Neutron skin of spherical nuclei in relativistic and nonrelativistic mean-field approaches

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We have studied the present theoretical understanding of size of neutron, proton, matter, and charge distributions in spherical nuclei. The ground-state nuclear radii have been evaluated in the framework of the relativistic mean-field approach within the Hartree approximation. A comparison has been done with the radii obtained with the nonrelativistic density-dependent Skyrme interactions. It is pointed out that with an increase in isospin of nuclei, the neutron rms radii in the relativistic mean-field approach are larger than both the empirical data and the predictions of the Skyrme mean field.

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The knowledge of radii and nucleon distributions in nuclei is important to understanding the nuclear structure and properties of ground state as well as excited states of nuclei. Precision value of the size of nuclei is required for extracting the nuclear compressibility from breathing-mode energies and for the sum-rule strengths of giant multipole resonances, to quote only a few examples. The ground-state nuclear properties have been widely studied and measured over last several decades. Various experimental approaches have been followed to map out the distribution of nucleons inside nuclei. The electron scattering has been employed amply to extract the charge distribution of nucleons in nuclei, and inelastic scattering of low- and medium-energy hadrons have been used to infer the matter distribution in nuclei. Isotopic shifts from muonic atoms and laser spectroscopy provide very accurate information on the charge distributions in nuclei. This is in many cases compatible with the charge distributions obtained from the elastic electron-scatering experiments. A detailed discussion of various nucleonic distributions obtained from different sources has recently been provided in the review of Ref. [1].

Theoretically, the ground-state root-mean-square (rms) radii of nuclei have been obtained using both the nonrelativistic density-dependent Skyrme interactions and the relativistic mean-field (RMF) approaches. Hartree-Fock calculations [2] have since long been used to provide a measure of the size of nuclei. Many density-dependent zero-range and finite-range Skyrme type of interactions have been used to calculate the ground-state properties of nuclei and nuclear matter. The Skyrme interaction [3] (SkM*) and the Gogny force [4] (D1), in particular, have been very successful in reproducing the charge radii of nuclei. On the other hand, the quantum hadrodynamical approach [5] in the relativistic mean-field approximation has recently been shown [6] to be successful for calculating the ground-state properties of spherical nuclei from light to the heavier ones. Especially, the parameter set NL1 has achieved a striking success in reproducing the binding energies and charge radii of nuclei. It may be noted that the RMF theory is very appealing in view of its built-in spin-orbit interaction and the density dependence. In contrast, the phenomenological Skyrme interactions assume a density dependence at the outset. In general, the ability to reproduce the experimentally obtained nucleon distributions provides a first and foremost criteria for the goodness of any nuclear interaction.

In both the nonrelativistic and relativistic approaches, the parameter sets (the forces) have been fitted to reproduce the empirical binding energy and the charge radii, the data known rather accurately. Empirical data on the matter as well as on the neutron distributions have been indirect. The theoretical calculations have thus largely compared the results on the charge radii. In this paper, we analyze the neutron and matter distributions in nuclei in the relativistic as well as in the nonrelativistic approaches. The calculations have been performed in the relativistic Hartree approximation with the parameter sets NL1 and NL2 for a set of closed- and open-shell spherical nuclei. A comparative study of nuclear sizes has been performed with the Skyrme interactions SkM* and SIII in the Hartree-Fock approach. We have focused our attention on the size of neutron skin in nuclei and a comparison has been made with the available empirical data.

We have calculated the ground-state properties of nuclei in the relativistic mean-field approach [5] in the Hartree approximation. The relativistic Hartree equations are solved using the method [6] based on an expansion in a basis of spherically symmetric harmonic-oscillator wave functions. The basis includes states up to a certain major shell quantum number N_F (N_B) in the expansion for fermions (bosons). For light nuclei,

$N_F = 10$ and $N_B = 10$ suffice to achieve a reliable convergence. For medium-heavy and heavy nuclei, we extend both the fermionic and bosonic basis upto 20 shells. The details of the expansion method have been provided in Ref. [6].

