

Statistical emission of deuterons and tritons from highly excited compound nuclei

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Statistical model calculations, employing optical model transmission coefficients for particle emission, have been able to describe in a satisfactory way n , p , d , t , and α emission properties of compound nuclei at excitation energies below 100 MeV. Recent experimental data have shown that the same model systematically overpredicts the deuteron and triton yields observed at higher excitation energies up to 405 MeV. The predictions of the statistical model with transmission coefficients derived from an ingoing-wave boundary condition and a direct reaction approach to fusion method are discussed. It is shown that a description of the deuteron and triton data is possible, provided that the corresponding inverse cross sections are reduced from the optical model predictions. For deuteron emission in particular, the required reduction is found to be consistent with experimental deuteron fusion cross sections. The breakdown of the traditional approach is attributed to the large percentage of nonfusion components contained in the optical model absorption cross section.

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I. INTRODUCTION

A number of heavy-ion physics studies have been devoted to the investigation of the properties of nuclear matter under the extreme conditions of high excitation energy and angular momentum. Experimental investigations employing heavy-ion-induced reactions have been able to isolate emission sources which are believed to have attained statistical equilibration with temperatures as high as 6 MeV [1]. Particle decay information, such as the energy distributions of the evaporated particles from the compound nucleus, is related to the properties of the emitting source. This information combined with the predictions of the statistical model has shown interesting properties of the nuclear matter, such as the reduction of the level density constant at high excitation energies [1,3,4].

Chbihi *et al.* [2,3] recently studied the evaporation spectra of equilibrium sources produced in the reaction $701\text{-MeV } ^{28}\text{Si} + ^{100}\text{Mo}$. They used two different methods to extract the level density constant at high excitation. It was found that, in order to reproduce the slopes of the light particle spectra, the level density constant of the excited nuclei must have a value of $A/10$ – $A/11$ at temperatures of 3.5–5.5 MeV; a conclusion also reached by other authors [4]. These results were based on statistical model calculations which presumed that the only parameter adjustment required to describe the data is the level density constant. However, statistical model calculations with a reduced level density constant lead to an overprediction (in some cases, by a factor of 2 or more) of the experimental deuteron and triton multiplicities. It was suggested that a more complicated parameter adjustment may be required to treat the decay of these highly excited nuclei. The present work follows up on this suggestion in order to obtain a consistent statistical model description of the data presented in Refs. [2,3].

In the statistical model, the decay of an excited nucleus

is determined by two factors: the level density of the populated nuclei and the appropriate transmission coefficients. The transmission coefficient for particle emission is related through the principle of detailed balance, to the one of the inverse process, namely, the capture of the particle by the excited daughter nucleus [5,6]. A commonly used set of transmission coefficients is the one describing the absorption of the incident particle by the target in the optical model [6].

Alexander, Magda, and Landowne [7] recently reviewed the logical basis of using optical model (OM) transmission coefficients (T_l) in statistical model calculations. A comparison of the OM T_l 's was made with the ones derived from an ingoing-wave boundary-condition (IWBC) calculation [8]. The latter T_l 's give the transmission probability through a real potential barrier. It was pointed out that processes like transparency, shape resonances, or peripheral absorption are normally present in the OM T_l 's. Such processes should not be included in a model describing the absorption of a particle by a nucleus, if one wishes to describe the process of particle emission with the same set of T_l 's. Alexander, Magda, and Landowne [7] suggested that simple barrier penetration T_l 's may be more appropriate for use in statistical model calculations.

In connection with the above-mentioned deuteron multiplicity problem, another observation came from analyses of deuteron-induced reaction data. In a recent study, West, Lanier, and Mustafa [9] measured excitation functions for the ground and an excited isomer state in the reaction $^{52}\text{Cr}(d,2n)^{52}\text{Mn}^{g,m}$. They found that the assumption of equating the fusion cross section of the reaction $d + ^{52}\text{Cr}$ to the OM absorption (reaction) cross section fails to explain the observed cross sections. Subsequently, Mustafa, Tamura, and Udagawa [10] were able to describe the above data using fusion cross sections from the direct-reaction approach to fusion (DRAF) method of Udagawa, Kim, and Tamura [12]. In the DRAF method,

the fusion part of the reaction process is incorporated in the inner region of the imaginary part of the optical potential. The outer part is associated with incomplete fusion or direct-reaction processes. This separation is achieved with the introduction of a fusion radius. A decomposition of the OM absorption cross section, σ_{abs} , into a complete fusion σ_{CF} and a direct interaction part σ_{DI} is then obtained: $\sigma_{\text{abs}} = \sigma_{\text{CF}} + \sigma_{\text{DI}}$. The usefulness of the method lies in the fact that once the fusion radius is determined, a description of the complete and direct interaction processes can be made on the same footing. Furthermore, it provides us with a method of creating T_l 's which are related to the fusion part of the absorption process in the OM description. Using this method, Mastroleo, Udagawa, and Mustafa [11] were able to describe the cross sections of various processes in $d + {}^{93}\text{Nb}$ reactions; a reaction system with a mass similar to the emission systems of Chbihi *et al.* [2,3].

Motivated by these observations, we examine the excitation energy dependence of particle multiplicities of Ref. [3] employing the above models for transmission coefficients in the statistical model. It is shown that the IWBC T_l 's provide a closer description of the data than the corresponding optical model T_l 's. However, discrepancies in the deuteron and triton multiplicities still remain. An improved description of the data is given with OM T_l 's for n , p , and α emission and T_l 's from the DRAF method for deuterons and tritons. For deuteron emission, the chosen fusion radius is consistent with the one required to describe the fusion process in $d + {}^{93}\text{Nb}$ reactions. For tritons, a fusion radius of a similar magnitude was found to be necessary to fit the data. However, the lack of a similar fusion data analysis prevents us from substantiating the employed value.

