmidt value. Indeed such mixing is required to bring us to the experimental value of $0.70\mu_N$. However, with the bare BA2 interaction, which has no medium effects, the value is $0.994\mu_N$. This indicates there is too much configuration mixing. Introducing medium corrections improves the situation so that with BA4 the value is $0.814\mu_N$.

As seen in Table I, the separation $E_{3/2_1^-}-E_{1/2_1^-}$ in ^{13}C with the bare BA2 interaction is 0.31 MeV, much smaller than the experimental value of 3.68 MeV (note that the Cohen-Kurath method, using empirical two-body matrix elements, gives a result very close to experiment). This again is indicative of the fact that the $p_{1/2}$ level is too close to $p_{3/2}$. In Table II using the BA4 interaction, the combination of phonon exchange and medium modification yields a reasonable result of 3.04 MeV.

The last entry in Table I is the famous allowed but nevertheless suppressed Gamow-Teller beta decay transition rate from ^{14}C (0_1^+1) to ^{14}N (1_1^+0). Experimentally, the value of B(GT) for this transition is essentially zero. However, in the LS limit the value of B(GT) for $(j^2)^{0^+1} \rightarrow (j^2)^{1^+0}$ is $(1.251)^2\times 6$. This transition is of *particular* interest because, as shown many years ago by Inglis [15], in a p-shell calculation it is not possible to get this matrix element to vanish unless there is a tensor interaction present in addition to the central and spin-orbit interactions. Now the value of B(GT) with BA1 is 4.795. This was previously analyzed by Zheng and Zamick [7] who concluded that the problem would be cured by making the tensor interaction weaker and/or the spin-orbit interaction stronger. Again this is consistent with what we have been saying about the $B(M1)$ for $0_1^+ \rightarrow 1^+$ transitions in ^{12}C. We recall that the above authors constructed a simplified interaction of the form $V_c + xV_{\text{s.o.}} + yV_t$ to model BA1. For $x=1$, $y=1$ they had a reasonable fit to Bonn A. Either by making x (i.e.,

TABLE II. Same as Table I but using renormalized two-body matrix elements which include the corrections due to the Bertsch-Kuo-Brown bubble only [Figs. 2(a) and 2(b)].

Nucleus	Quantity	BA1	BA2	BA3	BA4	BC4
^{12}C	$E_{1_1^+0}$	12.93	11.20	11.54	11.81	11.03
	$E_{1_1^+1}$	15.05	13.87	13.99	14.11	13.25
	$B(M1)_{1_1^+0}$	0.0040	0.0044	0.0081	0.0113	0.0114
	$B(M1)_{1_1^+1}$	1.177	1.295	1.890	2.358	2.503
	$N_{p_{1/2}}(0_1^+)$	1.956	1.915	1.693	1.535	1.500
	$N_{p_{1/2}}(1_1^+0)$	1.564	1.542	1.439	1.385	1.383
	$N_{p_{1/2}}(1_1^+1)$	1.679	1.658	1.532	1.464	1.453
^{13}C	$\mu_{1/2_1^-}$	0.866	0.841	0.766	0.721	0.683
	$\mu_{3/2_1^-}$	−0.944	−0.920	−0.853	−0.810	−0.797
	$E_{3/2_1^-}$	1.607	1.588	2.411	3.039	3.056
^{14}C	$B(\mathrm{GT})_{0_1^+\to 1_1^+0}$	0.156	0.011	0.069	0.151	0.073

the spin-orbit interaction) larger than one or by making y (i.e., the tensor interaction) smaller than one, one could cause B(GT) to vanish. It was also noted that if x was too small, e.g., $x=0.8$, there was no value of the tensor strength y for which B(GT) would vanish.

III. MEDIUM MODIFICATIONS

We now consider medium modifications in the relativistic Dirac spinor approach. In Table I we compare the results of BA1 (Blankenbecler-Sugar equation) and BA2 (Thomson equation with $m^*=m$) with BA3 and BA4 which are calculated with $m^*<m$.

Clearly one of the medium effects (i.e., using an effective mass m^* smaller than its free space value) is to make the spin-orbit interaction stronger. This can be seen in examining $\Delta\epsilon_p$. For the ^{4}He core the values of $\Delta\epsilon_p$ for the three interactions BA2, BA3, and BA4, are, respectively, 2.60, 3.45, and 4.06 MeV. For the ^{16}O core the corresponding values are 3.95, 5.24, and 6.16 MeV. We can be more precise. The strength of the spin-orbit interaction is inversely proportional to the effective mass, i.e., $V_{\mathrm{s.o.}}(m^*)=(m/m^*)V_{\mathrm{s.o.}}(m)$. This can be verified numerically from the values of $\Delta\epsilon_p$ which we just gave.

