Form factors and gauge invariance in pion photoproduction

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We consider the effect of strong-vertex form factors on the gauge invariance of Born diagrams for the pion photoproduction reaction. The minimal-substitution prescription of Ohta for the electromagnetic current operator in pion photoproduction yields the same result as the simplest Born approach for one particular invariant amplitude. This is incompatible with recipes that multiply the Born amplitude by an overall form factor.

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There has recently been a revival in the study of off-shell behavior and the use of form factors in electromagnetic interactions [1–3]. The emphasis has generally been on the constraint imposed by gauge invariance in such processes. Here we will consider the effect of strong-vertex form factors on the pion-photoproduction amplitude. The following will deal specifically with the modification of Born diagrams. Clearly, we are not the first to address this question. Our intention is to give a description of this process at the level of amplitudes rather than operators. The approach is very simple and thus may be of some pedagogical value. We will also comment on what can be learned from more microscopic investigations and comparisons with amplitudes found from data analysis.

As we wish to make our discussion as transparent and free from notation as is possible, we first consider the simplest (pseudoscalar) πNN vertex in a specific reaction, the process $\gamma p \to n\pi^+$. We have contributions from the three Born diagrams corresponding to the s-, t-, and u-channel exchanges:

$$\epsilon\cdot M_{fi}=ge\overline{u}_n\gamma_5\frac{(p_1+k)\cdot\gamma+m}{s-m^2}\left[\epsilon\cdot\gamma-\frac{\kappa_p}{2m}\epsilon\cdot\gamma k\cdot\gamma\right]u_p$$
$$+2ge\overline{u}_n\frac{q\cdot\epsilon}{t-\mu^2}\gamma_5 u_p$$
$$-ge\overline{u}_n\frac{\kappa_n}{2m}\epsilon\cdot\gamma k\cdot\gamma\frac{(p_2-k)\cdot\gamma+m}{u-m^2}\gamma_5 u_p\ . \qquad (1)$$

In the above, we have let k and q represent the photon and pion four-momenta, and have taken p_1 and p_2 to be the respective proton and neutron four-momenta. The quantities m and μ are the nucleon and pion masses, ϵ is the photon polarization vector, g is the pseudoscalar πNN coupling constant, κ_p and κ_n are the proton and neutron anomalous magnetic moments, and s, t, and u are the usual Mandelstam variables. Some pieces of the above amplitude are individually gauge invariant (satisfying the condition [4] $k\cdot M_{fi}=0$) specifically the magnetic moment terms. If one considers only those pieces which must cancel between diagrams in order to have gauge invariance, one is left with

$$\epsilon\cdot\overline{M}_{fi}=ge\overline{u}_n\left[\frac{2p_1\cdot\epsilon}{s-m^2}+\frac{2q\cdot\epsilon}{t-\mu^2}\right]\gamma_5 u_p\ , \qquad (2)$$

which indeed satisfies the condition $k\cdot\overline{M}_{fi}=0$.

If we put a strong form factor at the πNN vertex of each Born diagram, gauge invariance will be lost, since the s- and t- channel contributions to $\epsilon\cdot\overline{M}_{fi}$ will be multiplied by different momentum-dependent factors. In order to restore gauge invariance, Ohta has derived [2] an additional amplitude. (Furthermore, we may add here that Ohta addresses the problem on the operator level.) Consider the effect of adding strong form factors to the pseudoscalar Born calculation. Our relation for those pieces which must cancel between diagrams would read

$$\epsilon\cdot\overline{M}_F=ge\overline{u}_n\left[\frac{2p_1\cdot\epsilon}{s-m^2}G(s,\mu^2,m^2)+\frac{2q\cdot\epsilon}{t-\mu^2}G(m^2,t,m^2)\right]\gamma_5 u_p\ , \qquad (3)$$

where $G(s,t,u)$ is a general form factor. From Ohta's relations [2] we see that the term required to restore gauge invariance is

$$\epsilon\cdot\Delta M=ge\overline{u}_n\left[\frac{2p_1\cdot\epsilon}{s-m^2}[G(m^2,\mu^2,m^2)-G(s,\mu^2,m^2)]+\frac{2q\cdot\epsilon}{t-\mu^2}[G(m^2,\mu^2,m^2)-G(m^2,t,m^2)]\right]$$
$$\times\gamma_5 u_p\ , \qquad (4)$$

with the condition $G(m^2,\mu^2,m^2)=1$. From Eqs. (3) and (4) it is clear that the amplitude ΔM precisely cancels the form-factor modification of specific pieces of the Born amplitude. The combination $\epsilon\cdot(\overline{M}_F+\Delta M)$ gives back the result, displayed in Eq. (2), for a pointlike coupling.

