Slow proton production in semi-inclusive deep-inelastic neutrino scattering on hydrogen and deuterium

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The cross section for the production of slow protons in deep-inelastic (anti)neutrino scattering on hydrogen and deuterium is described in terms of fragmentation of the spectator diquark and the emission of nuclear spectators. We find that the softening of the x distribution, when going from $A=1$ to $A=2$, is due to spectator emission.

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I. INTRODUCTION

The effect of nuclear structure in deep-inelastic scattering (DIS) has been studied mostly by inclusive lepton scattering. From these experiments it has emerged that the ratio $F_2^A(x)/A$ differs from the structure function $F_2^N(x)$ of a free nucleon (the old EMC effect). For intermediate values of the Bjorken x ($x>0.2$) this effect can largely be explained in terms of fermi motion and binding of the nucleon and pionic contributions [1]. The inclusive DIS on nuclei is basically a convolution over the nucleon light-cone momentum. More detailed information on the nucleon momentum can be obtained from semi-inclusive processes. In particular, these processes permit the determination of the actual value of the light-cone momentum fraction carried by the hit nucleon.

Recently the results of two neutrino-induced deep-inelastic scattering experiments at Fermilab (E745 Collaboration) [2] and CERN (BEBC Collaboration) [3] have been published. In these experiments events were characterized by the presence (or absence) of short ionized tracks (dark tracks or stubs) at the interaction vertex. The latter were identified as slow protons with a momentum below approximately 1 GeV/c. The resulting EMC ratios for both types of processes differ: the dip at intermediate x is larger for events with dark tracks than for events without [2,4]. It was argued [2,5] that the events with dark tracks result from the absorption of the virtual boson by a deeply bound nucleon whereas the events without dark tracks correspond mainly to the scattering of loosely bound nucleons at the surface of the nucleus. However, the binding model of deep inelastic scattering cannot explain the production of backward protons without the inclusion of nuclear correlations.

The ratio $R(x)$ of the number of events with and without dark tracks was found to exhibit a strong A dependence. When $R(x)$ for neon and fluor is compared to the corresponding ratio for the deuteron, the latter turns out to be more sharply peaked for small x. An even greater effect is observed for the free proton. This softening of the x distribution with increasing mass number was interpreted [2] in terms of the emission of a spectator nucleon correlated with the struck one, a mechanism proposed by Frankfurt and Strikman [6].

However, in addition to the spectator emission the hadronization of the struck nucleon also plays a role. Recently Ishii, Saito, and Takagi [7] proposed a model that gives a qualitative description of the data of Ref. [2] using a combination of diquark fragmentation and rescattering of the struck nucleon.

In this paper we present a calculation of the so-called "tagged" structure functions for hydrogen and deuterium which are compared to the data of the BEBC Collaboration. In contrast to Ref. [7] we consider diquark fragmentation and the emission of nuclear spectators to explain the A dependence of the slow proton cross section. An earlier version of this work appeared in Ref. [8]. In Sec. II the formalism for semi-inclusive DIS is introduced. In Sec. III the competition between spectator emission and the direct fragmentation in the deuteron is studied.

II. SEMI-INCLUSIVE NEUTRINO-NUCLEON SCATTERING

The cross section for semi-inclusive deep inelastic neutrino scattering can be expressed generally as

$$\frac{d^5\sigma}{dE'd\Omega_\mu d^3\mathbf{p}_x}=\frac{G^2}{4\pi^2}\frac{E'}{EE_x}L_{\mu\nu}\tilde{W}^{\mu\nu} . \quad (1)$$

Here E and E' are the energies of the incoming neutrino and outgoing muon and E_x and $\mathbf{p}_x$ the energy and momentum of the observed outgoing particle, respectively. In inclusive scattering, the hadronic tensor $W^{\mu\nu}$ can be parametrized by three independent structure functions which in the Bjorken limit are related to the quark distribution in the nucleon:

$$\begin{aligned}F_2^{\nu p}(x,Q^2)&=\nu W_2(\nu,Q^2)=2x[d(x)+\bar{u}(x)] ,\\ F_3^{\nu p}(x,Q^2)&=\nu W_3(\nu,Q^2)=2[d(x)-\bar{u}(x)] ,\end{aligned} \quad (2)$$

with $Q^2=-q^2$ and $x=(Q^2/2P\cdot q)$ and $F_1=(1/2x)F_2$. In semi-inclusive scattering the hadronic tensor $\tilde{W}^{\mu\nu}$ involves extra structure functions [9]. These will in general depend on all Lorentz invariants that can be formed using the target momentum P, the momentum transfer q,

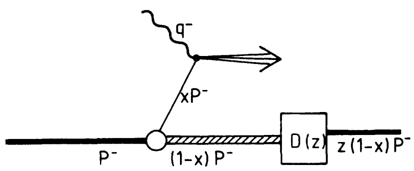

FIG. 1. Cluster fragmentation in DIS.

and the momentum p_x of the observed particle X, respectively. Following common practice we will assume that in the Bjorken limit the scattering amplitude factorizes into the scattering off a single quark and the hadronization of the resulting intermediate state.