The open-shell nuclei have been treated with a phenomenological pairing of the BCS type with a constant pairing gap taken from the difference of particle seperation energies of neighboring nuclei. The parameter sets NL1 and NL2 have been employed for all nuclei. The parameter set NL1 has been found to be very successful [6] for the ground-state properties of many nuclei all over the periodic table. It was fitted [7] to reproduce the ground-state properties of light and heavy nuclei including two isotopes of Ca and Sn. The effective mass of the set NL1 is 0.57 and the incompressibility of nuclear matter is 211 MeV. These two quantities determine the equation of state of nuclear matter. In contrast, the parameter set [8] NL2 represents an other extreme on the value of the incompressibility with a value of 400 MeV. The effective mass for this parameter set is also higher than that of NL1. The coupling constants and parameters of both these sets can be found in Ref. [8].

We have included several closed-shell and open-shell nuclei in our calculations. The closed-shell nuclei studied are ^{40}Ca, ^{48}Ca, and ^{208}Pb. The open-shell nuclei have been taken to be two isotopes of Ni (^{58}Ni and ^{64}Ni), ^{90}Zr, two isotopes of Sn (^{116}Sn and ^{124}Sn), and ^{140}Ce. This provides a representative spectrum of spherical nuclei over a large mass and isospin range. We have calculated the binding energies, charge radii, and matter radii of nuclei. Binding energies calculated with the parameter set NL1 show an overall good agreement with the experimental values. The parameter set NL2, on the other hand, overpredicts the binding by about 1–3 % in most of the cases.

The charge radii have been obtained by folding the proton distribution with its form factor as $r_c = ({r_p}^2 + 0.64)^{1/2}$. A comparison of the predictions has also been made with the available experimental charge radii [10]. It is worth noting that the parameter set NL1 reproduces the experimental charge radii of nuclei very well. The agreement is good, in particular, for the heavier nuclei. The parameter set NL2 also shows a reasonable agreement with the experimental data. It may be mentioned that the charge radii along with the binding energies have been the prime data which have been used to adjust the parameters of various interactions.

As a further example of the goodness of the paramater set NL1, we have calculated the charge radii of a chain of Sn isotopes. The results are shown in Table I. The solution of the relativistic Hartree equations has been obtained in the coordinate space as against the oscillator expansion method mentioned above. The Sn nuclei being superfluid, pairing correlations have been incorporated phenomenologically in the BCS framework. The pairing gap has been taken to be constant with its value derived from empirical data [9]. The experimental charge radii shown in Table I are from precision measurements [11] using muonic atoms. It is amazing that the theoretical charge radii agree remarkably with the experimental data. This demonstrates that the parameter set NL1 is well suited to reproducing the charge radii of nuclei.

TABLE I. The charge radii (r_c in fm) of various Sn isotopes obtained in the relativistic Hartree approach (calc.) with the parameter set NL1. The relativistic equations have been solved in the coordinate space as compared to the solutions obtained using the oscillator expansion method. The experimental (expt) charge radii (r_c) measured [11] from muonic atoms are shown for comparison. The agreement of the theoretical predictions with the empirical data is seen to be remarkable to the fourth decimal place.

Nucleus	r_c(calc)	r_c(expt)
^{112}Sn	4.5941	4.5958 ± 0.0005
^{114}Sn	4.6106	4.6103 ± 0.0005
^{116}Sn	4.6260	4.6261 ± 0.0005
^{118}Sn	4.6398	4.6395 ± 0.0005
^{120}Sn	4.6518	4.6522 ± 0.0006
^{122}Sn	4.6628	4.6633 ± 0.0006
^{124}Sn	4.6734	4.6736 ± 0.0006

The rms radii (r_n) of neutron distribution calculated with the relativistic parameter sets are shown in the second and third columns of Table II. For comparison with the experiments, we have taken r_n (shown by expt) from the data of 800 MeV polarized proton scattering as summarized in Ref. [1]. The parameter set NL1 gives neutron radii slightly larger than NL2 except in a few nuclei. Both the sets NL1 and NL2, on the other hand, overestimate the empirical r_n except for a few cases. As the neutron excess increases, the parameter set NL1 overestimates the empirical values by about 2–3 %. The difference between the two increases with increase in neutron asymmetry. For ^{40}Ca, the empirical value [1] of r_n seems to be larger than the value from the analysis of optical potential in inelastic scattering. The latter data give r_n of about 3.37 fm and is close to the value from NL1. Here we do not include the experimental data from medium-energy inelastic scattering due to large uncertainties in determining the neutron radii.

