Neutral pion condensation in quark matter including vacuum fluctuation effects

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Neutral pion condensation in quark matter is investigated including vacuum polarization effects. The vacuum instability is removed by eliminating the Landau ghost from the meson propagators.

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Relativistic field theoretical models describing fermions moving in background meson fields are frequently used to describe the structure of nucleons [1], nuclei [2], or infinite matter like quark matter [3] and nuclear matter [4]. Since these models are relativistic, they are capable of describing the change of the vacuum structure due to the presence of valence particles, i.e., the vacuum fluctuation effects. However, in a series of papers [5] it has been pointed out that, in these nonasymptotically free field theories, the vacuum by itself exhibits an instability in the following sense: If one applies the loop expansion and the standard renormalization procedure, the energy of the nontranslational invariant vacuum in which the background meson fields have finite momenta can be made arbitrarily low relative to the translational invariant vacuum by increasing the momenta of the meson fields. It has been pointed out that this instability is related to the "Landau ghost" [6] in the meson propagators, i.e., poles of the meson propagators at spacelike momenta of the order $\sqrt{-q^2}\approx 1$ GeV. Recently a method to remove the Landau ghost has been proposed [7] by extending the ideas of Redmond and Bogoliubov, Logunov, and Shirkov [8] to the case of finite baryon density. In this paper we will show that the vacuum instability discussed above can be removed by using the method of Ref. [7]. We will use the chiral σ model [9] as a model for quark matter. The mechanism by which the meson fields acquire finite momenta is the familiar pion condensation [10]. Recently [3], this model with a neutral pion condensate [11] has attracted attention because, due to its simplicity and transparency, it can give valuable insights into the more complicated calculations for finite solitons. In particular, we can use this model to get insight into the role of a classical pion field in quark systems [1], and this point, together with the problem of the vacuum instability, motivates our work.

The Lagrangian of the linear σ model is, assuming exact chiral symmetry ($m_\pi=0$),

$$\mathcal{L}=\bar{\psi}[i\partial\!\!\!/-g(\sigma+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})]\psi+\tfrac{1}{2}(\partial_\mu\sigma)^2+\tfrac{1}{2}(\partial_\mu\boldsymbol{\pi})^2-\frac{\lambda^2}{4}(\sigma^2+\boldsymbol{\pi}^2-v^2)^2\ , \quad (1)$$

where ψ is the quark field and σ and $\boldsymbol{\pi}$ are the chiral meson fields. Here $v=f_\pi$, where f_π is the pion decay constant, and λ^2 is related to the σ meson mass by $\lambda^2=m_\sigma^2/2v^2$.

We assume neutral pion condensation with the following standing wave configuration of the classical meson fields [3,11]:

$$\sigma=\tilde{v}\cos\mathbf{q}\cdot\mathbf{x}\ ,\quad \pi_1=\pi_2=0\ ,\quad \pi_3=\tilde{v}\sin\mathbf{q}\cdot\mathbf{x}\ . \quad (2)$$

The radius of the chiral circle $\tilde{v}$ and the momentum $\mathbf{q}$ are treated as variational parameters. The translational invariant vacuum (zero density) is characterized by $\tilde{v}=v$ and $\mathbf{q}=0$. In the case $\mathbf{q}=0$, the quantity $\tilde{m}=g\tilde{v}$ has the meaning of an effective quark mass. The parameters of the model are the free quark mass $m=gv$ and m_σ.

The meson fields (2) are obtained from the "normal state" ($\sigma=\tilde{v}$, $\boldsymbol{\pi}=0$) by applying a chiral rotation. Since this rotation is a local one, the energy changes and the pion condensed state might become the ground state.