In examining the effects of the medium modification we see that in general they are beneficial but there is one surprise. The value of $B(M1)_{1_1^+}$ increases from $0.693\mu_N^2$ to $1.139\mu_N^2$ and to $1.565\mu_N^2$ as we go from BA2 to BA3 and to BA4. We are moving closer to the experimental value of $2.85\mu_N^2$ in going from BA2 to BA4. The value of B(GT) in ^{14}C→^{14}N goes from 3.98 to 0.036 and to 0.060. The latter two numbers are very close to the experimental value of ~ 0.

What is at first surprising is, however, that the energies of the 1^+ states go down. For example, for the 1_1^+1 state, the excitation energy goes down from 12.45 MeV for BA2 to 12.18 MeV for BA3 and to 12.00 MeV for BA4. The same is true for the 1_1^+0 state. Why does the energy of the 1^+ state go down when the spin-orbit splitting is increased?

This behavior was previously noted by Zheng and Zamick [7]. It was explained as being due to the fact that for a smaller spin-orbit splitting, the configuration mixing in the ground state increases so that the 0_1^+ state gets pushed down more than the 1^+ states.

We note that the occupancies $N_{p_{1/2}}$ also improve when we go from BA2 to BA4. The value of $N_{p_{1/2}}$ for the 1_1^+1 state decreases from 1.826 to 1.654 and then to 1.550, the latter value being reasonably close to the Cohen-Kurath value of 1.492.

What is not evident in Table I is that the medium modification does not affect the tensor interaction in any significant way. This can be seen by looking at selected matrix elements where only the tensor interaction acts. For example, only the tensor interaction contributes to the matrix element

$$\langle (0s_{1/2}0s_{1/2})^{J=1,T=0}|V|(0s_{1/2}0d_{3/2})^{J=1,T=0}\rangle$$

because the orbital angular momentum of the initial state is $L=0$ and that of the final state is $L=2$. The value of this matrix element is 3.756 MeV for BA2, 3.722 MeV for BA3, and 3.696 MeV for BA4. We see that there is scarcely any difference in the three cases.

IV. CORE POLARIZATION

In Tables II, III, and IV we repeat the calculations done in Table I but we include effects of core polarization. The core polarization is performed for two nu-

TABLE III. Same as Table I but using renormalized two-body matrix elements which include the corrections due not only to the Bertsch-Kuo-Brown bubble but also to the hole-hole diagram [Figs. 2(a)–2(c)].

Nucleus	Quantity	BA1	BA2	BA3	BA4	BC4
^{2}C	$E_{1_1^+0}$	14.62	12.70	12.81	12.92	11.97
	$E_{1_1^+1}$	16.64	15.25	15.19	15.18	14.17
	$B(M1)_{1_1^+0}$	0.0024	0.0026	0.0051	0.0076	0.0080
	$B(M1)_{1_1^+1}$	0.832	0.911	1.395	1.797	1.955
	$N_{p_{1/2}}(0_1^+)$	2.097	2.065	1.863	1.714	1.661
	$N_{p_{1/2}}(1_1^+0)$	1.647	1.625	1.515	1.454	1.449
	$N_{p_{1/2}}(1_1^+1)$	1.761	1.742	1.609	1.533	1.524
^{13}C	$\mu_{1/2_1^-}$	0.947	0.926	0.831	0.770	0.725
	$\mu_{3/2_1^-}$	−0.998	−0.977	−0.906	−0.859	−0.850
	$E_{3/2_1^-}$	1.201	1.165	1.884	2.471	2.601
^{14}C	$B(\mathrm{GT})_{0_1^+1\to 1_1^+0}$	2.507	1.764	0.276	0.029	0.052

cleons in the p shell plus a ^{4}He core. However, the bare matrix elements are those appropriate to nuclei around the ^{16}O region. It should be added that the single-particle interaction between a p-shell nucleon and the $0s$ shell core is renormalized to the second order. The calculations are carried out in second order and with progressively more types of diagrams which can be summarized as follows [see also diagrams (a), (b), (c), and (d) in Fig. 2]:

$$\text{Table I;}\quad \text{bare } G \text{ matrix } 2(a)\ ,$$

$$\text{Table II;}\quad \text{bare+Bertsch-Kuo-Brown bubble } 2(a) \text{ and } 2(b)\ ,$$

$$\text{Table III;}\quad \text{bare+bubble+hole-hole } 2(a)\text{–}2(c)\ ,$$

$$\text{Table IV;}\quad \text{bare+bubble+hole-hole+particle-particle } 2(a)\text{–}2(d)\ .$$