The above study indicates that, for deuterons and tritons, the direct interaction part is a sizable fraction of the OM absorption cross section. Elimination of the direct interaction components is shown to be important for a successful description of deuteron and triton emission in statistical model calculations.

In the next section, we outline our statistical model calculations employing OM T_l 's and we show that the model is adequate for the description of the decay properties of low excited compound nuclei. A detailed comparison with the data of Ref. [3] shows the inadequacy of the model at high excitations. The introduction of IWBC and DRAF T_l 's for deuterons and tritons is made in Secs. III and IV. In Sec. V, we present a description in the context of a temperature-dependent level density constant. Section VI contains the conclusions of this research.

II. THE STATISTICAL MODEL WITH OPTICAL MODEL TRANSMISSION COEFFICIENTS

In the following sections, we present calculations performed with the code EVAP, described in Ref. [13]. EVAP is a Monte Carlo statistical model evaporation code which evolved from the code PACE [14] after extensive modifications. The features of the code which are relevant to the purpose of the present work include the

following: (a) the extended excitation energy range of validity of the code which makes possible the calculation of our excited systems up to $E^* \sim 405$ MeV, (b) the inclusion of decay channels involving n , p , d , t , α , ${}^3\text{He}$, and ${}^6\text{Li}$ emission, and (c) the use of transmission coefficients calculated individually for each relevant nucleus and type of emitted particle. This method eliminates cumulative errors from the earlier used extrapolation procedure for transmission coefficients [14]. (A discussion of these errors can be found in Ref. [13].) The transmission coefficients are stored into computer files and are read in by the code. This way, it becomes possible to create T_l sets under different assumptions and test their effect in a statistical model calculation.

We first examine statistical model calculations employing optical model T_l 's. As a test system, we consider the reaction of 121-MeV ${}^{14}\text{N} + {}^{103}\text{Rh}$ studied by Galin *et al.* [15], a system of mass close to those in the following discussion. This reaction produces the compound nucleus ${}^{117}\text{Te}$ at an initial excitation energy of 107 MeV. Our calculations assumed that ${}^{117}\text{Te}^*$ is formed at 107 MeV with a diffuse triangular angular momentum distribution determined by the fusion cross section of 1408 mb. This fusion cross section was derived from systematics [17] and the diffuseness parameter Δ of the angular momentum distribution was set equal to $2\hbar$. Emission of n , p , α , d , t , ${}^3\text{He}$, and ${}^6\text{Li}$ was taken into account with transmission coefficients resulting from optical model calculations. Optical model parameters were provided from Refs. [19, 20, 21, 22, 18, 18, and 18], respectively. The level densities were calculated using the Gilbert and Cameron composite formula [23,14] with a level density parameter of $a = A/6.0$ as suggested in Ref. [16]. The events from the statistical model calculation were stored in computer files and sorted with appropriate gates which simulated the experimental conditions.

In Fig. 1, we show the experimental p , α , d , and t spectra observed at 134.5° in the center-of-mass system. Our

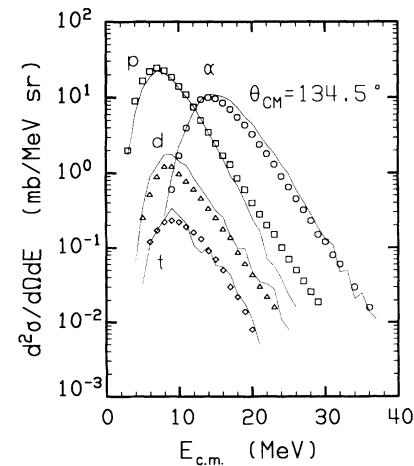


FIG. 1. Experimental [15] p , α , d , and t center-of-mass energy spectra (symbols) observed at $\theta_{\text{c.m.}} = 134.5^\circ$ in the deexcitation of ${}^{117}\text{Te}^*$ (107 MeV). The solid lines show the predictions of a statistical model calculation with optical model transmission coefficients.

TABLE I. Experimental [15] and calculated cross sections of the evaporated charged particles in the reaction 121-MeV $^{14}\text{N} + ^{103}\text{Rh} \rightarrow ^{117}\text{Te}^*$ ($E^* = 107$ MeV).

Particle (ν)	σ_ν (mb)	σ_{calc} (mb) (OM T_l)	σ_{calc} (mb) (IWBC T_l)
p	1894 ± 190	1753.9	1918.9
d	90 ± 14	135.5	74.7
t	21 ± 3	22.8	7.5
α	930 ± 90	1157.4	846.8

symbols correspond to interpolations through the data of the measured spectra [15]. The solid lines show the calculated spectra for the same center-of-mass angle. The agreement in the shapes and absolute magnitude of the spectra is good, besides a small overprediction of the deuteron yield. A comparison between the experimental and calculated angle-integrated cross sections is given in Table I. It should be noted that the proton to alpha ratio could be further improved if a smaller maximum angular momentum for fusion was used.