This result becomes more interesting if one considers the influence of off-shell effects at the reducible electromagnetic vertex, to be used with the bare (free) propagator. It can be shown [1] that those pieces of the γNN and $\gamma\pi\pi$ vertex which couple to the charge are not modified by an off-shell pion or nucleon leg when the $k^2=0$ condition holds. [These couplings give us the two terms in Eq. (2).] The remaining pieces in Eq. (1) certainly are modified by off-shell effects at the strong and electromagnetic vertices. However, since these terms are individually gauge invariant, the form of this modification is not constrained.

The above argument can be extended to include the most general [1] pion-nucleon vertex

$$\Lambda(p',p)=\gamma_5 G_1+\gamma_5\frac{p\cdot\gamma-M}{M}G_2+\frac{p'\cdot\gamma-M}{M}\gamma_5 G_3 +\frac{p'\cdot\gamma-M}{M}\gamma_5\frac{p\cdot\gamma-M}{M}G_4\ , \tag{5}$$

with four independent form factors. The general result is more easily expressed if we first decompose the photoproduction amplitude into invariant amplitudes [5]

$$\epsilon\cdot M_{fi}=\bar{u}_n\sum_{j=1}^{4}A_jM_ju_p\ , \tag{6}$$

with the explicitly gauge invariant representation

$$M_1=-\gamma_5\epsilon\cdot\gamma k\cdot\gamma\ , \tag{7}$$

$$M_2=2\gamma_5(\epsilon\cdot p_1 k\cdot p_2-\epsilon\cdot p_2 k\cdot p_1)\ , \tag{8}$$

$$M_3=\gamma_5(\epsilon\cdot\gamma k\cdot p_1-\epsilon\cdot p_1 k\cdot\gamma)\ , \tag{9}$$

$$M_4=\gamma_5(\epsilon\cdot\gamma k\cdot p_2-\epsilon\cdot p_2 k\cdot\gamma)\ . \tag{10}$$

Note that such a representation is only possible for a gauge invariant amplitude. For the Born terms we have, from Eq. (1),

$$A_1=\frac{ge}{s-m^2}(1+\kappa_p)+\frac{ge}{u-m^2}\kappa_n\ , \tag{11}$$

$$A_2=\frac{2ge}{(s-m^2)(t-\mu^2)}\ , \tag{12}$$

$$A_3=\frac{ge}{s-m^2}\frac{\kappa_p}{m}\ , \tag{13}$$

$$A_4=\frac{ge}{u-m^2}\frac{\kappa_n}{m}\ . \tag{14}$$

From the above, we can see that the amplitude given in Eq. (2) corresponds to A_2. In terms of invariant amplitudes, we can then state that A_2 is unaltered by form factors at the pseudoscalar pion-nucleon vertex, given Ohta's prescription for regaining gauge invariance.

A consideration of the pseudovector coupling provides a simple example of the above argument, applied to a more complicated pion-nucleon vertex. It is well known that the pseudovector set of Born terms can be written as the pseudoscalar set plus magnetic moment terms which contribute only to A_1. Thus, in a pseudovector coupling scheme, we also have an invariant amplitude A_2 which is unaltered by off-shell effects. An explicit calculation, using Ohta's minimal substitution prescription, shows that this amplitude A_2 does not change even if one uses the most general pion-nucleon vertex. [The unaltered Chew-Low-Goldberger-Nambu (CGLN) amplitude [5] is A_2. This corresponds to the Ball amplitudes [6] A and B.] This result holds for the photoproduction of both charged and neutral pions.

The above argument does not prove that A_2 is completely free of modfications due to internal structure. The result of Ohta is only unique up to individually gauge invariant terms. Thus, in principle one could add a term proportional to M_2, since M_2 is constructed to be gauge invariant. We are only demonstrating that a simple minimal-substitution prescription does not give us this term. In order to learn more one must go to a microscopic model [7].

Some recent dynamical calculations of meson photoproduction [8] have applied form factors to the Born amplitude. In these calculations, an "average" form factor [typically of the type $\Lambda^2/(\Lambda^2+Q^2)$, with Q being the relative meson-nucleon three-momentum] has been utilized. Such a form factor does not isolate A_2 as being in any way special, in contrast to Ohta's minimal-substitution prescription. Each multipole amplitude is also modified in the same manner. Studies of recent partial-wave analyses [9] of pion-photoproduction data may yield clues to the way multipole amplitudes behave at moderate energies.

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[4] At this point, we should remind the reader that we are dealing with amplitudes rather than operators. The reader may be more familiar with the basic operator constraint:

$$k^{\mu}M_{\mu}(p_2,q;\ p_1,k)=e[\Delta^{-1}(q)\Delta(q-k)\Lambda_5(p_2,p_1) - \Lambda_5(p_2,p_1+k)S(p_1+k)S^{-1}(p_1)]$$

considered in Ref. [1]. When all external legs are on shell, this reduces to the gauge invariance constraint which we are using.

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