The factorization for $\tilde{W}$ amounts to multiplying each term of the right-hand side of Eq. (2) by a fragmentation function D which parametrizes the hadronization process. In particular, the semi-inclusive analog of F_2 (denoted by $\mathcal{F}_2$) is given by

$$\mathcal{F}_2^{\nu p}(x,z_f,p_{x_\perp})=2x[d(x)D_d^x(x,z_f,p_{x_\perp})+\bar{u}(x)D_{\bar{u}}^x(x,z_f,p_{x_\perp})] , \tag{3}$$

with the light-cone momentum fraction $z_f=p_x^+/(P^++q^+)$. After integration over the neutrino scattering angle the semi-inclusive cross section becomes (cf. Fig. 1)

$$\frac{d^4\sigma_{\bar{\nu}}^{\nu}}{dx\,dz_f d^2p_{x_\perp}}=\frac{G^2m_NE}{\pi}\left\{\frac{x}{3}\mathcal{F}_1+\frac{1}{2}\mathcal{F}_2\pm\frac{x}{3}\mathcal{F}_3\right\} , \tag{4}$$

where the $\pm$ correspond to neutrino and antineutrino, respectively

A. Fragmentation functions

In deep-inelastic scattering in the extreme parton model a quark with light-cone momentum fraction x is knocked out from the nucleon. The final state of this process is that of a (leading) quark carrying a large momentum and a recoiling quark-cluster with a momentum fraction $1-x$ of the nucleon. Usually it is assumed that quark antiquark pairs are formed in the color field, eventually combining to color neutral particles. A simple way to describe fragmentation (relevant for the present case) is the one introduced by Field and Feynman (FF) [10,11]. In FF the leading quark and the quark cluster are identified with the "jets" observed in experiment. Each jet initially carriers the light-cone momentum of the associated quark (cluster). The probability for splitting off a hadron/meson carrying a fraction $1-\xi$ of the jet momentum is given by a splitting function $f(\xi)$. The remaining jet will fragment in exactly the same way as the "parent" jet, it only has a lower momentum. This is repeated until all momentum is converted. The resulting probability of creating a particle with a momentum fraction ξ of the initial jet is given by a fragmentation function $D(\xi)$ which satisfies an integral equation of the form [11]

$$D(\xi)=f(1-\xi)+\int_\xi^1\frac{d\eta}{\eta}f(\eta)D\left[\frac{\xi}{\eta}\right] . \tag{5}$$

In the FF picture the fragmentation function describes two different processes: the fragmentation of the hit quark leading to mostly fast particles, and the cluster fragmentation. Because in this paper we are interested in the production of slow hadrons, we will focus on the latter.

While z_f is the relevant variable for the leading (fast) quark fragmentation, the variable that enters in the cluster fragmentation function is $z_s=p_x^-/(P^-+q^-)$. The probability to create a fragment X with light-cone momentum p_x^- between $z_sp_{\text{jet}}^-$ and $(z_s+dz_s)p_{\text{jet}}^-$ from a jet with quark content $\{q\}$ is given by

$$D_{\{q\}}^x(z_s)dz_s\overset{Q^2\to\infty}{=}\frac{1}{1-x}D_{\{q\}}^x\left[\frac{z}{1-x}\right]dz , \tag{6}$$

where $z=p_x^-/m_N$. The last step is only valid in the Bjorken limit and a nucleon at rest in the laboratory system. Note that $z=1$ at $x=0$ corresponds to a hadron at rest.

Because the neutrino can interact with both valence and sea quarks, it is convenient to distinguish between two types of processes, namely the fragmentation of a diquark when the neutrino interacts with a valence quark, and a four-quark fragmentation when it interacts with a sea quark [12]. The diquark fragmentation function has been parametrized by Bartl, Fraas, and Majerotto [11] who extended the original prescription of Field and Feynman to include the flavor degree of freedom and diquarks.

In the present work we are mainly interested in the properties of $D_{qq}(z)$ near $z=1$. In this case the D functions can be approximated very well by the splitting functions weighted by the relative probabilities for creating the various types of baryons. Therefore we only keep the leading term in the integral equation (5) for D. The resulting diquark fragmentation functions are equal to the first term of those given in Ref. [11], which are proportional to the counting rule ansatz $f(\xi)=\xi$ [13,14].

Using the factorization assumption and Eq. (6), the semi-inclusive neutrino-proton cross section becomes

$$\sigma^{\nu p}=\sigma_{\text{val}}^{\nu p}+\sigma_{\text{sea}}^{\nu p} , \tag{7}$$

where the valence contribution is given by

$$\frac{d^2\sigma_{\text{val}}^{\nu p}(x,z)}{dx\,dz}=\frac{G^2m_NE}{\pi}\frac{2x}{1-x}d_v(x)D_{uu}^p\left[\frac{z}{1-x}\right] \tag{8}$$

and the sea contribution by

$$\frac{d^2\sigma_{\text{sea}}^{\nu p}(x,z)}{dx\,dz}=\frac{G^2m_NE}{\pi}\frac{2x}{1-x}\left[d_s(x)D_{uud\bar{d}}^p\left[\frac{z}{1-x}\right]+\frac{1}{3}\bar{u}_s(x)D_{uuud}^p\left[\frac{z}{1-x}\right]\right] . \tag{9}$$