In Fig. 2(b), particle p is in the $1s$-$0d$ shell, hole h in the $0s$ shell. In Fig. 2(c), two holes h1 and h2 are in the $0s$ shell. In Fig. 2(d), two particles p1 and p2 are in the $1s$-$0d$ shell, or one of them in the $0p$ shell and the other in the $0f$-$1p$ shell.

Looking at Table II we see that if we limit the core polarization to the phonon exchange (i.e., the bubble) between two nucleons, the results are very beneficial for the energies and $B(M1)$ in ^{12}C. For example, with the highly medium modified BA4 interaction ($m^*=630$ MeV/c^2), the excitation energies of the 1^+ $T=0$ and $T=1$ states, which for the bare G matrix were 10.22 and 12.00 MeV, respectively, are now higher: 11.81 and 14.11 MeV. The latter values are now close to the experimental values of 12.71 and 15.11 MeV. The value of $B(M1)_{0_1^+\to 1_1^+,T=1}$ increases from $1.565\mu_N^2$ to $2.358\mu_N^2$, the latter being quite close to the experimental value of $2.85\mu_N^2$. For the above $B(M1)$ rate, we go from a bare BA2 ($m^*=m$) value of $0.693\mu_N^2$ to the phonon exchange renormalized BA4 ($m^*=630$ MeV/c^2) value of $2.358\mu_N^2$. It would appear that we have gone a long way to solving the anomaly.

Unfortunately, as seen in Table III, when we add hole-hole diagrams the results get somewhat eroded. The value of $B(M1)$ (BA4 column) drops from $2.358\mu_N^2$ in Table II to $1.797\mu_N^2$ in Table III. The energies of the 1^+ states go up somewhat. The inclusion of the particle-

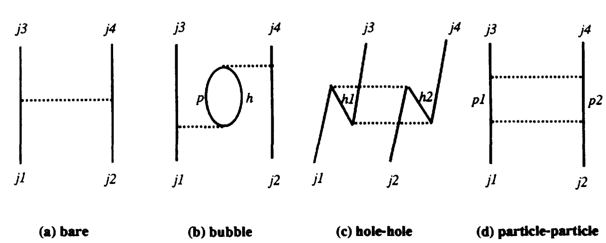

FIG. 2. Diagrams taken into account in the renormalization of two-body matrix elements: (a) the bare G matrix, results using bare G matrices are shown in Table I. (b) The Bertsch-Kuo-Brown bubble with $h=0s$ and $p=1s$-$0d$. Results using (a),(b) are shown in Table II. (c) The hole-hole diagram with $h1=h2=0s$. Results using (a)–(c) are shown in Table III. (c) The particle-particle diagram with $p1=1s$-$0d$ and $p2=1s$-$0d$ or $p1=0f$-$1p$ and $p2=0p$. Results using (a)–(d) are shown in Table IV.

TABLE IV. Same as Table I but using renormalized two-body matrix elements which include the corrections due not only to the Bertsch-Kuo-Brown bubble but also to the hole-hole diagram and the particle-particle ladder diagram [Figs. 2(a)–2(d)].

Nucleus	Quantity	BA1	BA2	BA3	BA4	BC4
^{12}C	$E_{1_1^+0}$	17.72	15.10	14.89	14.74	13.60
	$E_{1_1^+1}$	20.11	18.07	17.67	17.37	16.16
	$B(M1)_{1_1^+0}$	0.0012	0.0014	0.0030	0.0046	0.0049
	$B(M1)_{1_1^+1}$	0.514	0.597	0.952	1.249	1.374
	$N_{p_{1/2}}(0_1^+)$	2.256	2.214	2.045	1.918	1.873
	$N_{p_{1/2}}(1_1^+0)$	1.763	1.724	1.592	1.519	1.518
	$N_{p_{1/2}}(1_1^+1)$	1.894	1.871	1.724	1.639	1.633
^{13}C	$\mu_{1/2_1^-}$	1.011	0.987	0.921	0.870	0.837
	$\mu_{3/2_1^-}$	−1.042	−1.024	−0.967	−0.927	−0.927
	$E_{3/2^-}$	0.724	0.760	1.278	1.736	1.882
^{14}C	$B(\mathrm{GT})_{0_1^+1\to1_1^+0}$	6.099	5.178	1.595	0.280	0.295

