Hot nuclear and neutron matter with a density-dependent interaction

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The temperature dependence of the different thermodynamic quantities like entropy, equation of state, free energy, internal energy, effective mass, chemical potential, etc. are calculated for nuclear and neutron matter within a fully self-consistent model and compared with the results obtained by others. An effective interaction consisting of Sussex interaction plus an empirical density-dependent interaction is taken in the calculation. The latter is added so as to correctly reproduce the saturation property of nuclear matter. The calculated entropy is in excellent agreement with experiment.

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I. INTRODUCTION

Over the last few years, there has been an increased interest in the studies of excited nuclear matter because of the hope that such a hot and dense nuclear matter is likely to be produced (though for a very short time, $<10^{-22}$ sec) during the relativistic heavy-ion collisions. The studies of such matter under unusual conditions of densities and temperatures would also be useful in the understanding of stellar collapse, supernova explosions, neutron stars, etc. For the description of the bulk properties of such excited nuclear and neutron matter, a reliable equation of state is crucial. Entropy is one of the most important thermodynamical variables since it can provide information on the equation of state and phase transitions in nuclear matter. In the heavy-ion collisions, entropy is mainly produced during the formation of the fire ball. Since it does not change significantly during the expansion stage, it can provide information about the hot and compressed stage of collision. Experimentally it can be extracted with good accuracy from fragment yields in contrast to quantities like breakup temperature and densities. In addition, other thermodynamical quantities like chemical potential, free energy, effective mass, etc., also play an important role in the understanding of the hot matter. Hence a good theory should be capable of giving an accurate description of the equation of state, entropy, and other thermodynamic variables of hot nuclear and neutron matter.

Recently we [1] have successfully explained the experimental observations regarding entropy production in heavy-ion collisions within a fully self-consistent model. This model is a generalization of the Brueckner theory to low finite temperatures in which scattering to intermediate states is taken into account and the degeneracy and the single-particle potential are calculated self-consistently. The equation of state thus obtained is expected to be valid up to densities much higher than the nuclear matter densities. The entropy calculated using this model is in excellent agreement with recent experimental data. However, in the above calculation, Sussex interaction has been used which gives insufficient binding by about 3 MeV/nucleon and does not saturate correctly in nuclear matter. Hence Tripathi *et al.* [2] have suggested the addition of a density-dependent term to the original Sussex interaction matrix elements. The parameters of this density-dependent term are fixed empirically by fitting the binding energies and densities of nuclear matter and ^{16}O. The main aim of this paper is to study entropy, single-particle potential, internal energy, chemical potential, effective mass, etc. of nuclear and neutron matter at different temperatures. The organization of this paper is as follows: In Sec. II, the details of our model are given. Results of our calculation are discussed in Sec. III. Finally, Sec. IV contains the conclusions of our study.

II. THE DETAILS OF THE MODEL

The details of the formalism have already been discussed [1,3,4]. For completeness, we give a few important steps. The grand thermodynamic potential per unit volume is given by

$$\Omega=-P=-T\ln \operatorname{tr}\exp[-(H-\mu_n)/T]\ , \tag{1}$$

where H, P, T, μ, and n are the Hamiltonian, pressure, temperature, chemical potential, and number density, respectively. This thermodynamic potential can be expressed as linked cluster expansion analogous to zero-temperature Brueckner-Goldstone expansion, i.e.,

$$\Omega=\Omega_0+\Omega_1+\Omega_2+\cdots\ , \tag{2}$$

where $\Omega_0,\Omega_1,\Omega_2,\ldots$ are the contributions to the thermodynamic potential due to the unperturbed part, one-body part (single-particle potential), and two-body part (binary collision) of the Hamiltonian. Our formalism is limited up to Ω_2. In this formalism we have used the Brueckner reaction matrix instead of the bare *NN* force. The number density n is given by

$$n=\sum_{\tau} n_{\tau} , \tag{3}$$

where n_{τ} is the number density of nucleons with isospin τ (+ for protons and − for neutrons). The proton to neutron ratio is defined as $\gamma = n_+/n_-$. The single-particle energy is given by