The probability for observing slow proton ("dark track") in a certain momentum bin in the final state is obtained by integrating the double-differential cross section over the appropriate z interval and dividing by the total inclusive cross section:

$$P_{dt}(x,\Delta z)=\frac{1}{\sigma^{\rm incl}(x)}\int_{\Delta z}dz\left[\frac{d^2\sigma^{\nu p}_{\rm val}(x,z)}{dx\,dz}+\frac{d^2\sigma^{\nu p}_{\rm sea}(x,z)}{dx\,dz}\right]. \quad (10)$$

The cross section for scattering of a neutrino from neutron is obtained by interchanging $d\leftrightarrow u$ and $\bar{u}\leftrightarrow\bar{d}$ and the corresponding change of the indices of the D functions. The cross sections for scattering of an antineutrino on a proton or a neutron is obtained by replacing quark momentum distributions by antiquark distributions and vice versa.

We note that the formation and decay of resonances also gives rise to slow protons. This will be discussed in Sec. II C.

B. Four-quark fragmentation and pion contribution

The hadronization of four-quark states, in contrast to (di)quarks, has not been addressed in great detail in the literature. We consider two different prescriptions for this. The first is analogous to the treatment of the diquark, i.e., the four-quark fragmentation function is taken to be equal to the counting-rule estimate. In this way the four-quark and diquark fragmentation functions are equal, for instance $D^p_{uud\bar{d}}=D^p_{uu}$.

As an alternative treatment of the scattering off sea quarks we consider the pionic diagram of Fig. 2. The neutrino interacts with a virtual pion leaving a (slow) recoiling nucleon or nucleon resonance. This process was first considered by Sullivan [15] for the inclusive deep-inelastic cross section and by Lusignoli *et al.* [16,17] for the study of the structure function of the pion by measuring low momentum protons in the final state. More recently it has been considered in the discussion of the πNN form factor [18], the Gottfried sum rule [19,20], and semi-inclusive neutrino-nucleon scattering [21]. For simplicity, following Hwang, Speth, and Brown [22], we assume that the sea contribution in Eq. (7) can be represented by the diagrams in Fig. 2, i.e., we take the extreme picture in which the sea quarks are fully represented by the meson cloud.

The semi-inclusive cross section can be expressed as [17,23]

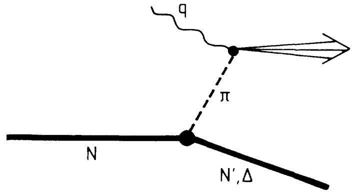

FIG. 2. Diagrammatic representation of the pion exchange contribution to the production of slow protons.

$$\frac{d^3\sigma}{dx\,dt\,dz}=\frac{G^2s}{2\pi}(1-z)\frac{g^2}{(4\pi)^2}f^2(t)\frac{t}{(t+m_\pi^2)^2}F_{2,\pi}(x_\pi)\;, \quad (11)$$

in which $t=-p_\pi^2$, $x_\pi=x/(1-z)$, g is the πNN coupling constant with $g^2/4\pi=14.2$, and $f(t)=(\Lambda^2-m_\pi^2)/(\Lambda^2+t)$ is the πNN form factor. Following Ref. [22] we use for the cutoff $\Lambda=1$ GeV.

As we need the cross section for protons in a given momentum bin, we integrate this cross section over the corresponding t,z domain. The t integration can be carried out analytically with boundaries $t_{\rm min}=m_N^2\times(1/z+z-2)$ and $t_{\rm max}=2m_N(E_{\rm max}-m_N)$ yielding

$$\frac{d^2\sigma}{dx\,dz}=\frac{G^2s}{2\pi}(1-z)\frac{g^2}{(4\pi)^2}F_{2,\pi}(x_\pi)\times[\Phi_N(t_{\rm min})-\Phi_N(t_{\rm max})]\;, \quad (12)$$

with

$$\Phi_N(t)=\frac{\Lambda^2+m_\pi^2}{\Lambda^2-m_\pi^2}\ln\frac{t+\Lambda^2}{t+m_\pi^2}-\frac{m_\pi^2}{t+m_\pi^2}-\frac{\Lambda^2}{t+\Lambda^2}\;. \quad (13)$$

To include the production (and subsequent decay) of deltas we replace $g^2/4\pi$ in Eq. (11) by $g_\Delta^2/4\pi=16.3$ and the factor t in the numerator by [17]

$$(M_+^2+t)^2(M_-^2+t)/(6m_\Delta^2m_N^2)$$

with $M_\pm=m_\Delta\pm m_N$ and the monopole form factor by a dipole, with the result:

$$\frac{d^2\sigma^\Delta}{dx\,dz}=\frac{G^2s}{2\pi}(1-x)\frac{\Gamma g_\Delta^2}{(4\pi)^2}F_{2,\pi}(x_\pi)\times\frac{\Phi_\Delta(t_{\rm max})-\Phi_\Delta(t_{\rm min})}{6m_N^2m_\Delta^2}\;, \quad (14)$$

with $t_{\rm min}=m_N^2(z-1)+(1/z-1)m_\Delta^2$, $t_{\rm max}=2m_NE_{\rm max}-m_N^2-m_\Delta^2$,

$$\Phi_\Delta(t)=\int dt\frac{(M_+^2+t)^2(M_-^2+t)}{(m_\pi^2+t)^2}\frac{(\Lambda^2-m_\pi^2)^4}{(\Lambda^2+t)^4}\;. \quad (15)$$

