α decay calculations with a realistic potential

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We have previously used a cluster model, employing a square-well nuclear potential plus a surface-charge Coulomb potential, to satisfactorily describe α-decay half-lives for more than 400 nuclei. Here, we investigate a more realistic version of this cluster model, in which the square-well nuclear potential is replaced by a "cosh" potential geometry having nonzero diffuseness, and the Coulomb potential by one appropriate to a point charge α particle interacting with a uniformly charged spherical core. By varying the adjustable parameters of this more realistic model, we find several potentials which give comparable fits to the α-decay data, and select the one which best tallies with information from other areas of nuclear physics. In addition, we find that the α-particle preformation probability needed to describe favored transitions in odd-mass nuclei is only 60% of the equivalent quantity required for the ground state to ground state transitions of even-even nuclei.

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I. INTRODUCTION

We have recently shown [1–3] that it is possible to describe satisfactorily the large body of data on favored α decay using a simple cluster model. The α-core potential used in this model consisted of a square-well nuclear potential of depth V_N, fixed for all decays, and radius R determined by the Q-value of each individual decay, complemented by a surface charge Coulomb potential of the same radius. With a fixed set of only three parameters we were able to reproduce the half-lives for most of the ~ 150 ground state to ground state decays of even-even nuclei [1,2] to within a factor of 2, and most of the ~ 250 favored decays of odd-mass nuclei [3] to within a factor of 3. Although we noted strongly correlated ambiguities in the best-fit values of our parameters, we were nevertheless able to conclude convincingly that the α-core relative motion wave function should contain a large number of nodes (which remains constant while a major neutron shell is being filled and increases sharply as the parent neutron number goes up through the magic value of 126), and that the α-particle preformation probability is very similar in all the decays examined.

Having set a benchmark for the level of agreement to be expected between calculation and experiment with minimum use of parameter variation, and determined the essential characteristics of a successful model, it is clear that more realistic forms of the α-core potential should be investigated. We thus replace the square-well nuclear potential by a "cosh" geometry

$$V_N(r) = -V_0 \frac{1+\cosh(R/a)}{\cosh(r/a)+\cosh(R/a)} \tag{1.1}$$

of depth V_0, nonzero diffuseness a, and radius R. This form of the cluster-core potential has been very useful in studies of clustering phenomena in light nuclei [4], and becomes closely similar to the commonly used Woods-Saxon potential for large values of R/a. We also replace the surface-charge Coulomb potential by a form appropriate to a point α particle interacting with a uniformly charged spherical core of radius R:

$$V_C(r) = \begin{cases} \dfrac{Z_1 Z_2 e^2}{r} & \text{for } r \geq R \,, \\[2ex] \dfrac{Z_1 Z_2 e^2}{2R}\left[3-\left[\dfrac{r}{R}\right]^2\right] & \text{for } r \leq R \,, \end{cases} \tag{1.2}$$

where Z_1 and Z_2 are the charges of the α particle and the core, respectively. There is no *a priori* reason to tie the Coulomb and nuclear radii together as we have done in Eqs. (1.1) and (1.2), but we impose this constraint so as to minimize the number of free parameters.

The use of a nuclear potential with a nonzero diffuseness enables us to show that the level of agreement with the data we obtained previously was not an artifact generated by the unphysical square-well geometry employed. Also, it allows us to make direct comparison with the more realistic potentials used in related areas of nuclear physics.

In Sec. II we outline the modified version of the cluster model, taking into account the changes in the nuclear and Coulomb potentials. In Sec. III we describe our searches for the best set of parameters to reproduce α-decay half-lives. We discuss our results in Sec. IV and draw our conclusions in Sec. V.