$$\epsilon_{\tau}=\frac{\hbar^2k^2}{2m_{\tau}}+U_{\tau}(k) , \tag{4}$$

where k is the momentum, and m_{τ} is the nucleon mass. The single-particle potential $U_{\tau}(k)$ is defined by

$$U_{+}(k_1)=\frac{1}{2\pi^2}\int_0^{\infty}dk_2[n_{+}(k_2)g_{++}(E_s,k_1,k_2)+n_{-}(k_2)g_{-+}(E_s,k_1,k_2)] , \tag{5}$$

where $n_{\tau}(k)$ is the Fermi distribution function given by

$$n_{\tau}(k)=(1+\exp\{[\epsilon_{\tau}(k)-\mu_{\tau}]/T\})^{-1} . \tag{6}$$

Here μ_{τ} is the chemical potential of the nucleons with isospin τ. We have put the Boltzmann constant equal to 1. The number density n_{τ} is obtained by integrating the distribution function $n_{\tau}(k)$ over all momenta and weighting it with spin degeneracy. The g's are the interaction matrices

$$g_{\tau\tau}(E_s,k_1,k_2)=\frac{\tan^{-1}[\pi\rho_E Q_{\tau\tau}K_{\tau\tau}(E_s)]}{\pi\rho_E Q_{\tau\tau}} , \tag{7}$$

where ρ_E is the single-particle level density and the K matrix satisfies the integral equation

$$K_{\tau\tau}(E_s)=V_{\tau\tau}+V_{\tau\tau}\frac{Q_{\tau\tau}}{E_s-H_0}K_{\tau\tau} . \tag{8}$$

Here $V_{\tau\tau}$ is the realistic nuclear interaction, $Q_{\tau\tau}$ is the Pauli operator

$$Q_{\tau\tau}=[1-n_{\tau}(k_1)][1-n_{\tau}(k_2)] , \tag{9}$$

and E_s is the starting energy of the two particles and is given by

$$E_s=\left[\frac{\hbar^2}{2m_{\tau}}\right](k_1^2+k_2^2)+U_{\tau}(k_1)+U_{\tau}(k_2) . \tag{10}$$

It may be noted that the single-particle potential is needed in calculating $n_{\tau}(k)$ [Eq. (6)] which is in turn required to calculate the single-particle potential itself. Hence the single-particle potential is calculated by iteration. Because of this self-consistency of the single-particle potential, the scattering to intermediate states are taken into account properly through the Pauli operator. The chemical potential μ_{τ} is determined by the number density constraint (3). It should be noticed that Eqs. (3) and (5) warrant double self-consistency which must be satisfied with respect to the single-particle potential and chemical potential. The entropy and the internal energy are calculated using the formulas

$$S=-\frac{1}{n}\sum_{\tau}\frac{2}{(2\pi)^3}\int d^3k\{n_{\tau}(k)\ln n_{\tau}(k)+[1-n_{\tau}(k)]\ln[1-n_{\tau}(k)]\} , \tag{11}$$

$$\frac{u}{A}=\frac{1}{n}\sum_{\tau}\frac{2}{(2\pi)^3}\int d^3k\, n_{\tau}(k)\left[\frac{\hbar^2k^2}{2m_{\tau}}+\frac{1}{2}U_{\tau}(k)\right] . \tag{12}$$

The pressure, the effective mass, and free energy are calculated using the formulas

$$P=\frac{1}{\pi^2}\sum_{\tau}\int_0^{\infty}dk\, k^2n_{\tau}(k)\left[\frac{1}{3}k\frac{d\epsilon_{\tau}}{dk}+\frac{1}{2}U_{\tau}(k)\right] , \tag{13}$$

$$\frac{m^*}{m}=\sum_{\tau}\left[1+\frac{2m_{\tau}}{\hbar^2}\left[\frac{dU_{\tau}}{dk^2}\right]\right]^{-1} , \tag{14}$$

and

$$F=u-TS . \tag{15}$$

As has been mentioned earlier, the Sussex interaction does not saturate correctly in nuclear matter. It gives insufficient binding energy by about 3 MeV/nucleon. Hence Tripathi *et al.* [2] have added an empirical density-dependent interaction to the original Sussex interaction so as to reproduce the saturation properties. If $V_{\rm SME}$ denotes the original tabulated Sussex interaction