Next, we turn our attention to the behavior of particle multiplicities at higher excitation energies: $100 \leq E^* \leq 405$ MeV. Chbihi *et al.* [2,3] made a detailed study of incomplete fusion reactions induced by 701-MeV ^{28}Si incident on ^{100}Mo . Their 4π experimental setup made possible the detection of forward recoil evaporation residues in coincidence with the emitted neutrons, charged particles, and γ rays. The spectrum of residue velocities was divided into six bins which correspond to six different regions of excitation energy of the emitting system. The excitation energy and primary mass were determined with a linear momentum reconstruction. Neutron, proton, deuteron, triton, and alpha particle spectra corresponding to these bins were analyzed using three-source Maxwellian fits to extract the evaporative, intermediate, and projectile-like source multiplicities. It was found that the above reaction proceeds mainly through incomplete fusion channels producing highly excited evaporation sources with $100 \leq E^* \leq 405$ MeV. The evaporation n , p , d , t , and α energy spectra were analyzed and the level density constant was extracted [3]. The slopes of the spectra implied a level density parameter of $A/10.0$ – $A/11.0$ for the highest excitation energy bins. The corresponding average particle multiplicities (M_i , $i = n, p, \alpha, d, t$) as functions of the excitation energy of the

emitting source are indicated by the symbols in Fig. 2.

Particle emission from the above sources was simulated with the reactions listed in Table II. Each reaction system was chosen in such a way that the compound nucleus mass and excitation energy matches the values found from the linear momentum reconstruction [2]. The critical angular momentum for fusion was adjusted according to the prediction of the sum-rule model of Wilczyński [25].

Calculations with EVAP were performed for these systems with optical model transmission coefficients. The calculations employed the Gilbert and Cameron composite level density formula with three values for the level density parameter. The results are shown in Fig. 2(a) with the dashed, dash-dotted, and solid lines corresponding to calculations with a level density parameter of $A/6.0$, $A/8.5$, and $A/11.0$, respectively.

We see that decreasing the level density parameter from $A/6.0$ to $A/11.0$ reduces the predicted number of emitted neutrons and protons, and increases the number of emitted alpha particles, deuterons, and tritons. This trend is expected from elementary considerations in the statistical model. A small level density parameter reduces the steepness in the variation of the level density with excitation energy and angular momentum. This enhances the emission of energetically expensive (in separation or emission energy) modes. Therefore, the emission of deuterons, tritons, and alpha particles increases as the level density parameter decreases.

Figure 2(a) also shows that the level density parameter of $A/11.0$ gives a close description of the M_n and M_p data; a result consistent with the level density parameter derived from the slopes of the emission spectra. The M_α data lie between the $A/6.0$ and $A/11.0$ calculations. However, the deuteron and triton yields are always overpredicted, no matter which level density parameter is used. The smallest level density parameter produces the largest discrepancy, which is greater than a factor of 2 at all excitation energies.

The above observations show a contradictory behavior between the trend of the experimental data and the predictions of the statistical model. Although the slopes of the evaporation particle spectra were found to be consistent with a value of $A/11.0$, the d and t multiplicities are in favor of $A/6.0$. However, even with the value of $A/6.0$, the predicted deuteron and triton yields are in excess of the experimental values.

TABLE II. Reaction systems employed in the simulation of different deexcitation bins in the reaction 701-MeV $^{28}\text{Si} + ^{100}\text{Mo}$.

Bin	Reaction	CN	E^* (MeV) ^a	l ($\hbar$) ^b
1	152-MeV $^6\text{Li} + ^{100}\text{Mo}$	^{106}Rh	158	18.2
2	197-MeV $^7\text{Li} + ^{100}\text{Mo}$	^{107}Rh	200	21.1
3	271-MeV $^{10}\text{B} + ^{100}\text{Mo}$	^{110}Ag	260	27.2
4	354-MeV $^{14}\text{N} + ^{100}\text{Mo}$	^{114}In	316	36.1
5	441-MeV $^{19}\text{F} + ^{100}\text{Mo}$	^{119}Sb	372	47.7
6	494-MeV $^{21}\text{Ne} + ^{100}\text{Mo}$	^{121}Te	405	48.8

^aInitial excitation energy.

^bAverage angular momentum estimated from the sum-rule model.