With the meson fields (2), the Dirac equation for the quarks can be solved analytically by undoing the chiral rotation mentioned above, and the spectrum is given by [11]

$$E(\mathbf{p})=\pm E^{\pm}(\mathbf{p})=\pm\left[\mathbf{p}^2+\tilde{m}^2+\frac{\mathbf{q}^2}{4}\pm\sqrt{(\mathbf{p}\cdot\mathbf{q})^2+\mathbf{q}^2\tilde{m}^2}\right]^{1/2}\ . \quad (3)$$

A useful concept in Lagrangian field theories is the effective action (Γ) which is a functional of the classical meson fields, from which the energy density of the system can be obtained directly. In actual calculations one has to specify an expansion scheme to approximate Γ, and the one used most commonly is the loop expansion. In this scheme, the one-loop approximation is identical to the familiar Hartree approximation, and the energy density of the system simply consists of the contribution due to the classical meson fields and a sum over the energies of all occupied fermion states with fixed background meson fields. In our case,

$$\epsilon=\epsilon_B+\epsilon_F+\epsilon_D\ , \quad (4)$$

where ϵ_B is the energy density of the classical meson fields,

$$\epsilon_B=\tfrac{1}{2}\bar{v}^2\mathbf{q}^2+\frac{m_\sigma^2}{8v^2}(\bar{v}^2-v^2)^2\ , \tag{5}$$

and ϵ_F and ϵ_D are the contributions of the Fermi and Dirac sea, respectively, given by

$$\epsilon_X=P_X\frac{\gamma}{2}\int\frac{d^3p}{(2\pi)^3}[E^+\theta(\Lambda_X-E^+)+E^-\theta(\Lambda_X-E^-)]\ , \tag{6}$$

where $X=F$ or D, $P_F=1$, $P_D=-1$, Λ_F equals the Fermi energy E_F, and Λ_D is an ultraviolet cutoff which we will let go eventually to infinity. $\gamma=12$ is the degeneracy factor. After adding the meson mass and wave-function renormalization counterterms, which are determined for zero density at the renormalization point $k^2=0$, ϵ_D becomes finite. Analytical forms can be found in Ref. [3]. The vacuum instability is due to the term ϵ_D, which for large q behaves like $-\mathbf{q}^2\ln|\mathbf{q}|$.

Next we remove the instability from the energy density. The prescription of Ref. [7] to accomplish this in the case of the familiar loop expansion is as follows: Calculate the effective action (or the energy density functional) in the usual way up to some order in the expansion. Take the Lorentz covariant part (i.e., that part which does not explicitly depend on the density) of the resulting expression and expand it further around the vacuum values of the meson fields. The coefficient functions of the quadratic terms, which are the inverse meson propagators at zero density in this approximation including the Landau ghost, should be replaced by their ghost-subtracted counter parts which satisfy the Källén-Lehmann representation. The resulting effective action is the generating functional of all one-particle irreducible Green's functions, where the two-point functions now satisfy the Källén-Lehmann representation. In the case of the loop expansion, this "overall ghost subtraction," which is done at the end of the calculation, is indeed sufficient in the following sense: it leads to total meson propagators (including the density-dependent parts) which are free of the Landau ghost, and to an expression for the energy density which is free of the corresponding instability due to the high-momentum components of the meson fields.

We now follow this procedure and expand $\epsilon_B+\epsilon_D$ in powers of $s(\mathbf{x})=\sigma-v=\bar{v}\cos\mathbf{q}\cdot\mathbf{x}-v$, and $\pi(\mathbf{x})=\bar{v}\sin\mathbf{q}\cdot\mathbf{x}$ (we suppress the isospin index 3). The second-order contribution becomes, in momentum space,

$$\epsilon^{(2)}=-\frac{1}{2V}\int\frac{d^3p}{(2\pi)^3}[s(\mathbf{p})G_\sigma^{-1}(-\mathbf{p}^2)s(-\mathbf{p})+\pi(\mathbf{p})G_\pi^{-1}(-\mathbf{p}^2)\pi(-\mathbf{p})]\ , \tag{7}$$

where V is the volume and G_σ^{-1} and G_π^{-1} are the σ and π inverse propagators, respectively, for zero density:

$$\begin{aligned}G_\sigma^{-1}(p^2)&=p^2-m_\sigma^2-\Sigma_\sigma(p^2)\ ,\\ G_\pi^{-1}(p^2)&=p^2-\Sigma_\sigma(p^2)\ .\end{aligned} \tag{8}$$