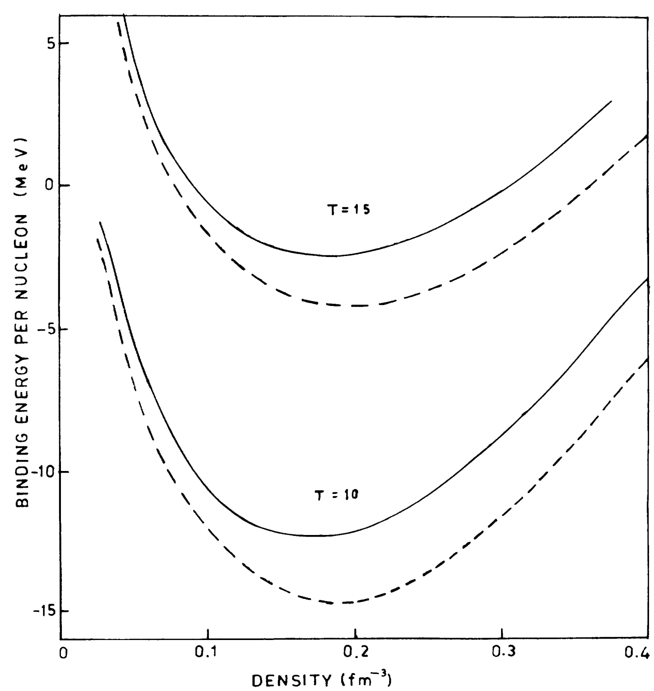

FIG. 1. Binding energy per nucleon versus density at $T=10$ and 15 MeV calculated with the Sussex and density-dependent interaction. The broken lines represent the result calculated only with the Sussex interaction.