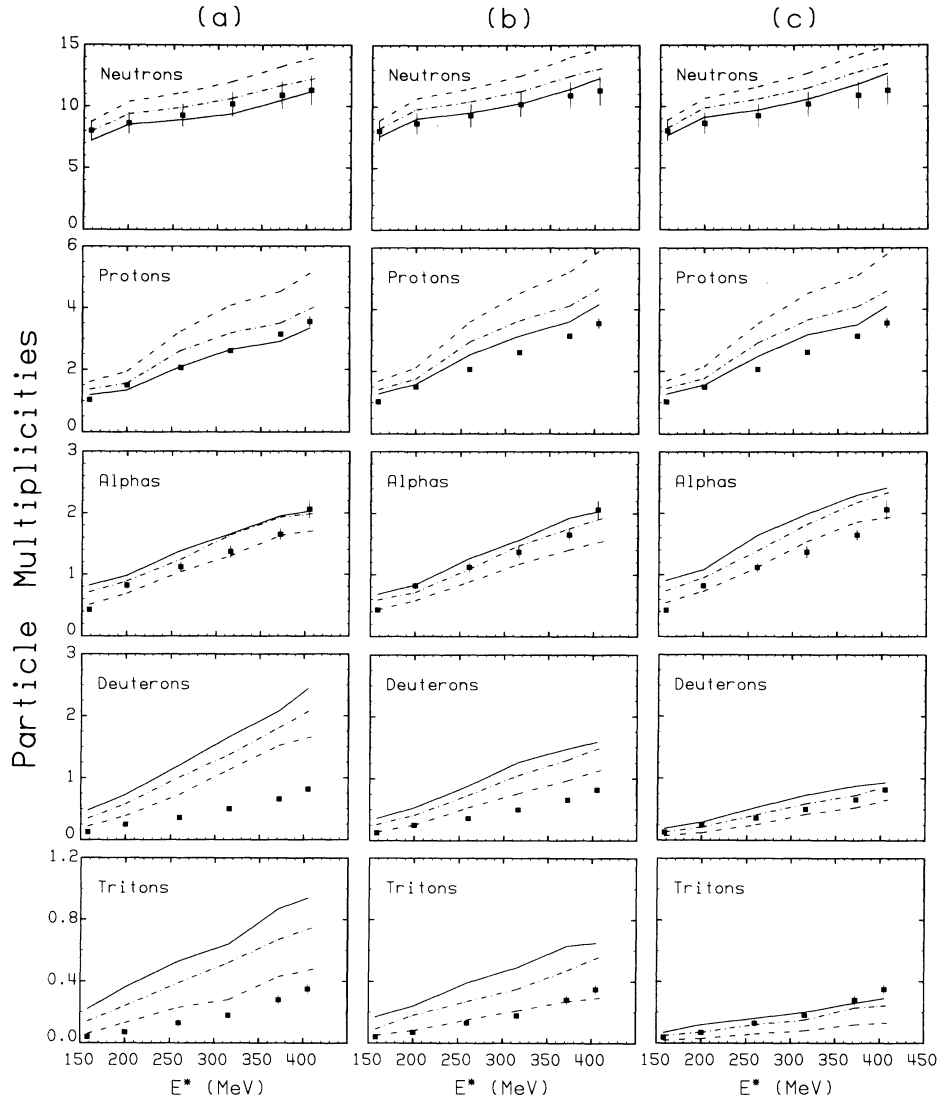


FIG. 2. (a) Experimental n , p , α , d , and t multiplicities (symbols) as a function of the excitation energy of the emitting source in reactions of 701-MeV ^{28}Si on ^{100}Mo . The predictions of the statistical model with optical model transmission coefficients and level density parameters of $a = A/6.0$, $A/8.5$, and $A/11.0$ are shown with the dashed, dash-dotted, and solid lines, respectively. (b) Same as in (a), with transmission coefficients derived from the ingoing-wave boundary-condition model. (c) Same as in (a), with DRAF transmission coefficients for deuterons and tritons.

Due to the high excitation energies involved, changes in the critical angular momentum for fusion do not greatly affect the calculated relative particle emission yields. Therefore, the only control we used previously over the predicted relative particle yields was the level density parameter. However, another important factor that determines particle emission is the transmission coefficients. The effect of different models for transmission coefficients in our calculations is examined in the following two sections.

III. TRANSMISSION COEFFICIENTS IN THE IWBC MODEL

In the statistical model, transmission coefficients for particle emission are related through the principle of de-

tailed balance [5,6] to those of the inverse process, i.e., particle capture by the daughter nucleus. A commonly used set of transmission coefficients is the one describing the absorption of the incident particle by the target in the optical model [6]. Alexander, Magda, and Landowne [7] recently examined the logical basis of using optical model transmission coefficients in statistical model calculations. A comparison between the OM T_l 's for neutrons, protons, deuterons, tritons, and α particles was made with corresponding ingoing-wave boundary-condition calculations [8]. The IWBC T_l 's give the probability of transmission through the real potential barrier. Alexander, Magda, and Landowne identified certain features of the OM which are included in the treatment of absorption, but are not necessarily related to fusion: (a) the OM

includes reactive scattering outside the real potential well, due to the tail of the imaginary potential, (b) the OM includes repenetration to the entrance channel (transparency), and (c) the OM supports size resonances due to standing waves inside the well.

The presence of the above features in the OM absorption cross sections and transmission coefficients raises questions on their applicability in describing the inverse process of particle evaporation. Hence, the IWBC model may provide a better logical choice for transmission coefficients since it describes simply the barrier penetrability for capture of a particle by the daughter nucleus.

The IWBC model differs from the OM in setting a boundary condition on the partial waves at distances smaller than the position of the potential maximum, instead of choosing wave functions which vanish at the origin. In the vicinity of this boundary, one requires that the partial-wave functions have the form of incoming waves. This implies the absence of reflected waves from the nuclear interior as would be expected for strong absorption.

In Ref. [7] the behavior of IWBC T_l 's as a function of the channel energy was compared to those of the OM. The IWBC T_l 's increase monotonically with channel energy and eventually approach unity. For alpha particles, this trend is similar to that predicted by the OM. However, at a given channel energy, the IWBC model predicts smaller values for the highest l waves. This is related to the absence of reactive scattering for distant collisions. For deuterons and tritons, the reduction in the high- T_l components is even larger. For neutrons and protons, the IWBC T_l values are significantly high even at low energies, in contrast to the OM values which may exhibit oscillatory structures. This difference was attributed to processes like transparency or size resonances inside the potential well, which are not present in the IWBC model.

The calculations of the previous section were repeated using IWBC T_l 's obtained with the real parts of the optical model potentials [18]. The role of the imaginary parts was replaced by the ingoing-wave boundary condition inside the barrier [7].

The results of this calculation for the total cross sections of the light particles emitted in the decay of $^{117}\text{Te}^*$ are given on Table I. The calculated proton and alpha cross sections are compatible with the experimental data. The deuteron cross section is slightly underestimated and the triton cross section is underestimated by a factor of 3. Compared to the calculation with OM T_l 's, we notice a small increase in the proton yield and a decrease in the alpha yield. The deuteron and triton yields are reduced by factors of 2 and 3, respectively.