The Skyrme interaction SkM* has been employed extensively to calculate the ground- and excited-state properties of nuclei in the Hartree-Fock approach [3, 12]. The Skyrme force SIII, in comparison, has a hard equation of state with incompressibility 356 MeV. It, however, reproduces the ground-state properties of nuclei well [2]. We have performed the Hartree-Fock calculations with both SkM* and SIII as two extreme interactions for the set of nuclei as discussed above. The pairing correlations have been included for the open-shell nuclei in the BCS formalism. Both SkM* and SIII reproduce the experimental binding energies reasonably well for all nuclei except ^{48}Ca which seems to be an exceptional case in many of its properties. The good agreement of the Skyrme interactions SkM* and SIII with the empirical data derives partly from the fact that their parameters have been fitted to reproduce the ground-state properties of nuclei [2, 3].

The force SkM* reproduces the empirical charge radii well. SIII, on the other hand, overestimates the charge radii for heavy nuclei slighly. It may be mentioned that

TABLE II. Neutron rms radii (r_n in fm) with different approaches. The experimental data in the last column (expt) refer to experiment on 800 MeV polarized proton scattering [1].

Nucleus	r_n (NL1)	r_n (NL2)	r_n (SkM*)	r_n (SIII)	r_n (expt)
^{40}Ca	3.363	3.310	3.377	3.362	3.491
^{48}Ca	3.645	3.626	3.600	3.600	3.625
^{58}Ni	3.690	3.722	3.699	3.735	3.700
^{64}Ni	3.983	3.961	3.899	3.913	3.912
^{90}Zr	4.340	4.314	4.284	4.312	4.289
^{116}Sn	4.758	4.743	4.658	4.659	4.692
^{124}Sn	4.917	4.925	4.786	4.803	4.851
^{140}Ce	5.060	5.02	4.942	4.971	4.971
^{208}Pb	5.799	5.744	5.619	5.646	5.593

SkM* is considered to be one of the best Skyrme interactions. It has been shown [12, 13] that SkM* succeeds in getting closer to the precision data [14] on breathing-mode energies of medium-mass nuclei within 1 MeV; SIII, however, fails drastically in predictions of the breathing-mode energies. Thus, we have chosen SIII only to illustrate its neutron radii though it provides bad excited-state properties.

We focus our attention on neutron radii of nuclei. Table II shows the neutron radii of nuclei for both the relativistic and the Skyrme approaches. The empirical data are from the analysis of 800 MeV polarized proton scattering [1]. A comparison of the SkM* and SIII values with the experimental data shows that SkM* achieves a reasonably good agreement with the the experimental data. Only in a few cases it underestimates the data. The predictions of SIII are also comparable to those of SkM* and the experiments.