particle ladders (Table IV) goes in the same direction as the hole-hole diagrams, thus making things even worse. The value of $B(M1)$ (BA4 column) goes down further to $1.249\mu_N^2$ and the energies of the 1^+ states become too high: 14.74 MeV for $T=0$ and 17.37 MeV for $T=1$.

It should be mentioned that the G matrices were constructed in such a way that the inclusion of ladder graphs up to $2\hbar\omega$ in energy is justified (see discussion at the beginning of Sec. II).

The results for $B(M1)$ are consistent with the $p_{1/2}$ occupancy of the ^{12}C ground state which in the naive spherical model would be zero. The values for the four tables with BA4 interaction are, respectively, 1.798, 1.535, 1.714, and 1.918. This should be compared with the Cohen-Kurath value of 1.470. Since the Cohen-Kurath calculation gives excellent fits to the data, we may assume that the value of 1.470 is quite reliable. We get close to this low occupancy when we include the phonon exchange only, but when we further introduce hole-hole diagrams and then particle-particle ladders the occupancy goes up and gets further away from the Cohen-Kurath "empirical result."

For ^{13}C, the magnetic moments of the ground state $\mu_{1/2_1^-}$ in Tables I to IV are, using the BA4 interaction, $0.814\mu_N$, $0.721\mu_N$, and $0.870\mu_N$, respectively. The excitation energies of the $J=3/2_1^-$ state $E_{3/2_1^-}$ in the four tables are 1.456, 3.039, 2.471, and 1.736 MeV, respectively. Experimentally, $\mu_{1/2_1^-}=0.7024\mu_N$ and $E_{3/2_1^-}=3.684$ MeV. Therefore in both cases, the results from Table II corresponding to the case in which the phonon exchange is included are the best. The overall situation is qualitatively similar to what happens in ^{12}C.

The value of B(GT) for $^{14}\mathrm{C}\to{}^{14}\mathrm{N}$ is reasonably small for all cases with the BA4 interaction. The respective values with the BA4 interaction in Tables I to IV are, respectively, 0.060, 0.151, 0.029, and 0.280. Actually there appears to be an overshoot (i.e., change of sign) when we go from BA2 to BA4. Still at the above numbers are much smaller than the bare BA2 value of 3.980.

V. THE HOSAKA-TOKI INTERACTION WITH LIGHT MESON EXCHANGE

As mentioned in the preceding sections there seems to be a need either to strengthen the spin-orbit interaction or to weaken the tensor interaction in a nucleus. We have already discussed the medium modifications which are achieved by assigning a Dirac mass $m^* < m$ to the lower components of the Dirac spinors. If, however, we consider a change of the effective mass for the nucleons in the nuclear medium, we might as well consider a change of the meson masses as well. Brown and collaborators [16] assume that inside the nucleus all mesons except for the pion should have effective masses less than the free values. Consequently, when a meson with a lighter mass is exchanged between two nucleons, the range of the interaction is larger. Such ideas are motivated from studies of the Nambu–Jona-Lasinio model [18] applied to quarks and meson excitations in a nuclear medium [19].

The ρ exchange between two nucleons gives a short-range repulsive contribution to the tensor interaction. With a lighter ρ mass the range of the repulsive part increases, thus canceling the attraction due to the pion exchange. This leads to an effectively weaker tensor interaction.

We give the results of the Hosaka-Toki interaction [17] with free space and reduced meson masses in Table V. We give the results both for bare G matrices and for matrices in which the phonon exchange is included [but not

TABLE V. Results for energies and transition rates of $M1$ excitations to the 1^+ states in ^{12}C with the Hosaka-Toki (HT) interaction. BHT (BHT*) represents the bare HT interaction with free space (reduced) meson masses, RHT (RHT*) represents the renormalized (only the Bertsch-Kuo-Brown bubble is included) HT interaction with free space (reduced) meson masses.