In Eq. (14) Γ depends on the specific decay process: $\Gamma_{p\pi^-\Delta^{++}}=\Gamma_{n\pi^+\Delta^-}=1, \Gamma_{p\pi^0\Delta^+}=\Gamma_{n\pi^0\Delta^-}=2/3$, and $\Gamma_{p\pi^+\Delta^0}=\Gamma_{n\pi^-\Delta^+}=1/3$. The total semi-inclusive cross section is given by Eq. (7) in which the "sea part" is replaced by Eq. (12).

C. Results

So far we neglected the transverse momentum of the fragments, hence the cross section depends on the momentum fraction z only. The transverse momentum is taken into account by multiplying the fragmentation functions with a Gaussian transverse momentum distribution $\rho(p_\perp)$ with the experimental width of 300 MeV/c [24] yielding

$$\frac{d^3\sigma}{dx\,dz\,dt}=\frac{d^2\sigma}{dx\,dz}\left|\frac{\partial p_\perp}{\partial t}\right|\rho(p_\perp)\;, \quad (16)$$

with $t=-(p'-p)^2$. The cross section for producing a proton with a momentum less than $p_{\rm max}$ from a nucleon thus becomes

$$\frac{d\sigma_{\rm dark}^{\rm nucl}}{dx}=\int_{\Delta z}dz\int_{\Delta t}dt\frac{d^3\sigma}{dx\,dz\,dt}\ , \tag{17}$$

with

$$\Delta z=[(E_{\rm max}-p_{\rm max})/m_N,1-x]$$

and

$$\Delta t=[m_N^2(1/z-z-2),2m_N(E_{\rm max}-m_N)]\ .$$

The total cross section for slow protons will also involve the decay of resonances, of which we only consider the delta. The fragmentation function for a delta is taken to be the same as for a nucleon. Therefore, the cross section for the production of a delta is given by Eqs. (8) and (9). Assuming that the delta decays isotropically in its rest frame, the probability that the decay of a delta with energy $E_\Delta=\sqrt{m_\Delta^2+p_\Delta^2}$ yields a nucleon with an energy less than a given $E_{\rm max}$ is given by

$$P_{E<E_{\rm max}}(E_\Delta)=\frac{1}{2}\left\{1-\frac{E_0u-E_{\rm max}}{p_0(u^2-1)^{1/2}}\right\}, \tag{18}$$

with $u=E_\Delta/M_\Delta$, p_0, and E_0 the momentum and energy of the nucleon (in the delta rest frame), respectively. The contribution of the delta decay to slow protons in the final state is given by the integral

$$\frac{d\sigma_{\rm dark}^{\rm res}}{dx}=\int dz_\Delta\int dt_\Delta\frac{d^3\sigma}{dx\,dz_\Delta dt_\Delta}P_{E<E_{\rm max}}(E_\Delta)\ , \tag{19}$$

with $t_\Delta=-(p_\Delta-p)^2$.

In Fig. 3 we show $P_{dt}(x,\Delta z)$ with and without the delta contribution for the pionic diagram. This figure shows

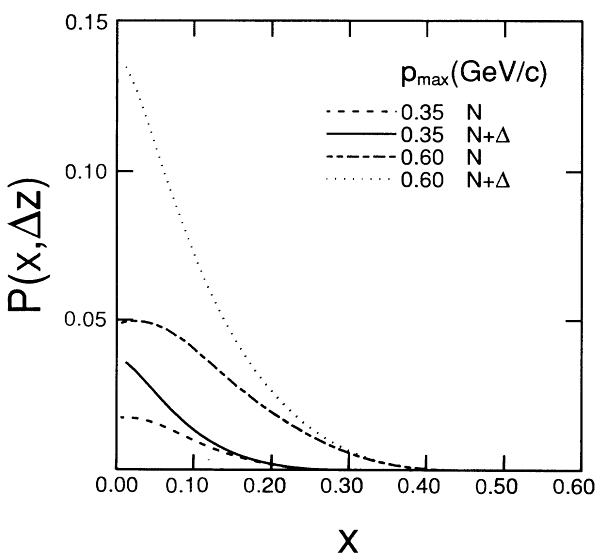

FIG. 3. $P(x,\Delta z)$ for the pionic process in neutrino scattering without (dashed and dash-dotted lines) and with (solid and dotted lines) the delta contribution.