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II. MODIFIED CLUSTER MODEL WITH A REALISTIC POTENTIAL

The method of calculating an α-decay half-life with a more realistic form of the α-core potential is similar to that presented previously [1–3], with the various manipulations carried out numerically instead of analytically. We write the cluster-core potential $V(r)$ as the sum of a realistic nuclear term [see Eq. (1.1)], a realistic Coulomb term [see Eq. (1.2)], and a Langer modified centrifugal barrier [in which $L(L+1)$ is replaced by $(L+\frac{1}{2})^2$]. Thus we put

$$V(r)=V_N(r)+V_C(r)+\frac{\hbar^2}{2\mu r^2}(L+\tfrac{1}{2})^2 , \tag{2.1}$$

where μ is the reduced mass of the cluster-core system. The nuclear potential depth V_0 and diffuseness a are fixed for all decays (see Sec. III), so that the radius R is the only unknown in this expression.

The classical turning points (r_1, r_2, and r_3 in order of increasing distance from the origin) are found by numerical solution of the equation $V(r)=Q$, where Q is the energy appropriate to the decay under consideration. We deduce Q from the measured α-particle kinetic energy E_α by applying a standard recoil correction, as well as an electron shielding correction in the systematic manner suggested by Perlman and Rasmussen [5]:

$$Q=\frac{A_p}{A_p-4}E_\alpha+(65.3Z_p^{7/5}-80.0Z_p^{2/5})10^{-6}\ \text{MeV} , \tag{2.2}$$

where Z_p and A_p are the charge and mass numbers, respectively, of the parent nucleus. It is worth noting that for small L the value of r_1 is close to zero. Also, the nuclear term $V_N(r)$ can be neglected in the asymptotic region, and r_3 can be found by solving the resulting quadratic equation.

We next choose a value (see Sec. III) for the orbit global quantum number $G=2n+L$, where n is the number of nodes in the radial wave function, and $L=0$ for the favored s-wave transitions considered here. The radius parameter R, which appears in both $V_N(r)$ and $V_C(r)$, can then be evaluated separately for each decay by applying the Bohr-Sommerfeld quantization condition to place the quasibound state of relative motion with n nodes at the energy Q. Hence,

$$\int_{r_1}^{r_2} dr \left[\frac{2\mu}{\hbar^2}[Q-V_N(r)-V_C(r)]-\frac{(L+1/2)^2}{r^2}\right]^{1/2} =(2n+1)\frac{\pi}{2}=(G-L+1)\frac{\pi}{2} . \tag{2.3}$$

This system of equations for R, involving the turning points and the Bohr-Sommerfeld condition, is readily solved to an acceptable level of accuracy by a few iterations of the Newton-Raphson method. For small values of the diffuseness the radius obtained from the earlier square-well treatment [1–3] provides a good starting value.

Once the value of R has been determined, we can calculate the width Γ of the quasibound state, in semiclassical approximation, by following the procedure of Gurvitz and Kälbermann [6]. Thus

$$\Gamma=PF\frac{\hbar^2}{4\mu}\exp\left[-2\int_{r_2}^{r_3}dr\,k(r)\right] , \tag{2.4}$$

where P is the α-particle preformation probability. The normalization factor F is given by

$$F\int_{r_1}^{r_2}dr\,\frac{1}{k(r)}\cos^2\left[\int_{r_1}^{r}dr'\,k(r')-\frac{\pi}{4}\right]=1 , \tag{2.5}$$

where the squared cosine term may be replaced by its average value of $\frac{1}{2}$ without significant loss of accuracy, so that

$$F\int_{r_1}^{r_2}\frac{dr}{2k(r)}=1 , \tag{2.6}$$

with the wave number $k(r)$ given by

$$k(r)=\left[\frac{2\mu}{\hbar^2}|Q-V(r)|\right]^{1/2} . \tag{2.7}$$

The α-decay half-life is then related to the width by $T_{1/2}=\hbar\ln 2/\Gamma$.

The relevant charges, masses, and Q-value are defined by the decay under consideration. The radius R appropriate to the decay is determined by the procedure outlined above. This leaves four free parameters (to be determined by a best fit to the available data): the α-particle preformation probability P, the nuclear potential depth and diffuseness, V_0 and a, and the global quantum number G.