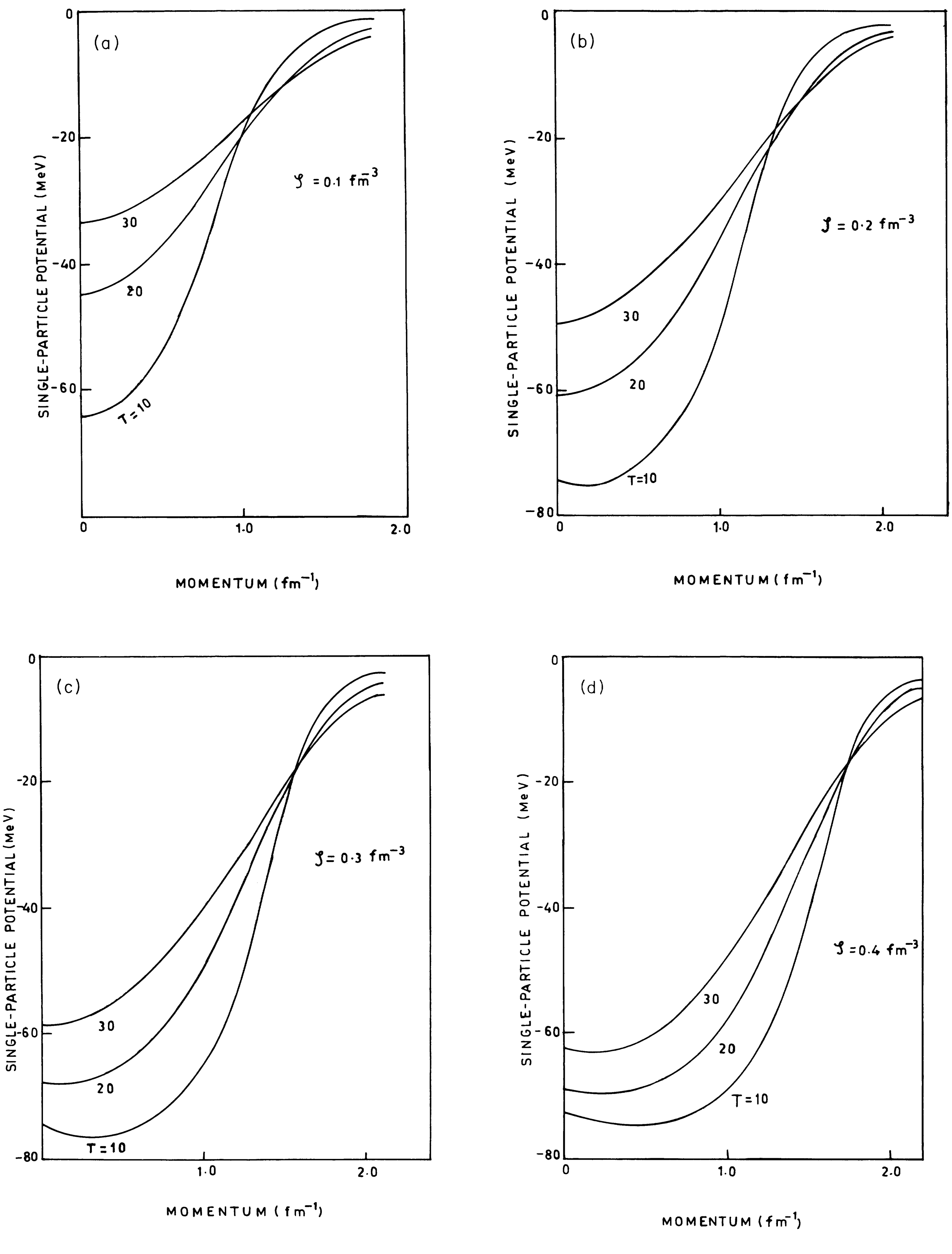

FIG. 2. The single-particle potential versus momentum for nuclear matter at $T=10$, 20, and 30 MeV. (a) $\rho=0.1$ fm^{-3}, (b) $\rho=0.2$ fm^{-3}, (c) $\rho=0.3$ fm^{-3}, and (d) $\rho=0.4$ fm^{-3}.

matrix elements, they define an effective interaction

$$V_{\text{eff}} = V_{\text{SME}} + W \ , \tag{16}$$

where W is a simple density-dependent addition, whose parameters are fixed empirically by fitting the binding energies and densities of nuclear matter and ^{16}O. Specifically

$$W = \sum_{i<j} \{ A\delta(r)\rho^{\alpha}(R) + \tfrac{1}{2}[B(1+P_{ij})+C(1-P_{ij})]\exp(-r^2/a^2)\} \ , \tag{17}$$

where $r = r_i - r_j$, $R = \frac{1}{2}(r_i + r_j)$, ρ is the density, and P_{ij} the space exchange operator. The values of the different