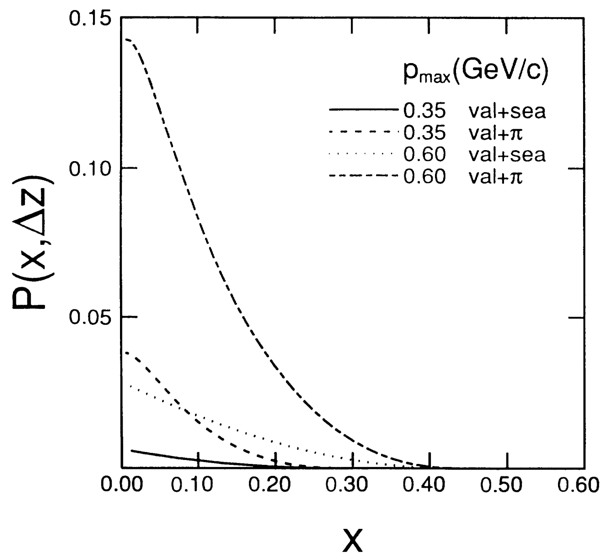

FIG. 4. $P(x,\Delta z)$ for neutrino proton scattering calculated using the pionic- (dashed and dash-dotted lines) and the pure fragmentation description (solid and dotted lines) of four-quark states (including the resonance contribution).

that the decay of deltas gives an appreciable contribution to the cross section for slow protons.

In Fig. 4 we compare the cross sections for slow protons using the two different prescriptions for the sea-quark contribution. It is seen that the cross section for the pure fragmentation description is much smaller in magnitude than for the pionic one. This is mainly due to the larger number of possible final states in the fragmentation process, whereas the pionic process produces nucleons and deltas only. We note that, when neglecting deltas, the pionic contribution is equal for both neutrino and antineutrino scattering.

In order to compare with the data of Ref. [3] we plot in Fig. 5 the normalized ratio

$$R(x,\Delta z)=\frac{\sigma_d(x)/\int dx\,\sigma_d(x)}{\sigma_{nd}(x)/\int dx\,\sigma_{nd}(x)} \tag{20}$$

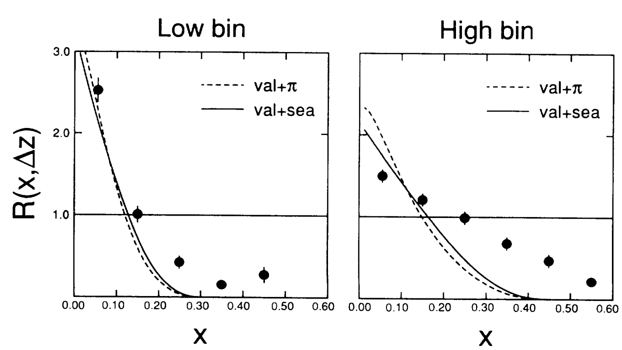

FIG. 5. $R(x,\Delta z)$ for neutrino proton scattering, using the fragmentation and pionic description of sea processes, for the momentum bins $150<p<350$ MeV/c and $350<p<600$ MeV/c, compared to the data of Ref. [3].

of events with (d) and without (nd) dark tracks using the two different prescriptions for sea processes. It is seen that there is only a small difference in the x dependence. However, we note that $R(x,\Delta z)$ is not very sensitive to the magnitude of the cross section, which is quite different for both descriptions. A possible explanation for the discrepancy with the data for large x is the uncertainty in the experimental value of x [21].

To describe slow proton production off nuclei (see Sec. III) it is necessary to have a good description of the process on a free nucleon. It is most convenient to use an effective fragmentation picture because of the factorization property. To this end we adjust the strength of the fragmentation functions for diquarks and tetraquarks such that for low x the cross section for the free nucleon agrees with the data. This procedure amounts to multiplying the normalization of the diquark and tetraquark fragmentation functions by 6.

III. PRODUCTION OF SLOW PROTONS OFF NUCLEI

To study the effect of the nuclear medium on the semi-inclusive cross section we consider the production of slow protons in neutrino-nucleus scattering. In the impulse approximation there are two processes that can give rise to the production of slow protons. They are shown in the two diagrams in Fig. 6. The first contribution (a), which we will call the "direct" one, corresponds to the detection of a fragment from the struck nucleon. The second (b), called the "spectator," corresponds to a process in which one observes a different nucleon which was correlated with the struck nucleon [6]. We assume that the diagrams in Fig. 6 are the only diagrams that contribute and that there is no interference between the two.

In the next section we derive the cross sections for the direct and spectator process for a general nucleus. In Sec. III B we restrict ourselves to the special case of the deuteron.

A. General convolution formulas

The cross section for the direct process is directly related to the free nucleon cross section. The only difference is that the neutrino does not scatter on a free nucleon but on a nucleon bound in a nucleus, i.e., its momentum is nonzero and its energy is not equal to the rest mass. This gives rise to effective values of the Bjorken x and the fragmentation parameter z_s. The conventional method for

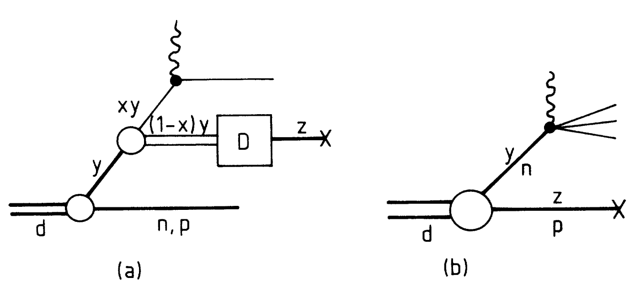

FIG. 6. The (a) direct and (b) spectator contributions.

dealing with these effective variables is the convolution model (valid in the impulse approximation).