III. CHOICE OF MODEL PARAMETERS

In this section we describe how we assigned values to the parameters P, V_0, a, and G so as to best reproduce the measured half-lives of ground state to ground state α decays in even-even nuclei. We include neither the very lightest nor the very heaviest of these nuclei in our fit, since their production and measurement is at the limits of present experimental capabilities, and their half-lives are significantly less well known than is typically the case. We therefore include in our fitting exercise only the decays of even-even nuclei listed in Tables 1–3 of Ref. [2] which have been measured to good accuracy and do not contain a magic or near-magic number of neutrons.

We have restricted attention to even-even parent α-decay data for two main reasons. On the one hand, we can be sure that we are dealing with pure $L=0$ transitions. On the other hand, the ground state of the daughter nucleus is always the most heavily populated following the decay, and is a natural candidate for the core in our model. Both these simplifications are absent when considering the decays of odd-mass nuclei, making the choice of which odd-A decay data to include in a set for fitting much more ambiguous.

Although we fit the parameters of our more realistic model to the same data set as was used for the square-well fit [2], we have modified our fitting criterion somewhat. Previously, in the square-well fit, we minimized the quantity

$$S_E=\sum\left[\frac{\log_{10}T^{\rm exp}_{1/2}-\log_{10}T^{\rm cal}_{1/2}}{\Delta(\log_{10}T^{\rm exp}_{1/2})}\right]^2 , \tag{3.1}$$

thus weighting the sum of the squared deviations of the logarithms of the calculated and measured half-lives by the *experimental* errors in the logarithms of the half-lives.

This is certainly the generally adopted procedure in parameter fitting exercises, but is not entirely appropriate in our case. For example, although we find strong evidence for rather similar α-preformation factors P, these are unlikely to be *exactly* the same in all cases, and the consequent scatter limits the best fit that can be achieved by optimizing the potential parameters alone. From the internal evidence, e.g., Geiger-Nuttall linear fits to series of isotopes, this scatter is at the level of a factor of about 1.5. It is therefore unreasonable to weight the deviations with $\Delta(\log_{10}T^{\rm exp}_{1/2})$, when $\Delta(\log_{10}T^{\rm cal}_{1/2})\approx 0.2$ is typically much larger. Also, a relatively small number of nuclei which are especially amenable to precise measurement, including ^{208,210,214}Po, ^{220,224}Rn and ^{222,224,226}Ra dominate the expression in Eq. (3.1) because of the extremely small experimental errors on their measured half-lives. These values certainly provide valuable information, but should not be allowed to overwhelm the fit, virtually excluding contributions from all other data.

We thus replace S_E by the quantity

$$S_C=\sum\left[\frac{\log_{10}T^{\rm exp}_{1/2}-\log_{10}T^{\rm cal}_{1/2}}{\Delta(\log_{10}T^{\rm cal}_{1/2})}\right]^2 , \tag{3.2}$$

where we take $\Delta(\log_{10}T^{\rm cal}_{1/2})$ to be equal for all nuclei included in the fit. Since we shall not try to draw any conclusions from the absolute value of S_C, the precise value that we attach to $\Delta(\log_{10}T^{\rm cal}_{1/2})$ is not important. Given that it is equal in all cases, it may be simply factored out of the expression for S_C.

There are certain features of the earlier square-well fitting exercise [1,2] which we also encounter here. The α-preformation parameter P remains essentially undetermined by our fits; we again eliminate it at the outset by taking the limiting value $P=1$ for convenience, in line with previous treatments [2,7]. We can make no statement about the absolute value of P since whatever value we choose, provided it is $\gtrsim 0.001$, can be readily accommodated by compensatory changes in the other parameters. However, whatever value for P is adopted, it can be maintained essentially fixed (to within about a factor of 2) for all the decays we have examined.

Apart from the difficulty of fixing P, we again find that several families of parameter values are capable of giving equally good descriptions of the α-decay data. The only unchanging feature of the fits is that the global quantum number G should be large (in the range 18–24), and increase by two units as N increases from below the magic number of 126 to above it. Thus, although we cannot uniquely fix a value for G appropriate to $N>126$ ($G_>$ say) we can certainly affirm that the corresponding value of G for $82<N\leq 126$ ($G_<$ say) should be given by $G_>-G_<=\Delta G=2$.