Here Σ_σ and Σ_π are the one-fermion loop self-energies of the sigma and the pion. For large spacelike momenta they behave like $p^2\ln(-p^2)$, which leads to the Landau ghosts in G_σ, G_π and to the instability of the vacuum. Due to the above discussion we replace ϵ of Eq. (4) according to

$$\epsilon\to\epsilon_{\rm KL}=\epsilon+(\epsilon_{\rm KL}^{(2)}-\epsilon^{(2)})\ , \tag{9}$$

where $\epsilon_{\rm KL}^{(2)}$ is obtained from (7) by replacing the propagators G_α by the Källén-Lehmann ones Δ_α, which are free of the Landau ghosts and satisfy a Källén-Lehmann representation. We wish to point out that, although our method can be applied systematically to stabilize systems against high-frequency fluctuations of the boson fields, it is rather formal in the sense that the new form of the energy density cannot be simply reinterpreted physically by, say, specifying a modified way to fill up the fermion levels. This is due to the fact that the ghost subtraction is done for the effective action *after* integrating out the fermion fields [7]. The method thus corrects directly the meson spectra and affects the fermionic motion only indirectly.

We now discuss the ghost eliminated propagators Δ_α. In the case of the nonchiral model considered in Ref. [7], the ghosts are subtracted from the meson propagators separately because the meson fields are independent. But now the σ and π fields are not independent because of the chiral symmetry, and the propagators are related by the Ward identity [9]

$$-ivT(-q;q,0)=\Delta_\sigma^{-1}(q^2)-\Delta_\pi^{-1}(q^2)\ . \tag{10}$$

Here $T(-q;q,0)$ is the $\sigma\pi^2$ vertex including the one-fermion loop correction, and the arguments denote the incoming meson momenta ($-q$ for the sigma and q, 0 for the pions). In the spirit of Ref. [7] we amend only the two-point functions of the theory, that is, we leave T unchanged, construct the ghost eliminated pion propagator Δ_π, and then use Eq. (10) to obtain Δ_σ. The ghost-eliminated pion propagator is given by

$$\Delta_\pi(p^2)=G_\pi(p^2)-\frac{Z_g}{p^2+m_g^2}\ , \tag{11}$$

where m_g is the ghost mass and Z_g is the residue of this pole ($Z_g<0$). From Eqs. (10) and (11) it follows that our Källén-Lehmann propagators satisfy the same renormalization conditions as the original ones. The pion inverse propagator before and after ghost elimination is shown in Fig. 1. We note that the ghost mass $m_g\approx 1$ GeV is rather low, and therefore we expect that the ghost subtraction will significantly affect physical quantities.

The final form of the ghost-eliminated energy density is

$$\epsilon_{\rm KL}=\epsilon+\frac{\bar{v}^2}{2}[G_\pi^{-1}(-\mathbf{q}^2)-\Delta_\pi^{-1}(-\mathbf{q}^2)]\ , \tag{12}$$

where we used Eq. (10).

The results are shown in Figs. 2–5. Here we use the parameters $m=372$ MeV and $m_\sigma=800$ MeV. In Fig. 2 the energy per quark is shown as a function of the baryon density for three cases, namely, the "normal" phase ($q=0$), the pion condensed phase including the Landau ghost, and without the Landau ghost. The pion conden-

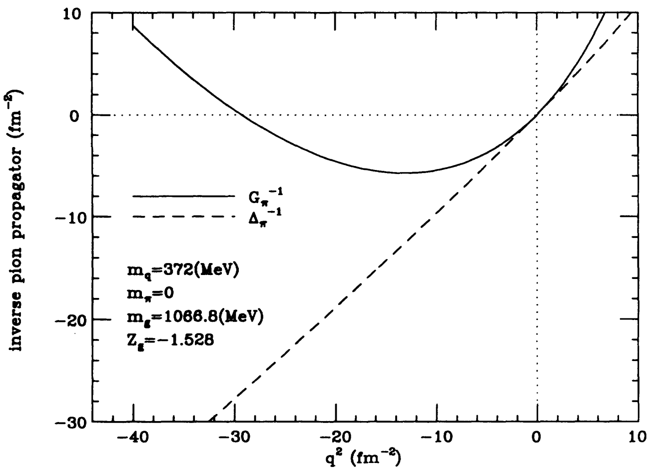

FIG. 1. Inverse pion propagator in free space before and after ghost elimination. The full line is the ghost-included one (G_π^{-1}) and the dashed line is the ghost-eliminated one (Δ_π^{-1}). m_g is the ghost mass and Z_g is the residue of the ghost pole.