The comparison of the high excitation energy multiplicity data with the calculations employing IWBC T_l 's is given in Fig. 2(b). Calculations with the three level density constants of $A/11.0$, $A/8.5$, and $A/6.0$ are again shown by the solid, dash-dotted, and dashed lines. The trend of the curves with level density constant is the same as in Fig. 2(a). The overall agreement with the data is improved. The calculation tends to reproduce better the alpha particle and, to a lesser extent, the deuteron and triton multiplicities. However, the overprediction of M_d

and M_t still remains for all three choices of the level density constant.

IV. DEUTERON AND TRITON TRANSMISSION COEFFICIENTS IN THE DRAF METHOD

The calculations of the previous section showed an improvement in the description of the deuteron and triton multiplicities, over the ones employing OM T_l 's. We attribute this improvement to the elimination of absorption processes in the optical model which are not related to fusion. In this section, we examine the elimination of direct reaction contributions in the optical model absorption cross section.

Udagawa, Kim, and Tamura [12], have developed a technique which allows us to calculate cross sections for complete fusion and direct reaction processes on the same footing within the direct reaction theory. The basic ingredients of the method, called the direct-reaction approach to fusion, are as follows.

In the optical model description, absorption is represented by the imaginary part of the potential and is associated with all processes that remove flux from the elastic channel. These processes include complete fusion, incomplete fusion, direct interactions, etc.

It can be shown [24] that, for the motion of particles in a complex potential $U = V + iW$, the continuity equation with a sink term ($W < 0$) is satisfied, i.e.,

$$\frac{\partial}{\partial t}(\Psi^* \Psi) + \nabla \cdot \mathbf{J} = \frac{2}{\hbar} \Psi^* W \Psi, \quad (1)$$

where $\mathbf{J}$ is the probability current density vector and Ψ is the (OM) wave function of the particle. The sink term represents the loss of particles per unit volume per unit time. Integration of this term in a sufficiently large volume V (enclosing the nucleus) yields the number of particles lost per unit time. The absorption cross section σ_{abs} is then obtained as the ratio of this integral to the incident flux, i.e.,

$$\sigma_{\text{abs}} = \frac{2}{\hbar v} \int_V \Psi^* W \Psi dV, \quad (2)$$

where v is the asymptotic velocity in the entrance channel.

Substituting the partial-wave expansion for the wave function

$$\Psi = \sum_{l=0}^{\infty} (2l+1) i^l \frac{u_l(kr)}{kr} P_l(\cos\theta) \quad (3)$$

into Eq. (2), we find

$$\sigma_{\text{abs}} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) T_l. \quad (4)$$

The transmission coefficient T_l is expressed as

$$T_l = \frac{8}{\hbar v} \int_0^{\infty} u_l^* W u_l dr \quad (5)$$

in terms of the radial wave function u_l . This integral actually extends up to the point where W is negligible.

Following Ref. [12], we define the fusion potential W_F

as the inner region of W , i.e.,

$$W_F = \begin{cases} W & \text{for } r \leq R_F = r_F A^{1/3}, \\ 0 & \text{for } r > R_F, \end{cases} \quad (6)$$

where $R_F = r_F A^{1/3}$ is a (sharp cutoff) fusion radius that separates the inner and outer regions of W . It is assumed [12] that the fusion potential W_F is responsible for the complete fusion (CF) processes. The outer part, called the direct-interaction (DI) part of W , is associated with the nonfusion (but still absorption in the OM description) processes. The decomposition $W = W_F + W_{DI}$ leads to a separation of the OM absorption cross section σ_{abs} into a complete fusion (σ_{CF}) and a direct-interaction part (σ_{DI}): $\sigma_{abs} = \sigma_{CF} + \sigma_{DI}$.

The usefulness of the method lies in the fact that once the fusion radius R_F is specified, we obtain a unified description of σ_{CF} and σ_{DI} . Furthermore, we can obtain an estimate of the nonfusion processes contained in the OM absorption cross section, together with a transmission coefficient set related to the complete fusion process.

The above method, called the direct-reaction approach to fusion, has been successfully applied in the description of the heavy-ion sub-barrier fusion cross sections and the associated angular momentum distributions [12]. Two applications of the method have recently been made in deuteron-induced reactions.

West, Lanier, and Mustafa [9] measured excitation functions for the ground and an excited isomer state in the reaction $^{52}\text{Cr}(d, 2n)^{52}\text{Mn}^{g,m}$. An analysis of the data was made under the commonly made assumption that the fusion cross section is equal to the OM absorption cross section for $d + ^{52}\text{Cr}$. It was found that this assumption fails to explain the observed cross sections. Mustafa, Tamura, and Udagawa [10] further showed that one can successfully explain these data using a σ_{fus} value obtained from the DRAF method.

In an attempt to systematize the method, Mastroleo, Udagawa, and Mustafa [11] recently presented an analysis which shows a complete description of particle cross sections in $d + ^{93}\text{Nb}$ reactions. They used the DRAF and a breakup fusion approach. An agreement between the two methods was established, indicating that the DRAF method can be used rather reliably to evaluate σ_{CF} and σ_{DI} . A comparison of these cross sections showed that a large fraction of the OM absorption cross section involves direct reaction components. It becomes apparent that the DRAF method provides a meaningful way of obtaining transmission coefficients associated with complete fusion.

A transmission coefficient set was created with OM T_l 's for n , p , and α emission and DRAF T_l 's for d and t emission. This choice was based on the assumption that the most important correction in the OM T_l 's is in the deuteron and triton channels. The DRAF T_l 's were created with the procedure described above, using the OM potentials of Sec. II. The fusion radius parameters for deuterons and tritons were chosen equal to 1.7 and 1.6 fm, respectively. The deuteron fusion radius parameter is consistent with the analysis of Mastroleo, Udagawa, and Mustafa [11], as discussed below. The corresponding

parameter for tritons was introduced as a fit parameter in order to bring agreement with the data.