Adopting $P=1$ and $\Delta G=2$, we obtain essentially identical quality fits to the half-life data with the four sets of parameter values shown in Table I. Our strategy has been to select a value for $G_>$ and a small starting value for the diffuseness a, and then to search for a value of V_0 to minimize the quantity S given by

$$S=\sum(\log_{10}T^{\rm exp}_{1/2}-\log_{10}T^{\rm cal}_{1/2})^2 . \tag{3.3}$$

Keeping $G_>$ fixed, we increased a by 0.05 fm and optimized V_0 again. The incremental increases in a and optimizations of V_0 were continued and the least of these minima of S, and corresponding values of a and V_0, determined. Once these had been established, $G_>$ was changed, and the sequence of stepping through a and optimizing V_0 repeated for this new global quantum number. Reducing $G_>$ below 18 or increasing it above 24 leads to worse fits, as manifested by progressively larger values of S.

On numerical grounds alone, we have no strong preference for any particular parameter set given in Table I. Nevertheless, we eventually chose to continue our analysis with the values

$$V_0=162.3\ \text{MeV},\quad a=0.40\ \text{fm},\quad G_>=22 ,\quad G_<=20,\quad P=1 \tag{3.4}$$

for the following three physical reasons. A straightforward application of the Wildermuth condition to the valence protons and neutrons outside a ^{208}Pb core yields $G_>=22$. The real part of the global α-nucleus optical potential of McFadden and Satchler [8] has $a=0.52$ fm, and a depth of $V_0=185$ MeV. This depth was subsequently updated, in a 1983 communication to the NEA, to a value of $V_0=164.7$ MeV (and the radius parameter increased somewhat). Note that although this potential has a Woods-Saxon geometry it is nearly identical to our nuclear potential, Eq. (1.1). A calculation of the ground state rotational band of ^{204}Hg, treated as an α hole in a ^{208}Pb core, gives a good reproduction of the energies and electromagnetic properties [9] of the 0^+, 2^+, 4^+, and 6^+ states using the potential prescription of Eq. (3.4). Thus, although we cannot uniquely determine an α-nucleus potential simply by its ability to reproduce α-decay data, the parameter set of Eq. (3.4) does give an excellent description of such data and is simultaneously compatible with expectations from several other areas of nuclear physics.

TABLE I. Values of V_0, a, and $G_>$ which yield good fits to α-decay data. In all cases we take $P=1$ and $\Delta G=G_>-G_<=2$. See text for details.

V_0 (MeV)	a (fm)	$G_>$ (for $N>126$)	S [see Eq. (3.3)]
141.9	0.70	18	1.11
152.5	0.55	20	1.07
162.3	0.40	22	1.11
172.0	0.25	24	1.21

IV. CALCULATIONS OF α-DECAY HALF LIVES

Using the parameters of Eq. (3.4) we have calculated the half-lives of all the 154 ground state to ground state decays of even-even nuclei listed in Tables 1–5 of Ref. [2], where a square-well nuclear potential and a surface charge Coulomb potential were employed. Our results are qualitatively similar to those earlier ones, and in particular, the discussions given there of uncertainties and difficulties in the interpretation of some of the data points are still highly relevant. Rather than presenting our extensive results in tabular form, which we will supply to the interested reader on request, we give them in graphical form. Figure 1 shows our results for those decays in which the even-even parent nuclei contain more than 82 neutrons. We hasten to add that there is no problem in applying our model to nuclei having $N<82$, though the value of G must be reduced by a further two units [10] (to 18 in this case) to obtain agreement between measured and calculated half-lives at the factor of 2 level. We omit them here for convenience in choosing the horizontal scale of our diagram.

Figure 1 shows $\log_{10}(T_{1/2}^{\rm exp}/T_{1/2}^{\rm cal})$ as a function of parent nucleus neutron number for 147 even-even nuclei. Broken lines indicate that the deviation between experimental and calculated half-lives is within a factor of 2 for the great majority of decays. A few cases, such as $^{166}_{74}\mathrm{W}_{92}$ and $^{254}_{104}\mathrm{Rf}_{150}$, are not very well reproduced (as before [2]); however, in general, an ability to predict closely a wide range of α-decay half-lives with a simple four-parameter model, given only the measured α-particle energy E_α, is clearly demonstrated.