sation sets in at $\rho_1 \approx 0.05$ fm^{-3}, and at $\rho_2 \approx 0.52$ fm^{-3} the chiral phase transition ($\tilde{v} \rightarrow 0$) occurs for the case where the ghost is eliminated. Between these two densities the pion condensed phase has a lower energy than the normal phase. Although the energy for the case where the ghost is included goes to minus infinity for large values of $|\mathbf{q}|$, it is possible to find a local minimum up to $\rho_3 \approx 1.1$ fm^{-3}. Around this density $|\mathbf{q}|$ becomes comparable to the ghost mass, and for higher densities there does not exist even a local minimum. As can be expected from Eq. (12), the ghost subtraction introduces additional repulsion. An important point to note is that, if the ghost is not subtracted, there occurs no chiral phase transition. This point will be discussed further below.

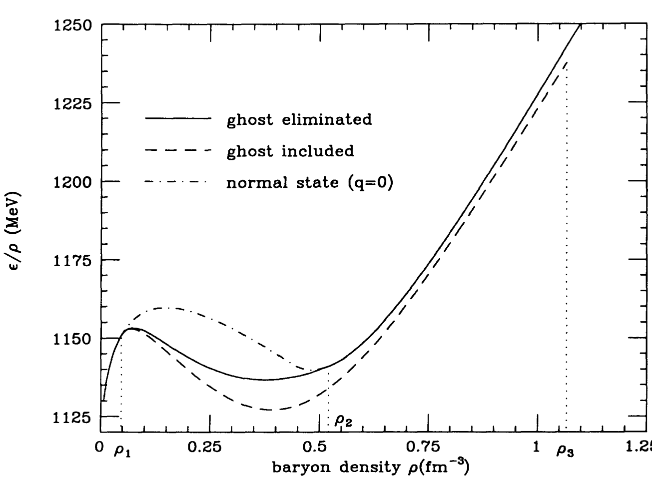

FIG. 2. Energy per quark as a function of the baryon density in three cases. The full line is the ghost-eliminated case, the dashed line is the ghost-included one, and the dash-dotted line corresponds to the normal (no pion condensate) phase. For explanation of the ρ's, see text.

In Fig. 3(a) the optimized momentum is shown and in Fig. 3(b) the optimized effective quark mass (or, strictly speaking, the radius of the chiral circle) is shown. Consider first Fig. 3(a). In both cases with or without the ghost $|\mathbf{q}|$ starts to increase from zero at the critical density. (Note that we use zero pion mass.) In the ghost-eliminated case, the chiral phase transition occurs at ρ_2 and above this density the energy no longer depends on $|\mathbf{q}|$. In the other case, $|\mathbf{q}|$ eventually goes into the ghost region where no stable state can be found. Figure 3(b) shows that the chiral phase transition in the ghost-eliminated case is a continuous (second-order) one. In the case including the Landau ghost, $\tilde{m}$ remains finite. This behavior can be understood as follows: For high $\mathbf{q}^2$ the energy density behaves as [see Eq. (7)]

$$\epsilon \rightarrow \frac{\tilde{v}^2}{2}\mathbf{q}^2 Z_\pi \tag{13}$$

with

$$Z_\pi = [-\mathbf{q}^2 G_\pi(-\mathbf{q}^2)]^{-1} \rightarrow -\frac{\gamma}{16\pi^2} g^2 \ln\left[\frac{\mathbf{q}^2}{m^2}\right]. \tag{14}$$