In the impulse approximation the "inclusive" hadronic tensor $W^A_{\mu\nu}$ for the nucleus can be written as a convolution of the hadronic tensor $W^N_{\mu\nu}$ for the nucleon and the spectral function S_A [25–27]:

$$W^A_{\mu\nu}(P_A,q) = \sum_{p,n} \int d^4p \frac{Am_N}{m_A} S_A(p) W^N_{\mu\nu}[q_N, p_N(p,P_A)] \ , \tag{21}$$

where p_N is a model-dependent function of the nucleon four-momentum p and nuclear four-momentum P_A, $q_N = q + p - p_N$ is the momentum transfer experienced by the nucleon, and S_A is a spectral function which satisfies the normalization

$$\sum_{p,n} \int d^4p \frac{AE_N}{m_A} S_A(p) = 1 \ , \tag{22}$$

with $E_N = \sqrt{\mathbf{p}^2 + m_N^2}$. Because of the special form of the semi-inclusive current tensor the convolution formulas for $F_{1,2,3}$ which follow from Eq. (21) can be extended directly to their semi-inclusive counterparts. The resulting convolution formulas for $\mathcal{F}_{1,2,3}$ depend on the choice of dynamics. We use the instant-form dynamics [25,26] and assume that the nucleon structure functions are slowly varying functions of the nucleon four-momentum squared p_N^2. The nucleon momentum is defined as $p_N = P_A - P^*_{A-1}$ where P^*_{A-1} is the momentum of the residual nucleus. In addition to the convolution for x used by Oelfke, Sauer, and Coester [26], we introduce a convolution for the fragmentation variable z_s. As a result we express the nuclear structure functions $\mathcal{F}^A_{1,2,3}(x,z_s)$ as a double convolution

$$\mathcal{F}^A_{1,3}(x,z_s) = \sum_{p,n} \int dy\, dy_z \frac{1}{y} g^N(y,y_z) \mathcal{F}^N_{1,3}(x/y, z_s/y_z) \ , \tag{23}$$

$$\mathcal{F}^A_2(x,z_s) = \sum_{p,n} \int dy\, dy_z\, g^N(y,y_z) \mathcal{F}^N_2(x/y, z_s/y_z) \tag{24}$$

with

$$g^N(y,y_z) = \frac{Am_N}{m_A} \int d^4p\, y\, \delta\left[y - \frac{x}{x_N}\right] \delta\left[y_z - \frac{z_s}{z_{s,N}}\right] S_A(p) \ , \tag{25}$$

where we defined the convolution variables y and y_z to be the ratio of the measured and the effective x and z_s:

$$x = \frac{Q^2}{2P_A \cdot q} \frac{m_A}{m_N} \ , \quad x_N = \frac{Q^2}{2p \cdot q} \ ,$$
$$z_s = \frac{p_x^-}{P_A^- + q^-} \ , \quad z_{s,N} = \frac{p_x^-}{p^- + q^-} \ . \tag{26}$$

The relativistic spectral function in Eq. (25) can be related to the nonrelativistic spectral function [26]:

$$S_A(p)=\frac{m_A}{AE_N}\int dE\,\delta(p^0+\sqrt{m_{A-1}^2(E)+\mathbf{p}^2}-m_A)\times S_A^{NR}(\vec{p},E)\ . \qquad (27)$$

where E is the excitation energy of the residual nucleus.

Using that in the Bjorken limit

$$y=\frac{x}{x_N}\rightarrow\frac{p^-}{P_A^-}\frac{m_A}{m_N}\rightarrow\frac{p^0-p^3}{m_N}\ ,$$
$$y_z=\frac{z_s}{z_{s,N}}=\frac{p^-+q^-}{P_A^-+q^-}\rightarrow\frac{(y-x)m_N}{m_A-xm_N}\ , \qquad (28)$$

the convolution function (25) can be written as

$$g^N(y,y_z)=\int d^3\mathbf{p}\int dE\frac{m_A}{AE_N}yS_A^{NR}(\mathbf{p},E)\times\delta\left[y-\frac{p^0(E,\mathbf{p})-p^3}{m_N}\right]\times\delta\left[y_z-\frac{p^0(E,\mathbf{p})-p^3-xm_N}{m_A-xm_N}\right]\ , \qquad (29)$$

with

$$p^0(E)=m_A-\sqrt{m_{A-1}^2(E)+\mathbf{p}^2}\ . \qquad (30)$$

In general the convolution function violates the sum rule $\int dy\,dy_z g^N(y,y_z)=1$ because of the normalization of the spectral function. It is therefore necessary to introduce an *ad hoc* normalization factor in practical calculations such that the sum rule is satisfied.