Apart from exhibiting the sheer proximity of a large number of calculated and measured half-lives, Fig. 1 can

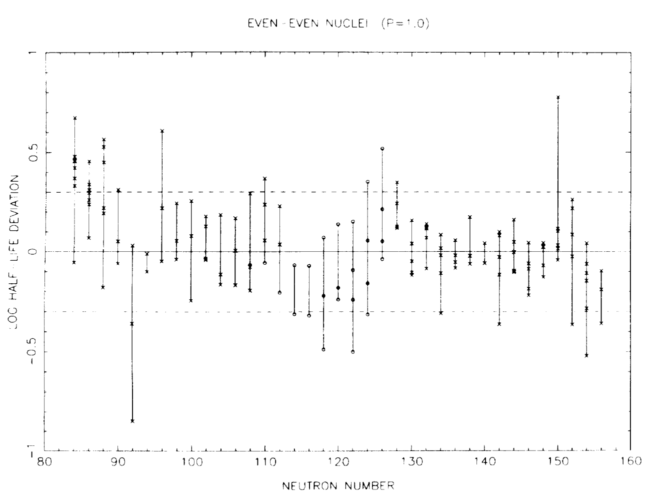

FIG. 1. Plots of $(\log_{10}T_{1/2}^{\rm exp}-\log_{10}T_{1/2}^{\rm calc})$ as a function of parent neutron number N for even-even nuclei. Results for individual parent nuclei with $Z>82$ and $N\le 126$ are labeled by open circles, and the remainder by crosses. Lines are drawn connecting each set of isotones. Factor of 2 deviations between theory and experiment are indicated by broken lines. The model parameters are $V_N=162.3$ MeV, $G_>=22$ (for $N>126$), $G_<=20$ (for $N\le 126$), $a=0.4$ fm, and $P=1$.

be used to search for systematic trends in the deviations of these two quantities. There do not seem to be any strong tendencies in evidence. However, there is an indication that our calculated half-lives are consistently lower than the measured values for the lightest nuclei having $82\le N\le 88$. Also, we have plotted decays for parent nuclei having $Z>82$ and $N\le 126$ by open circles and the remainder by crosses. We hoped in this way to highlight any effects of the $Z=82$ proton shell closure, which is conspicuous by the absence of its influence on our half-life calculations. There is a tendency for the calculated half-lives to be larger than the measured values for those nuclei having $Z>82$ but $N\le 126$, as manifested by the number of circles below the $\log_{10}(T_{1/2}^{\rm exp}/T_{1/2}^{\rm cal})=0$ line; this might be a weak effect of the $Z=82$ proton shell closure [1].

α decays from odd-mass nuclei are not as straightforward to categorize as those between the ground states of even-even nuclei. It is quite common for several states of the daughter nucleus to be strongly populated as a result of α decay, and the choice of favored state is not always clear-cut. Furthermore, the orbital angular momentum of the emitted α particle is not necessarily zero. Even when the parent and daughter states have equal spin-parity values, it is still generally possible for the α particle to be emitted in a variety of L states, and in principle a linear superposition of these states should be considered in order to treat the process rigorously.

It is well known from the experimental investigation of α decays in odd-mass nuclei that those decays which can proceed by s-wave transitions, and have the odd nucleon of both parent and daughter in the same orbital, proceed at a rate compatible with that of the neighboring even-even nuclei. It is conventional to introduce a hindrance factor (HF), calculated from the spin-independent equations of Preston [11], to describe this similarity. By convention HF $=1$ for the decays of even-even nuclei, and low hindrance factors of 1–4 in odd-mass nuclei may be taken to designate those transitions which we might reasonably expect to predict to good accuracy, within our model, without any further parameter adjustments. In addition, the problem of L admixtures may not be too serious, since preliminary estimates show that contributions from $L\ge 4$ only affect our calculated half-lives by $\lesssim 10\%$. The only serious competition may therefore be expected from $L=2$, which we estimate to be capable of making typically $\sim 30\%$ changes to $T_{1/2}^{\rm cal}$. Thus, we calculate the half-lives for α decays of odd-mass nuclei which are characterized by low hindrance factors using the unchanged parameter set of Eq. (3.4) and assuming pure s-wave decays. We anticipate a slight reduction in the quality of our fits with respect to those of the even-even nuclei, but consider the exercise to be useful, since we know [3] that our calculations can still correlate a large number of half-life measurements at the level of about a factor of 3.