In accordance with the general criterion for the existence

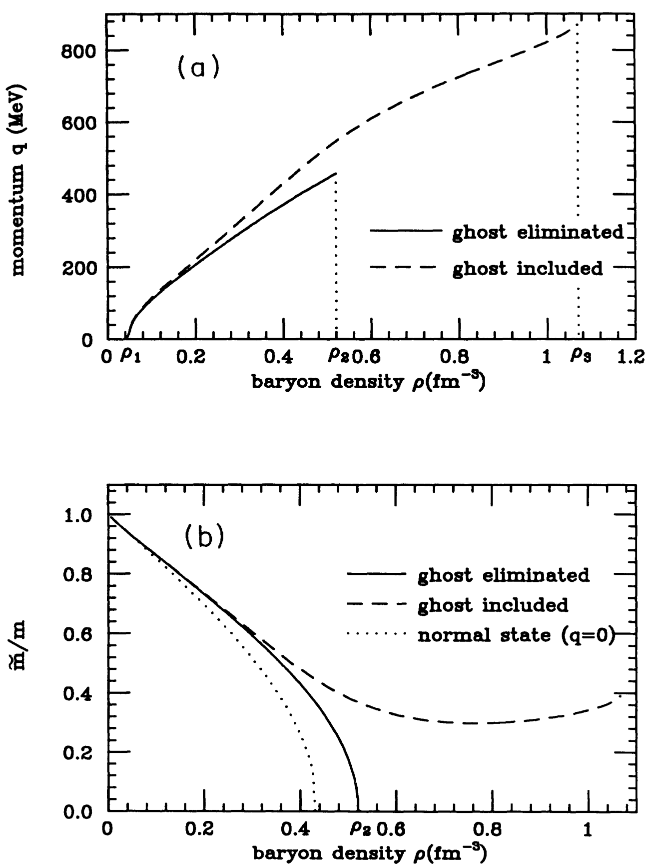

FIG. 3. (a) The optimized momentum. The full line shows the ghost-eliminated case and the dashed line shows the ghost-included case. (b) The optimized effective quark mass. The full line shows the ghost-eliminated case and the dashed line shows the ghost-included case, and the dotted line corresponds to the normal (no pion condensate) phase.

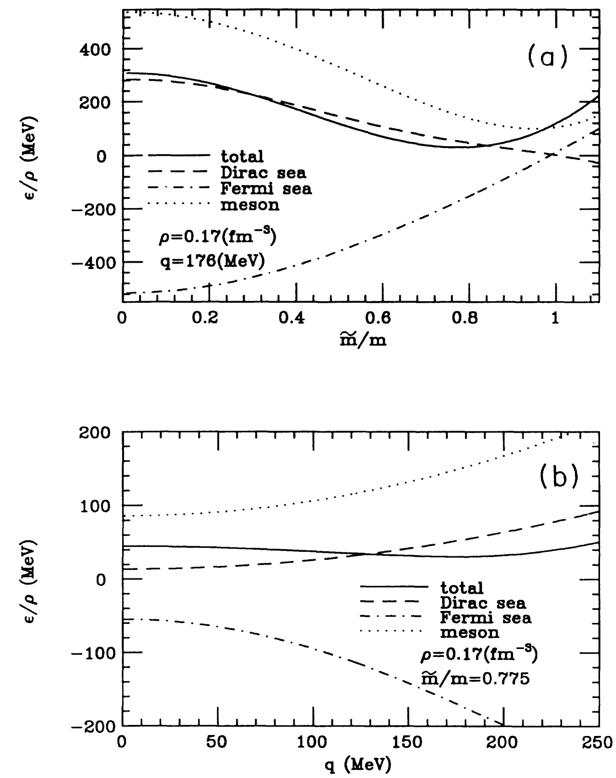

FIG. 4. The various contributions to the energy per quark for fixed density as a function of (a) the effective quark mass and (b) momentum. In both cases the full line is the total result and the dashed, dash-dotted, and dotted lines are the contributions of the Dirac sea (ϵ_D), Fermi sea (ϵ_F), and the classical meson fields (ϵ_B), respectively. The value of ρ equals the normal nuclear matter density (0.17 fm^{-3}), and the parameters q in (a) and $\tilde{m}$ in (b) are the optimized ones at this density.

of the ghost pole [7], Z_π becomes negative for high $\mathbf{q}^2$, and Eq. (13) shows that then the point $\epsilon(\tilde{v}=0)$ is always a maximum. On the other hand, if the ghost subtraction is performed, Z_π is positive and finite for high $\mathbf{q}^2$. (In the limit $\mathbf{q}^2 \to \infty$, it becomes the wave-function normalization factor for the pion field.)