For the relativistic parameter sets the situation is different. The neutron rms radii from NL1 are slightly larger than those of NL2. Both NL1 and NL2 show a reasonable agreement with the data on near symmetric nuclei. For neutron-rich nuclei, the neutron rms radius for the relativistic mean field is consistently larger than those from Skyrme interactions and the experimental data. For the best available parameter set NL1, the predicted neutron radii for asymmetric nuclei are 2–3 % larger than the empirical data. For the heaviest nucleus ^{208}Pb considered here, NL1 predicts the neutron rms radii about 3% larger than that in SkM* and the empirical data. This difference persists even for the parameter set NL2, but on a lesser scale. Thus, the relativistic mean field that reproduces the binding energies and charge radii of the neutron asymmetric nuclei correctly gives larger neutron radii. This is clearly illustrated by Fig. 1, where we plot the difference in neutron and proton radii ($r_n - r_p$) against $(N - Z)/A$. The relativistic and nonrelativistic results form two separate bands of data. For ^{40}Ca [$(N - Z)/A = 0$], both the relativistic and nonrelativistic results coincide. The neutron-proton radii differences for the sets NL1 and NL2 follow each other, NL1 giving higher $r_n - r_p$ than NL2 occasionally and vice versa. The radii difference for SIII is, in most cases, smaller than for SkM*. The radii differences for the Gogny force [4] D1 (not shown) are notably closer to those of SIII for heavy nuclei. As the asymmetry increases, the gap between the results of the relativistic and nonrelativistic sets increases, the relativistic sets predicting larger $r_n - r_p$ values. A larger slope for the parameter sets NL1 and NL2 as compared to that for Skyrme and the empirical data reveals inevitably that the relativistic mean field provides larger neutron radii when extra neutrons are added to nuclei. It may be noted that the asymmetry energy for both sets NL1 and NL2 is about 43 MeV in contrast with the empirical value of about 32 MeV and its value for the Skyrme interactions is about 30–32 MeV. A reduction in the ρ-meson strength in order to adjust the asymmetry energy brings about only a minor change in the differences in radii.

In Fig. 2 we show $(r_n - r_p)/r_m$ as a function of $(N - Z)/A$, where r_m is the matter radius. The theoretical values are shown by empty symbols for both the relativistic sets (joined by solid lines) and the Skyrme interactions (joined by dots). We display the results only for asymmetry parameter $(N - Z)/A > 0.1$. A clear distinction between the bands of the relativistic and Skyrme results emerges once again. NL1 (circle) and NL2 (triangle) results follow each other. In most of the cases, the results of NL1 and NL2 coincide, and only for a few nuclei

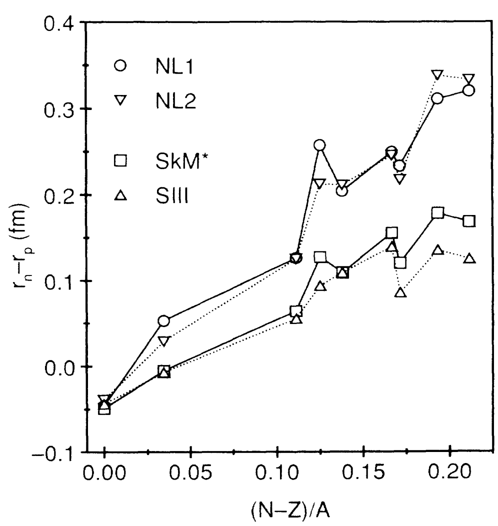

FIG. 1. Neutron-proton radii diiference $r_n - r_p$ for the relativistic parameter sets (NL1 and NL2) and the Skyrme interactions (SkM* and SIII).

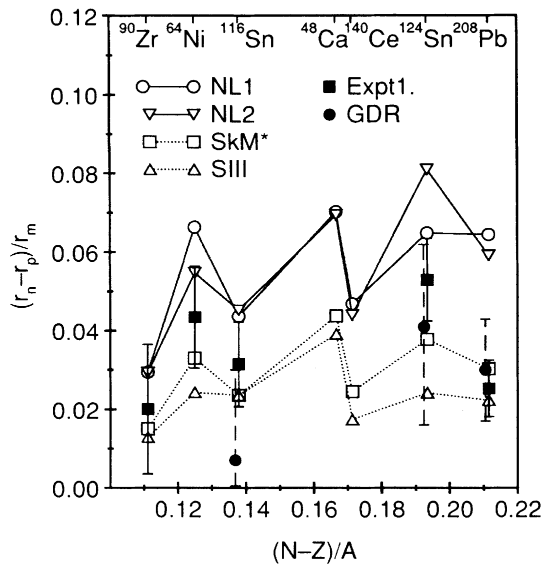

FIG. 2. $(r_n - r_p)/r_m$ for relativistic parameter sets (NL1 and NL2) and the Skyrme interactions (SkM* and SIII). The experimental data (Exptl) from 800 MeV polarized proton scattering [1] are also shown. The recent results [15] on neutron skin thickness from isovector giant dipole resonance (GDR) are shown by full circles.