The results of this calculation are compared to the particle multiplicity data in Fig. 2(c). For neutrons and protons we get a degree of agreement similar to that obtained with OM or IWBC T_l 's. However, the agreement in the deuteron multiplicities is now remarkably good. The triton multiplicities are reproduced with a fusion radius of a similar magnitude to that used for the deuterons. The calculated alpha particle multiplicities are, on the average, higher than those produced in the previous two calculations. For example, the curve corresponding to $A/11.0$ overestimates systematically the experimental data by a factor of ~ 1.3 . This trend can be explained with the reaction energetics and the level density argument of Sec. II. Deuteron emission is almost as energetically costly as alpha emission. Therefore, it makes a big contribution in the cooling off of the deexciting system. When OM T_l 's were used, the predicted deuteron yield was found comparable to the alpha yield. A reduction of the deuteron yield by almost 50% was made with the introduction of DRAF T_l 's. This leaves alpha emission as the next most probable energetically costly mode, whose competition becomes stronger for a low level density constant. The overprediction of the alpha particle yields in the calculation with $A/11.0$ is attributed to the use of a level density parameter which was kept constant throughout the deexcitation. (See Sec. V.)

Table III shows cross sections for the $d + ^{93}\text{Nb}$ reactions at the bombarding energies of 15 and 25.5 MeV. The fusion cross section $\sigma_{fus}(\text{DRAF})$ was determined with the DRAF method in the analysis of cross sections from this reaction [11]. These values are consistent with breakup-fusion calculations. For comparison, we show on Table III the absorption cross section calculated with the OM and the IWBC models using the parameters of Ref. [18]. We see that less than half of the OM cross section results in fusion. The IWBC model predicts 10 and 18 % lower values than the OM, which are still in excess of the DRAF values by almost a factor of 2.

The differences between the three T_l -model predictions are explained in Fig. 3 for the 15-MeV $d + ^{93}\text{Nb}$ reaction. On the top of this figure, we show plots of the radial absorption probability dP_l/dr [integrand of Eq. (5)] as a function of the distance r , for the indicated l waves. The total integral of each curve gives the corresponding OM T_l . Absorption occurs in the outer region of the nuclear volume, as a result of the Woods-Saxon derivative term of the imaginary potential W (shown on the bottom). This reflects the suppression of available states for

TABLE III. Fusion cross sections in $d + ^{93}\text{Nb}$ reactions described by the DRAF method. The corresponding predictions of the absorption cross sections in the optical and IWBC models are also given, for comparison.

E_d (MeV)	σ_R (OM) (mb)	σ (IWBC) (mb)	σ_{fus} (DRAF) (mb)
15.0	1565.6	1415.0	675
25.5	1924.9	1581.8	895

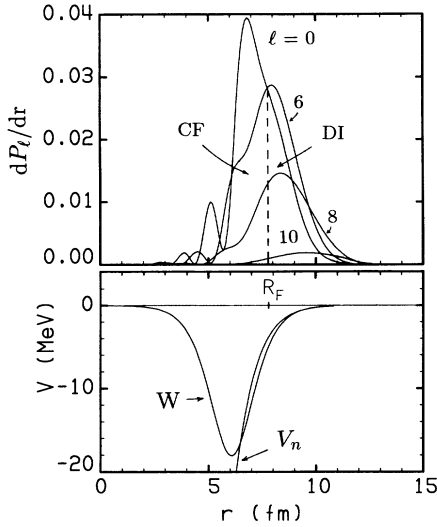


FIG. 3. Top: Radial absorption probability dP_l/dr as a function of r , for 15-MeV deuterons with the indicated l 's incident on ^{93}Nb . The fusion radius R_F separates the regions associated with complete fusion (CF) and direct interactions (DI). The associated real (V_n) and imaginary part (W) of the nuclear optical potential are shown on the bottom.

scattering in the central region due to Pauli principle effects. The fusion radius R_F (where $R_F = 1.7 A^{1/3}$) separates the regions corresponding to complete fusion and direct interactions. The integrals over each region give the transmission coefficients associated with each individual process. The partition of the areas, under the dP_l/dr curves, made by R_F are consistent with a $\sigma_{\text{fus}}(\text{DRAF})$ smaller than $\sigma_{\text{abs}}(\text{OM})$ by a factor of 2. We also notice an increase of the direct-interaction contribution with increasing l , as expected for a peripheral reaction. On the bottom of Fig. 3, the imaginary potential W together with the tail of the real optical model potential V_n are shown. The imaginary potential extends to distances larger than the tail of the real potential. This partly explains the lower $\sigma(\text{IWBC})$ values which exclude absorption resulting from the tail of the imaginary potential.

The basic parameter in the DRAF approach is the fusion radius parameter r_F . Application of this method to heavy-ion fusion data [12] has been shown that excitation functions can be reproduced quite well, both below and above the Coulomb barrier, with $r_F \approx 1.4$ fm. The study of the $^{52}\text{Cr}(d, 2n)^{52}\text{Mn}$ of Mustafa, Tamura, and Udagawa [10] determines an $r_F \approx 1.96$ fm. However, the analysis of the $d + ^{93}\text{Nb}$ reaction was found consistent with $r_F = 1.58$ fm. Clearly, there is a mass dependence in the extracted r_F parameters. It is also possible that an energy dependence or even an optical model parameter dependence may be present for a given system. Our choice of $r_F = 1.7$ fm for deuterons is equivalent to the $r_F = 1.58$ fm of Mastroleo, Udagawa, and Mustafa [11]. The apparent difference is due to the fact that the OM parameters we used are not exactly the same as those re-

ported in Ref. [11] for the same system. However, our calculations were found consistent with the parameters of Ref. [11]. The selected value of $r_F = 1.7$ fm was chosen on the basis of agreement with the calculated $\sigma_{\text{fus}}(\text{DRAF})$ of Table III.