We have therefore calculated 231 α-decay half-lives for odd-A nuclei with our modified potential model and the parameters of Eq. (3.4). The decays considered are those listed in Tables 1–6 of Ref. [3], and the same provisos and reservations about the data selection discussed in

Ref. [3] still hold well here. Figure 2 shows our results, in the same format as for the even-even nuclei, except that the broken lines indicate agreement between theory and experiment at the factor of 3 level (instead of factor of 2). Although a large majority of the decays fall within these limits, it is immediately apparent that there is a strong preponderance of points above the $\log_{10}(T_{1/2}^{\text{exp}}/T_{1/2}^{\text{cal}})=0$ line. This indicates a systematic underestimation of the half-lives of the favored α decays in odd-mass nuclei.

One possible explanation for this deviation is that the effective α-particle preformation probability in odd-mass nuclei, P_{OE}, may be smaller than the equivalent quantity for even-even nuclei P_{EE}. To consider this further, we leave all other parameters unchanged, but adjust P_{OE} so that the average of $\log_{10}(T_{1/2}^{\text{exp}}/T_{1/2}^{\text{cal}})$ for the odd-A nuclei considered in Fig. 2 becomes zero. We find that we need a value of $P_{OE}=0.6P_{EE}$ to achieve this. We then obtain the results shown in Fig. 3. The suggestion that P_{OE} should be smaller than P_{EE} is in line with the work of Blendowske *et al.* [12,13] who take $P_{OE}=0.5P_{EE}$. We remain a little cautious about this explanation, since our systematic underestimates of the odd-A half-lives using $P=1$ may be a reflection of some other neglected phenomenon, such as the contributions to $T_{1/2}^{\text{cal}}$ from emitted α particles having $L\geq 2$.

The overall description of the favored odd-A α-decay half-lives, although inferior to that of the even-even nuclei, is still very satisfactory. It is also noticeable that the two weak trends discussed above in connection with the even-even decays of Fig. 1 are also present for the odd-A decays of Fig. 3. Thus there appears to be a weak (but systematic) underestimate of the α-decay half-lives as the $N=82$ shell closure is approached and a similar overestimate for parents with $Z>82$ and $N\leq 126$.

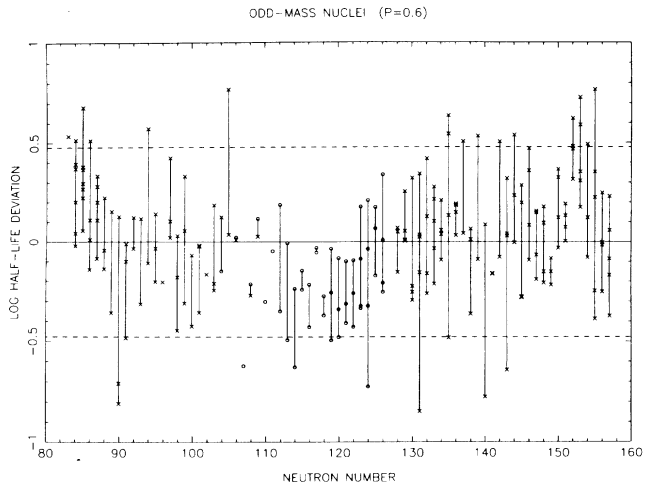

FIG. 3. Plots of $(\log_{10}T_{1/2}^{\text{exp}}-\log_{10}T_{1/2}^{\text{calc}})$ as a function of parent neutron number N for odd-mass nuclei. Results for individual parent nuclei with $Z>82$ and $N\leq 126$ are labeled by open circles, and the remainder by crosses. Lines are drawn connecting each set of isotones. Factor of 3 deviations between theory and experiment are indicated by broken lines. The model parameters are $V_N=162.3$ MeV, $G_>=22$ (for $N>126$), $G_<=20$ (for $N\leq 126$), $a=0.4$ fm, and $P=0.6$.