do NL1 values exceed those of NL2. The Skyrme results also follow each other. SkM* (square) values overestimate those of SIII (delta). It is worth noting that there is a parallel between the up and down behavior in the predictions of relativistic sets and those of Skyrme sets. For the Skyrme interactions, the overall slope of the band remains more or less constant as the asymmetry parameter $(N - Z)/A$ increases. This slope of the band for the relativistic sets seems, however, to increase with the asymmetry, thus implying that the relativistic mean field predicts larger neutron radii as the neutron asymmetry increases.

The experimental data available on the radii differences are shown in Fig. 2 by solid points. The data derived from 800 MeV polarized proton scattering [1] (exptl) are shown by a solid square with error bars indicated by a solid line. For ^{90}Zr, ^{116}Sn, and ^{208}Pb, the empirical data are consistent with the Skyrme interactions. The data on ^{64}Ni and ^{124}Sn fall, however, between the predictions of the relativisitic and Skyrme sets. For the well studied case of ^{208}Pb, the relativistic sets clearly overpredict the experimental data. We also show for comparison the recent experimental results [15] (solid circle with dashed error bars) on radii differences extracted from the study of isovector giant dipole resonance (GDR) excited in ^{116}Sn, ^{124}Sn, and ^{208}Pb in inelastic alpha scattering. The results within the error bars fall in the domain of the Skyrme interactions. Especially for ^{208}Pb, the size of the neutron skin is consistent with that from Skyrme interactions. On this nucleus, both the experimental data presented in Fig. 2 give smaller skin thickness than the relativistic sets. Thus, within the uncertainties the relativistic mean field provides higher values of the neutron skin thickness in neutron-rich nuclei.

In conclusion, we have studied the neutron rms radii of nuclei in the relativistic and the Skyrme mean-field approaches. It has been shown that the relativistic mean-field theory, which is capable of reproducing the ground-state binding energies and charge radii of nuclei extremely well, overestimates the neutron skin thickness of asymmetric nuclei over that of the Skyrme interactions and the empirical data. This behavior of the relativistic mean-field is not yet clear and futher work is in progress.

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[1] C.J. Batty *et al.*, Adv. Nucl. Phys. **19**, 1 (1989).
[2] P. Quentin and H. Flocard, Annu. Rev. Nucl. Part. Sci. **28**, 523 (1978).
[3] M. Brack, C. Guet and H.-B. Håkansson, Phys. Rep. **123**, 275 (1985).
[4] J. Dechargé and D. Gogny, Phys. Rev. C **21**, 1568 (1980).
[5] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. **16** 1 (1986).
[6] Y.K. Gambhir, P. Ring, and A. Thimet, Ann. Phys. (N.Y.) **511**, 129 (1990).
[7] P.G. Reinhard, M. Rufa, J. Maruhn, W. Greiner, and J. Friedrich, Z. Phys. A **323**, 13 (1986).
[8] P.G. Reinhard, Rep. Prog. Phys. **52**, 439 (1989).
[9] A.H. Wapstra and G. Audi, Nucl. Phys. **A432**, 1 (1985).
[10] H. de Vries, C.W. de Jager, and C. de Vries, At. Data Nucl. Data Tables **36**, 495 (1987).
[11] C. Piller *et al.*, Phys. Rev. C **42**, 182 (1990).
[12] P. Gleissl, M. Brack, J. Meyer, and P. Quentin, Ann. Phys. (N.Y.) **197**, 205 (1990).
[13] M.M. Sharma, W. Stocker, P. Gleissl, and M. Brack, Nucl. Phys. **A504**, 337 (1989).
[14] M.M. Sharma, W.T.A. Borghols, S. Brandenburg, S. Crona, J.H. Meier, M.N. Harakeh, and A. van der Woude, Phys. Rev. C **38**, 2562 (1988).
[15] A. Krasznahorkay *et al.*, Phys. Rev. Lett. **66**, 1287 (1991).