The consistency of the employed deuteron fusion radius parameter with the published analysis of fusion cross sections strengthens our confidence in the method. The lack of experimental data and a similar analysis of triton-induced reactions prevents us from substantiating the fitted fusion radius parameter for tritons ($r_{F,t} = 1.6$ fm), which was found similar to the value for deuterons.

V. THE USE OF A TEMPERATURE-DEPENDENT LEVEL DENSITY CONSTANT

It is known from a number of studies that the level density constant ($a = A/k$) at high temperatures acquires an increased value compared to the value for ground-state nuclei. The level density constant k is determined by the ratio of the effective to the nucleon mass (m^*/m) at the Fermi energy. For cold nuclei, m^*/m exhibits a peak as a function of momentum at the Fermi momentum. This enhancement of the effective mass is predicted to disappear at high temperatures ($\sim 4-5$ MeV) [3,26-28]. We therefore expect the level density parameter $a = A/k$ to decrease with excitation energy. Experimental evidence for such a decrease has been given (see references cited in [1,3,4]).

The use of a level density parameter independent of the excitation energy in the previous calculations provided a simplified description of the decay process. In order to make a more realistic treatment, we introduced a temperature-dependent level density parameter $a = A/k(T)$, where

$$k(T) = k(0) \frac{1.4}{1 + 0.4 \exp[-(T/3)^2]} \text{ MeV} . \quad (7)$$

This temperature dependence is based on the parametrization of Ormand *et al.* [28] and accounts for an inclusion of thermal and quantal fluctuations on a low-temperature level density constant $k(0)$. Choosing $k(0) = 8.0$ in Eq. (7) brings consistency with the level density constants extracted from the analysis of the evaporative spectra of Ref. [3].

The transmission coefficient set of the previous section combined with the $k(T)$ parametrization of Eq. (7) leads to an improved description of the data of the present study. Figure 4 shows the results of this calculation for the excitation energy dependence of the particle multiplicities. A good agreement is obtained in the reproduction of the data throughout the considered excitation energy range. In particular, the alpha particle multiplicities are better reproduced [cf., Fig. 2(c), solid line].

The same calculation reproduces the slopes of the energy spectra for each type of emitted particle in the above excitation energy range. In Fig. 5, the histograms show the calculated proton, alpha, deuteron, and triton energy spectra in the center-of-mass system. The spectra have been displaced on the vertical axis for pictorial reasons. The initial excitation energies of the emitting sources in

Figs. 5(a) and (b) are $E^* = 158$ and 405 MeV, respectively. They correspond to the excitation energy bins 1 and 6 of Table II. The solid lines in the same figure represent the slopes of the evaporative source fits to the data [3]. The calculated slopes are consistent with the experimental ones. When OM transmission coefficients are used for deuterons and tritons, the calculated spectra are softer than those obtained from the DRAF method (Fig. 5). This difference is related to the fact that the DRAF T_l 's do not exhaust the unitarity limit as implied by the discussion of Fig. 3. Therefore, the introduction of DRAF T_l 's for deuterons and tritons improves not only the pre-

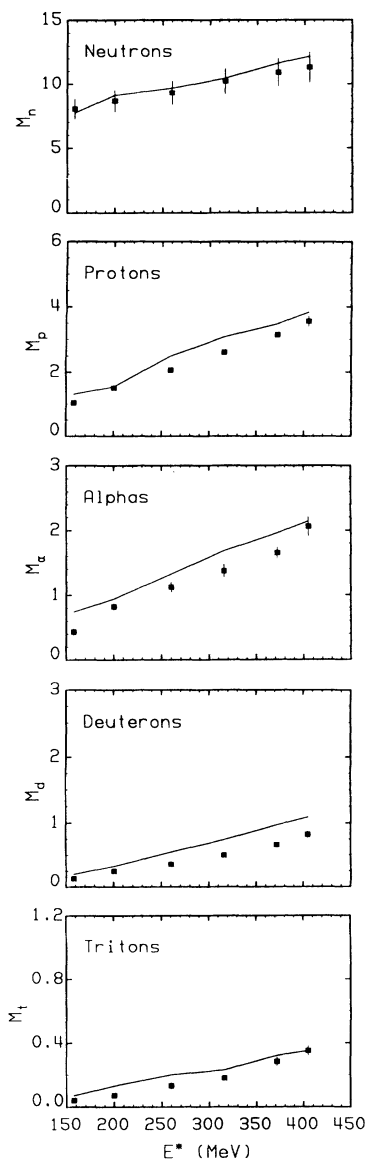


FIG. 4. Excitation energy dependence of n , p , α , d , and t multiplicities (symbols) associated with emissions from equilibrated sources in the reaction 701-MeV $^{28}\text{Si} + ^{100}\text{Mo}$. The solid lines show the results of a statistical model calculation with a temperature-dependent level density constant and DRAF transmission coefficients for deuterons and tritons.