The spectator mechanism corresponds to the emission of a nucleon correlated to the hit nucleon. When the spectator nucleon does not interact with the recoiling nucleus the spectator cross section is expressed in terms of the two-nucleon momentum distribution $n(z,y)$ which gives the joint probability for finding the observed proton with momentum fraction z and another nucleon with a momentum fraction y correlated in the nucleus. The spectator cross section thus reads

$$\frac{d^2\sigma_{\rm spec}^{\nu}}{dx\,dz}=\int dy\frac{d\sigma^{\nu n\rightarrow X}(x/y)}{dx}n(z,y)\ . \qquad (31)$$

B. The deuteron

In the deuteron the recoiling nucleus is just a free proton or neutron. Therefore $m_{A-1}(E)$ is just the proton or neutron mass. Also the spectral function is just the nucleon momentum distribution $n(p)=|\phi(p)|^2$ multiplied by a delta function which arises from the fact that the residual nucleus is just a free nucleon. It is simple to evaluate the integrals over E and $\mathbf{p}_\perp$ in Eq. (29):

$$g^N(y,y_z)=2\pi m_N^2 y\,\delta\left[y_z-\frac{m_N(y-x)}{m_d-xm_N}\right]\int_{p_{\min}}\frac{p\,dp}{E_N}n(p)\ , \qquad (32)$$

with

$$p_{\min}=\left|\frac{(m_Ny-m_d)^2-m_N^2}{2(m_d-m_Ny)}\right|\ .$$

Substituting Eq. (32) into Eq. (24) we find that

$$\mathcal{F}_2^d(x,z_s)=\sum_{p,n}\int dy\,f^N(y)\mathcal{F}_2^N\left[\frac{x}{y},\frac{z}{y-x}\right] \qquad (33)$$

with the convolution function

$$f^N(y)=2\pi m_N^2 y\int_{p_{\min}}\frac{p\,dp}{E_N}n(p)\ , \qquad (34)$$

and $z=p_x^-/m_N$.

The resulting cross section for the direct process thus reads

$$\frac{d^2\sigma_{\rm dir}^{\nu d}}{dx\,dz}=\frac{G^2m_NE}{\pi}\frac{1}{1-x}\sum_{\tau=p,n}\int_{x+z}^{m_d/m_N}dy\,f^N(y)\left\{\frac{2}{3}\mathcal{F}_2^{\nu\tau}\left[\frac{x}{y},\frac{z}{y-x}\right]+\frac{x}{3y}\mathcal{F}_3^{\nu\tau}\left[\frac{x}{y},\frac{z}{y-x}\right]\right\}\ . \qquad (35)$$

The spectator contribution of Fig. 6(b) can be calculated, giving

$$\frac{d^2\sigma_{\rm spec}^{\nu d}}{dx\,dz}=\int_x^{m_d/m_N}dy\frac{d\sigma^{\nu n\rightarrow X}(x/y)}{dx}\times f^N(y)\delta\left[z+y-\frac{m_d}{m_N}\right]\ . \qquad (36)$$

We note that the spectator process yields protons in both the forward ($z<1$) and the backward ($z>1$) hemisphere with equal probabilities. As a consequence of Eq. (36) the maximum value of z is m_d/m_N-x, which leads to a maximum p_3 momentum for backward protons:

$$p_3^{\max}=\frac{m_N^2-(m_d-xm_N)^2}{2(m_d-xm_N)}\ . \qquad (37)$$

C. Results

The cross section for slow protons is given by the sum of the direct (35) and the spectator cross section (36). The convolution function (34) is calculated using the deuteron momentum distribution as parametrized by Machledit, Holinde, and Elster [28]. To investigate the A dependence of $R(x,\Delta z)$, we calculated R for the free proton (see Fig. 7) using the renormalized fragmentation functions. In Fig. 8 we show the contribution of the direct and the spectator process to the ratio $R(x,\Delta z)$ of

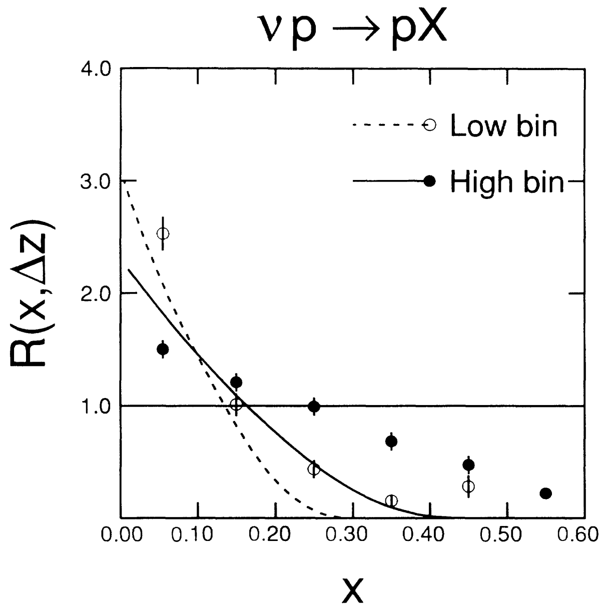

FIG. 7. $R(x,\Delta z)$ for neutrino scattering on hydrogen using the renormalized four-quark fragmentation functions compared to the data of Ref. [3].