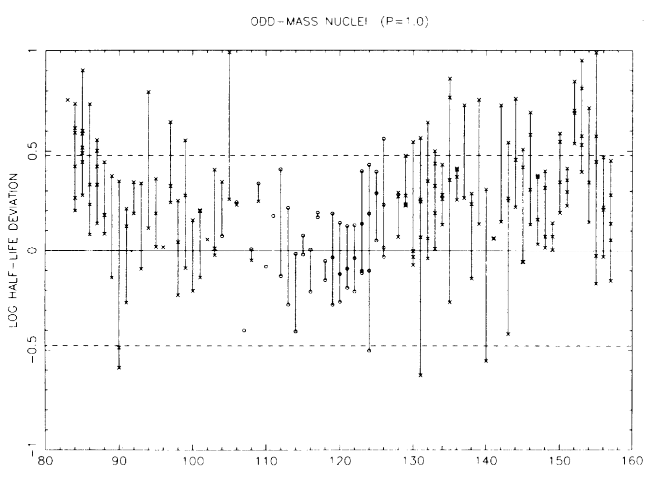

FIG. 2. Plots of $(\log_{10}T_{1/2}^{\text{exp}}-\log_{10}T_{1/2}^{\text{calc}})$ as a function of parent neutron number N for odd-mass nuclei. Results for individual parent nuclei with $Z>82$ and $N\leq 126$ are labeled by open circles, and the remainder by crosses. Lines are drawn connecting each set of isotones. Factor of 3 deviations between theory and experiment are indicated by broken lines. The model parameters are $V_N=162.3$ MeV, $G_>+22$ (for $N>126$), $a=0.4$ fm, and $P=1$.

V. CONCLUSIONS

Our original applications [1–3,10] of the cluster model discussed above, utilizing a square-well nuclear potential and a surface-charge Coulomb potential, examined 409 unhindered (or favored) α decays in both even-even and odd-mass nuclei with a set of three fixed parameters and were able to reproduce the half-lives of 363 of them to within a factor of 3. In view of the lack of detailed input specific to any given nucleus (just the charge, mass, and Q-value are required), this broad level of agreement with an extensive database is remarkable.

There are two fundamental features of our model which are responsible for its success. Firstly, we assume that the α-core relative motion wave function contains many nodes, so that the motion is characterized by a large value of the global quantum number G. This is not a particularly new idea, and is suggested by elementary considerations of how an α cluster could be formed at the nuclear surface by correlated motions of the underlying valence nucleons in shell model orbitals. Indeed, this is an assumption often made in analyzing α-transfer reactions onto heavy targets [14] and has even been applied to α decay itself in earlier studies [15]. We do not fix G *a priori* by appeal to any particular nuclear structure model, but rather treat it as a free parameter and find that several values in the range 18–24 are admissible if appropriate changes are made in the other parameters. Secondly, we fix the potential depth throughout, and adjust the radius of our potential so as to produce a quasi-bound state at the Q-value of each individual decay. This radius is determined quite independently of any reference to the half-life, yet its subsequent use in Eqs. (2.4)–(2.7) leads to an excellent estimate for the half-life.

We can summarize by saying that the square-well fits provide a very useful benchmark for the level of agreement that can be obtained between calculated and measured α-decay half-lives with no more explicit nuclear structure information than that contained in a knowledge of the Q-value for each individual decay. In fact, the Q-value alone contains a lot, *but by no means all*, of the essential nuclear structure physics relevant to the decay.