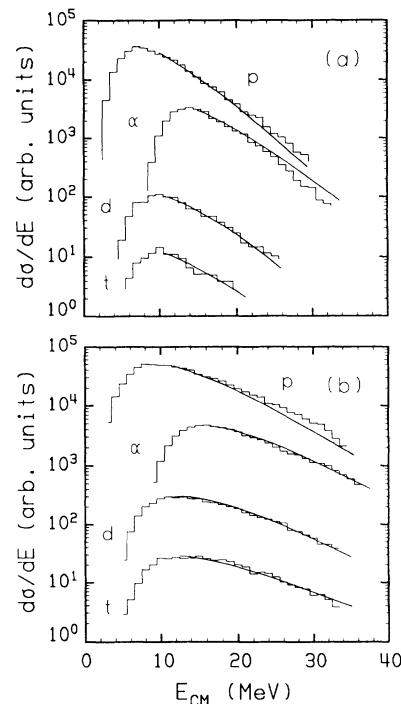


FIG. 5. Calculated p , α , d , and t energy spectra (histograms) using a temperature-dependent level density constant. The solid lines represent the experimental slopes obtained from evaporative moving source fits. (a) and (b) correspond to emissions from a source excited at $E^* = 158$ and 405 MeV, respectively.

diction of particle multiplicities but the slopes of the spectra as well. A detailed account of the implications of different transmission coefficient sets in the shapes of the calculated spectra is given in Ref. [29].

A number of tests were made in order to examine the sensitivity of the calculated particle multiplicities and slopes of the evaporation spectra with an energy-dependent level density parameter. A simple linear function for an increasing k with excitation energy was used for this purpose. The results of these calculations are summarized below.

(1) An increasing k from 6 to 11 MeV between $E^* = 0$ and 405 MeV: The calculated multiplicities were found to agree very well with the data. The slopes of the particle spectra for the first two excitation energy bins were close to the experimental ones. However, at higher excitations, the calculated slopes were much smaller (harder spectra) than the experimental ones or those of the $A/11.0$ calculations of the previous section.

(2) An increasing k from 6 to 8.5 MeV between $E^* = 0$ and 405 MeV: The calculated n and p multiplicities overestimated the data, but the α , d , and t multiplicities were found in a good agreement. The slopes of the spectra were close to the experimental ones, with the exception of the first two excitation bins where they were found softer.

(3) An increasing k from 8 to 10.5 MeV between $E^* = 0$ and 405 MeV: This calculation reproduces the n , p , d , and t multiplicities but overestimates slightly the α multi-

plicities. The trend of the slopes of the spectra was similar to that of case 2.

Certain remarks can be made on the use of a $k = k(E^*)$ [or equivalently $k(T)$] in the above calculations. The particle yields and spectral slopes depend both on k and on its rate of change with E^* . The particle multiplicities seem to be sensitive in the absolute value of k at each E^* . The slopes of the particle spectra are more sensitive in the rate of change of k with excitation energy. We conclude that a complete analysis for level density determinations requires a simultaneous knowledge of the slopes of the evaporation particle spectra and the corresponding multiplicities or cross sections.

A combination of properly matched dependences in (1) and (3) could provide a good description of the data at all energies. Interestingly enough, such an excitation energy dependence leads to a temperature variation for k similar to that of Eq. (7). However, it has to be noted that the temperature factor of $T/3$ in the denominator of Eq. (7) was based on calculations [27,28] on ^{208}Pb . This is expected to depend weakly on the nuclear mass. Our statistical model calculations seem to be insensitive in variations from $T/3$ to $T/4$ in Eq. (7). A weak temperature dependence in k is needed in order to describe the experimental data of the present study. Calculations of the temperature variation of k in the mass $A = 160$ region by Hasse and Schuck [26] are consistent with such a weak temperature dependence.

VI. SUMMARY

In the present work, we applied an extended version of a statistical model code to the description of particle emission properties of compound nuclei in a wide range of excitation energies. On the basis of agreement with available data, different penetrability models for particle emission were tested. Particular attention was paid to the case of deuteron and triton evaporation.

Our analysis shows that the statistical model with optical model transmission coefficients for n , p , α , d , and t emission provides a good description of particle evaporation in the case of low excited compound nuclei (E^* up to 100 MeV). However, this model systematically overpredicts the deuteron and triton emission yields at high excitation energies (E^* up to 405 MeV). The above discrepancies were attributed to absorption processes in the optical model inverse cross section which are not related to fusion. It seems that such processes represent a

small fraction of the OM absorption cross section for neutrons, protons, and alphas. This explains the success of the OM T_I 's in statistical model calculations when n , p , and α emissions dominate. However, this method breaks down for deuterons and tritons whose emission becomes more important at high excitation energies.

The elimination of OM absorption processes not related to fusion was made with two different approaches. In the first one, the introduction of IWBC T_I 's was made in order to eliminate OM processes like transparency, reactive scattering outside the real potential, and size resonances. In the second approach, an elimination of the direct-reaction components from the OM absorption cross section was made with the DRAF method. Based on studies of deuteron-induced reactions, it was realized that the direct-interaction part is a sizable fraction of the OM absorption cross section.

The present work demonstrates that the elimination of the direct-interaction components from the OM absorption is necessary for a successful description of deuteron and triton emission in statistical model calculations. The DRAF method provides a quantitative means of isolating these processes and extracting a transmission coefficient set consistent with the fusion process. With this method, it became possible to reproduce the deuteron and triton emission yields from highly excited compound nuclei; a result which could not be achieved with any other statistical model parameter change. For deuteron emission in particular, the employed fusion radius is consistent with a deuteron fusion cross-section analysis in the mass region of the present study. A global description of particle emission in 701-MeV reactions of ^{26}Si on ^{100}Mo reactions was given in the context of a temperature-dependent level density constant.

We believe that systematic studies of deuteron- and triton-induced reactions are needed in order to extract and systematize the fusion radii. This may provide valuable information for parameters to be used in the description of statistical deuteron and triton emission from highly excited nuclei.

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