events with and without slow protons for (anti)neutrino scattering on the deuteron. The ratio is plotted for the same momentum bins as in Sec. II. It is seen that the qualitative features of the data are reproduced. Note that in contrast to Ref. [8] we have fitted our fragmentation model to the free nucleon data, which results in a better agreement with experiment for the deuteron case. The direct contribution, which does not depend strongly on

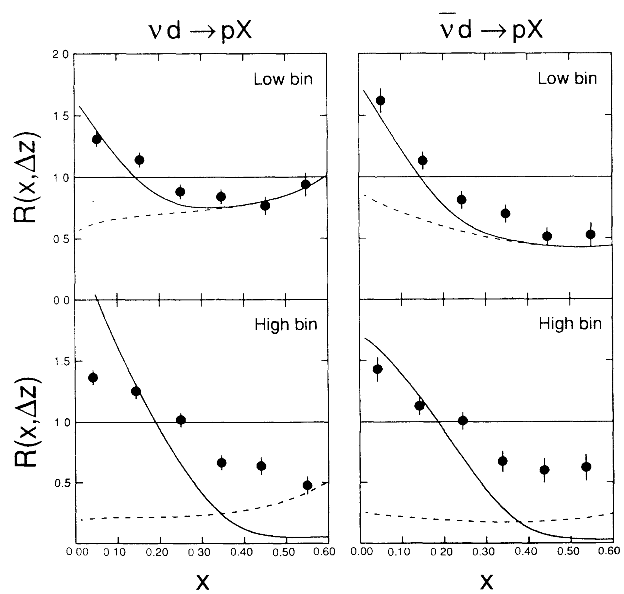

FIG. 8. Ratio $R(x,\Delta z)$ for neutrino and antineutrino scattering on deuterium compared to the data of Ref. [3]. Also shown are the separate contributions of the direct (dotted) and spectator (dashed) process for the low bin. (For the low bin the spectator contribution has been multiplied by 10.)

z, is only slightly modified by Fermi smearing. It still falls off rapidly with increasing x and can be neglected for $x > 0.5$. The spectator contribution is only significant in the low momentum bin where it is comparable to the direct process. (In Fig. 8 the spectator contribution in the low bin has been multiplied by 10.) The x dependence of R is governed by the structure of the convolution function $f(y)$, which is sharply peaked at $y = 1$. The momentum bins are such that the low bin is sensitive to the (sharp) edges of the deuteron momentum distribution and the high bin to the tails. This implies that the spectator contribution is extremely sensitive to the lower cutoff in the proton momentum. For example, a 10% higher value than the one reported in Ref. [3] reduces the spectator contribution by 80% in the low bin.

The difference in the x dependence of R for the neutrino and antineutrino is due to the difference of the quark distributions in the proton and the neutron which appear in the spectator contribution: In contrast to inclusive scattering where one averages over neutrons and protons, the observation of a spectator proton selects events where the (anti)neutrino scatters a neutron.

IV. DISCUSSION AND SUMMARY

Motivated by recent experimental data we have investigated the production of slow protons in semi-inclusive deep-inelastic scattering on $A = 1$ and 2 systems. For the free proton we compared a fragmentation and a pionic description for sea-initiated processes. The latter yields a much smaller cross section for small x values. This is in apparent agreement with the data [23].

Using the renormalized fragmentation functions we can qualitatively explain the dilution of the x dependence when going from $A = 1$ to 2. For the low momentum bin there is a competition between the direct and the spectator process, whereas protons in the high bin come predominantly from the fragmentation.

There remains an important problem to be investigated: we have not considered the rescattering of the struck nucleon with the residual nucleus as well as the rescattering of produced fragments. We do not expect the rescattering of fragments to be of great influence, as the hadronization length in the laboratory system is larger than the deuteron size [29]. The rescattering of the struck nucleon, however, can be important. Both the magnitude of the cross section and the x and z dependence will be modified. The precise effect of this cannot easily be calculated since we would need more information on the interaction of a highly excited nucleon with a nucleon. A very simple estimate of the rescattering cross section of the struck nucleon in a geometrical model shows that the role of prehadronization rescattering is non-negligible. However, the resulting ratio $R(x,\Delta z)$ is not changed drastically. It only flattened to give a slightly better description of the data.

Because the direct process contributes less than 1% to the cross section for slow backward protons, it is interesting to look separately at the forward and backward hemispheres [30]. Especially in the deuteron the backward protons will predominantly come from the spectator emission. Therefore, the spectrum of backward protons can provide us with valuable information on the high momentum part of the relativistic deuteron wave function and the convolution model used to calculate the spectator cross section.

In this paper we considered the simple cases of $A=1$ and 2. When going to heavier nuclei it is obvious that the effect of rescattering will become more important. But apart from the role of rescattering there are other properties to be studied in the higher A region. For instance, when deuterons are detected in neutrino scattering on ^{3}He scattering these will predominantly come from the spectator process. Because the momentum distribution and the nucleon structure functions are known, this process can serve as a test for the spectator mechanism. Also it can give information on the "deuteron content" of the ^{3}He wave function (that is assuming the spectator description is accurate).

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