Despite the successes of the square-well version of the cluster model treatment of α decay, it is open to the criticism that the potential is unrealistic, and, that the half-lives deduced from it may somehow be an artifact of the simplified geometry employed. To counter this criticism, we have reexamined our model, replacing the square-well nuclear potential by a realistic "cosh" potential geometry, which has been widely used in α-clustering studies of light nuclei [4], and replacing the surface charge Coulomb potential by a form appropriate to the interaction of a point α particle with a spherical uniformly charged core. We have taken the nuclear and Coulomb radii to be equal to keep the number of free parameters to a minimum, but this is not an essential requirement.

We have fitted the free parameters of our refined model to the same set of half-lives for ground state to ground state decays of even-even nuclei as were used in the original square-well fitting exercise [2]. We have changed our fitting criterion so that the deviations of all calculated and measured values are treated on an equal footing and no longer weighted by the corresponding experimental errors. Two features of the square-well fit are again evident. The absolute value of the alpha-preformation factor P is poorly determined, but may be held fixed at some arbitrary constant value, and, whatever large value is chosen for G (within the limits of 18–24 indicated by our fit), it should be increased by two units as the parent nucleus neutron number rises up through the $N=126$ shell closure.

We are unable to determine unique values for the free parameters of our model by fitting considerations alone. There are highly correlated ambiguities between them, and the four parameter sets given in Table I are all able to reproduce the even-even α decay data as well as, or even better than, the original square-well prescription. Although the parameter set of Eq. (3.4) is not rigorously demanded by the fitting criterion, we express a preference for it by appeal to physical arguments concerning the following: (i) The value of $G_>$ expected from application of the Wildermuth condition to the formation of an α cluster from the valence nucleons outside a ^{208}Pb core; (ii) proximity to the values of the depth V_0 and diffuseness a proposed by McFadden and Satchler [8] for the real part of their global alpha-nucleus optical potential (even though it is fitted to scattering data on nuclei predominantly much lighter than those of interest to us here); and (iii) the spectrum of ^{204}Hg considered as an α hole interacting with a ^{208}Pb core, for which our potential correctly reproduces the energies and electromagnetic properties [9] of those states in ^{204}Hg which are strongly excited in the ^{208}Pb$(d,{}^6\mathrm{Li})^{204}$Hg reaction [14]. Indeed, the authors of [14] find spectroscopic factors close to unity. We conclude that we can find a realistic α-nucleus potential which is able to generate a good fit to the α-decay data and is compatible with the constraints imposed by several other areas of nuclear physics.

The potential radii are determined separately for each decay and exhibit the same general behavior that was found for the square-well fits [1–3,10]. The value of R increases rather slowly while a neutron shell is being filled, jumps abruptly at the shell closure, and resumes its slow increase as the next neutron shell is filled. Although the average increase of R between the extremes of the mass range is roughly proportional to $A_p^{1/3}$, its value within a given major neutron shell is nearly constant. Using the parameters of Eq. (3.4), we find that R increases from 6.8 to 7.0 fm while the $82<N\le 126$ shell is being filled, jumps to about 7.5 fm at the shell closure, and then rises from 7.5 to 7.7 fm for the largest values of A_p. If we divide out a factor of $A_p^{1/3}$, we find that our radii lie in the range $(1.2-1.3)A_p^{1/3}$. For comparison, the radii of the Woods-Saxon optical potential of McFadden and Satchler are taken as $1.4A_p^{1/3}$. On the other hand, it has been reported [14] that smaller values $R=1.2A_p^{1/3}$ significantly improve the fit to ^{208}Pb$(d,{}^6\mathrm{Li})$ α-pickup data.

The final points to emerge from this study concern general trends in the behavior of $(T_{1/2}^{\mathrm{calc}}/T_{1/2}^{\mathrm{exp}})$ which may reflect corresponding trends in the α-preformation factor P. Although we can adequately describe the α decays of odd-A nuclei with the same parameters as for even-even nuclei, we underestimate their half-lives systematically. This can be corrected by reducing P to 60% of its previous value, in agreement with the work of Blendowske *et al.* [12,13]. In addition, there is an indication of a small systematic increase in P as the $N=82$ neutron shell closure is approached and also some further indication of a small systematic decrease in P at the $Z=82$ proton shell closure, as manifested by the decay data for both even-even and odd-A nuclei in Figs. 1 and 3, respectively.

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