Interaction	$E_{1_1^+0}$	$E_{1_1^+1}$	$B(M1)_{0_1^+\rightarrow 1_1^+0}$	$B(M1)_{0_1^+\rightarrow 1_1^+1}$
BHT	17.31	18.06	0.0016	0.339
BHT*	16.90	17.55	0.0105	1.463
RHT	20.53	22.82	0.0034	0.651
RHT*	21.96	23.35	0.0148	2.234

the hole-hole (hh) or particle-particle (pp) diagrams].

We feel that the bare Hosaka-Toki G matrix is somewhat too crude, i.e., it leads to significant overbinind and hence the results for energies of the 1^+ states cannot be taken too literally. What is more significant is the *difference* in the results with reduced and free meson masses. We note that there is a vast improvement in the value of $B(M1)$ when light meson masses are used. When the phonon exchange is added, the value of $B(M1)$ changes from 0.65 to 2.23 in the case of reduced meson masses.

VI. CONCLUSIONS AND RECENT DEVELOPMENTS

Quite different models predict changes for the effective masses of nucleons and mesons inside a nuclear medium[1,16,18,19]. It has been one aim of the present contribution to explore the effects of such a medium dependence of hadron properties on the structure of low-energy excitations of nuclei. As an example we consider several observables which are sensitive to the change of the spin-orbit splitting in the $0p$ shell. It had been observed [7,8] that realistic NN interactions yield results of nuclear structure calculations which are too close to the LS coupling limit. Empirically, it had been observed that these problems could be cured by an enhancement of the spin-orbit interaction and/or a reduction of the effective tensor interaction.

Our present investigation demonstrates that the relativistic effects of the DBHF approach, which yields a change of the Dirac spinors in the nuclear medium that may be described by a decrease of the effective mass of the nucleons, increase the spin-orbit interaction. On the other hand, the reduction of the meson masses (except the pion mass), which is motivated, e.g., by the Nambu–Jona-Lasinio model [18], also yields a weaker tensor force. Therefore both of these medium effects tend to improve the results of the nuclear structure calculation.

We close with some recent developments and some future considerations. Banerjee [20] uses a "toy model of baryon bag formation to study the changes in the structure of a nucleon in nuclear matter." While his model supports part of the hypothesis of Brown *et al.* [16] that the vector masses and the nuclear mass vary with F_π^+ in the same manner, his results "do not support the conjecture that the masses scale with F_π^+, i.e., $M/m = F_\pi^+/F_\pi$." His results do not support a weaker tensor interaction in nuclei.

Core polarization should also be reconsidered and there is no room for optimism. It has been suggested to us by Kirson [21] that the results from Table II, where only the Bertsch-Kuo-Brown (BKB) bubble has been included (but not pp and hh graphs), may be the most relevant ones. The argument is that the pp and hh ladders, roughly speaking, increase the magnitude of all matrix elements—a ballooning effect. Now if these *larger* matrix elements are used to calculate the BKB bubble diagram, then it will become larger as well.

The BKB graph goes in the right direction and enhancing this diagram will offset the bad effect of the pp and hh graphs of Table IV. Thus if we optimistically take the results of Table II as being the most definitive we see that the BA4 results ($m^* = 630.0$ MeV $/c^2$) are quite good. The calculated value of B(M1) from the ground state to the $J=1^+$ $T=1$ state is $2.36\mu_N^2$, which is close to the experimental value $2.85\mu_N^2$. Obviously, more careful calculations will have to be done to justify the above remarks.

Note added in proof: The increase in spin-orbit splitting with decreasing Dirac mass that we have obtained and that we have shown to be responsible for an enhanced $B(M1)$ to the first $T=1$ $J=1^+$ state in ^{12}C is in accord with early works on this subject, in the contest of scattering problems by Shepard, McNeil, and Wallace [22]. A discussion of the spin-orbit interaction by Celenza and Shakin [23] is particularly illuminating. They note that this interaction is enhanced because the velocity of a nucleon for a given momentum is greater when a Dirac mass less than unity is used. The nonrelativistic expression $\mathbf{v} = \mathbf{P}/m$ goes over into $\mathbf{v} = \mathbf{P}/E^*$, which is approximately $\mathbf{P}/m^*$. In retrospect to see the effects of the spin-orbit interaction in *nuclear structure* one should look at the so-called "spin-orbit" states, e.g., $P_{1/2}P_{3/2}^{-1}$ as indeed we have done here. However, we must take into account that in real life ^{12}C is deformed and therefore large shell-model calculations are vital.

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