In Figs. 4(a) and 4(b), the contributions from the various parts are shown as functions of $\tilde{m}$ and $\mathbf{q}$ for fixed density for the ghost-eliminated case. From these figures one can see that the contribution from the Dirac sea is repulsive and the pion condensation (as well as the chiral phase transition at higher densities) occurs as a result of a cancellation between the Fermi sea contribution and the others.

The pion condensation is related to the instability of the normal state. At the critical density for pion condensation, the normal-state pion propagator for $q_0=0$ changes its sign (has a zero energy pole at finite $|\mathbf{q}|$). This is shown in Fig. 5, where the inverse of the full pion propagator in the medium is plotted as a function of $q^2=-\mathbf{q}^2$ for three different densities. The values of $\tilde{m}$ used in this figure are the self-consistent solutions for the normal state ($q=0$). If one expands the full energy density ϵ_{KL} of Eq. (12) with respect to $\tilde{s}=\tilde{v}\cos\mathbf{q}\cdot\mathbf{x}-\tilde{v}$, $\pi=\tilde{v}\sin\mathbf{q}\cdot\mathbf{x}$, the inverse pion propagator shown in Fig. 5 appears as the coefficient of $(-\frac{1}{2}\pi^2)$ [compare with the expansion for zero density, Eq. (7)]. Using this relation, we see the following: For low densities we have $\tilde{\Delta}_\pi^{-1}(q^2)=\alpha q^2$, $\alpha>0$ for small spacelike momenta, and the normal state is stable, i.e., ϵ_{KL} has a minimum at $\mathbf{q}=0$. With growing density α decreases and becomes zero at the critical density (ρ_1 of Fig. 2). For higher densities $\alpha<0$ and the normal state is unstable [see Fig. 4(b)].

In Fig. 6 we compare the result of the full calculation

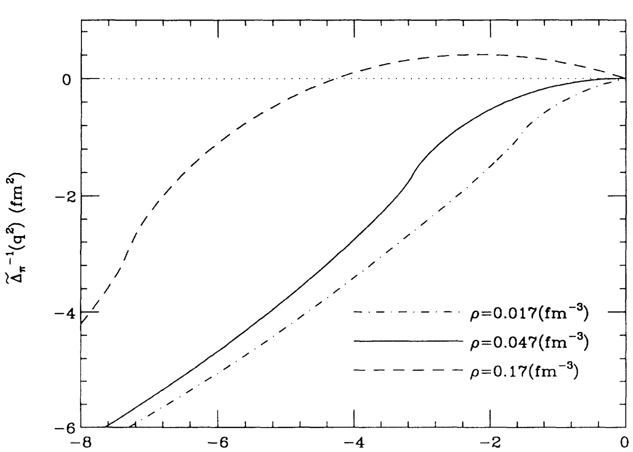

FIG. 5. Density-dependent inverse pion propagator in the normal phase without the Landau ghost for three different densities. The dash-dotted line shows the low density case ($\rho<\rho_1$), the full line corresponds to the critical density ($\rho=\rho_1$), and the dashed line shows the higher density case ($\rho>\rho_1$). The values of $\tilde{m}$ for each density are the self-consistent ones for the normal state.

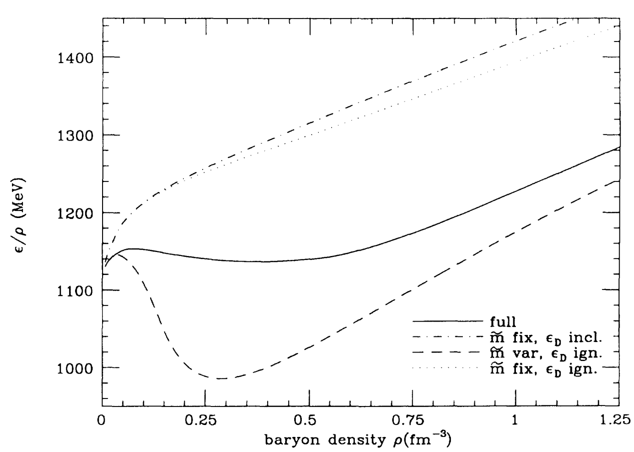

FIG. 6. Energy per quark as a function of the baryon density for four cases. The full line shows the result of the full calculation ($\tilde{m}$ varied and the contribution of the Dirac sea included) and agrees with the full line in Fig. 2. The dashed line refers to the case where $\tilde{m}$ is varied but the contribution of the Dirac sea is ignored. The dash-dotted line shows the result when $\tilde{m}$ is fixed to m (only q varied) and the contribution of the Dirac sea is included. The dotted line refers to the case where $\tilde{m}$ is fixed to m and the contribution of the Dirac sea is ignored.

with that where the Dirac sea contribution (ϵ_D) is ignored. The vacuum fluctuation effect is seen to be sizable and repulsive. We note that, in the calculation where ϵ_D is ignored, the chiral phase transition sets in at a much higher density ($\rho \approx 2.7$ fm^{-3}). In order to compare with the results in Ref. [3], we also show the corresponding two curves which are obtained in a calculation with fixed $\tilde{m} = m$, where only **q** is varied. We see that, by allowing the radius of the chiral circle to change, a sizable attraction is obtained.

The dependence of the energy density on the parameters m_σ and m is as follows: By increasing m_σ or decreasing m, one obtains more repulsion. For example, if we use $m_\sigma = 600$ MeV and the same m as in Fig. 2, there appears a bound state of quark matter at $\rho \approx 0.35$ fm^{-3}.

Finally, we wish to address the question of appropriate expansion schemes in Lagrangian field theories like the present one. Our calculation was based on the Hartree approximation [Eq. (4)], which has been generally quite successful in relativistic many-body theories [2] and therefore should presumably be the starting point in any sensible expansion scheme. The familiar loop expansion, however, has been shown to be unsuccessful for the case of a nonchiral field theory [12] because the inclusion of two-loop terms leads to a description of the system which is radically different from the Hartree one. There is, however, the possibility of alternative expansion schemes which start from the Hartree approximation. One candidate is the $1/N$ expansion, where N denotes the numbers of fermion species and must be eventually identified with 2 for the case of the isospin SU(2). In this scheme the Hartree term is the leading one, and by explicitly calculating the next-to-leading term, it has been shown [13] for a nonchiral model that there emerges at least a "weak convergence" in the sense that the qualitative features of the solution do not change. It has also been shown that the procedure of the elimination of the Landau ghost can be systematically applied in this case, too. (In the $1/N$ expansion, one also has to perform the ghost subtraction from the subgraphs in addition to the overall ghost subtraction discussed above. For details, see Ref. [13].) There is thus the possibility that chiral models like the one used in this paper can also be treated successfully in the $1/N$ expansion scheme.

To summarize, in this paper we proposed a useful method to eliminate the vacuum instability due to the Landau ghost in quark-meson theories where the classical meson fields have finite momenta. We exemplified this method for quark matter, but it can also be readily applied to finite solitons. We found that, besides removing the instability for large momenta, the ghost elimination has the further important effect of rendering possible a chiral phase transition at some value of the baryon density. The properties of quark matter which emerged in our study are as follows: There are two kinds of continuous phase transitions, one at low density from the normal phase to the pion condensed phase and another at higher density from the pion condensed phase to the Wigner phase. Both phase transitions emerge as a result of an interplay between the attractive Fermi sea contribution [ϵ_F of Eq. (4)] and the repulsive bosonic (ϵ_B) and Dirac sea (ϵ_D) contributions.

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