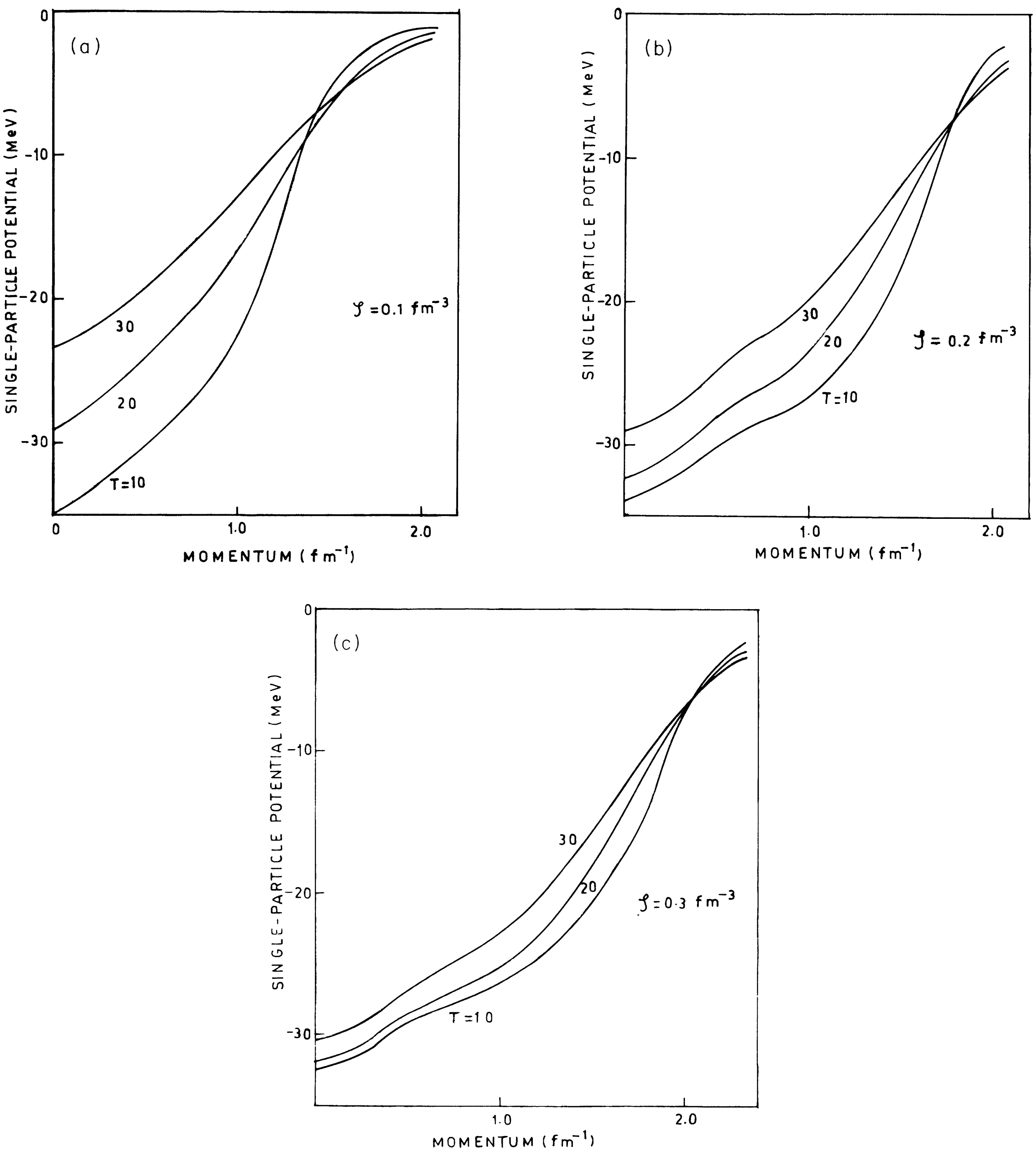

FIG. 3. The single-particle potential versus momentum for neutron matter at $T = 10$, 20, and 30 MeV. (a) $\rho = 0.1$ fm^{-3}, (b) $\rho = 0.2$ fm^{-3}, and (c) $\rho = 0.3$ fm^{-3}.

parameters of this equation are taken from Ref. [6] and are $A=568$ MeV fm$^{3+3\alpha}$, $B=-101.7$ MeV, $C=90.6$ MeV, and $\alpha=\frac{1}{3}$.

III. RESULTS AND DISCUSSIONS

The calculations are performed taking the effective interaction given by Eq. (16). As has been mentioned above, this effective interaction is obtained by adding a density-dependent term to the original Sussex interaction so as to correctly reproduce the saturation properties of nuclear matter. We restrict ourselves to symmetric nuclear matter. We have calculated the binding energy per nucleon at different densities and temperatures with the help of Eq. (12). In Fig. 1, we plot the binding energy per nucleon versus density at $T=10$ and 15 MeV. This is represented by the solid curve in the figure. For comparison, we also plot the binding energy per nucleon versus density at these two temperatures using only the original Sussex interaction (represented by the dashed lines). As expected, the binding energy per nucleon increases with the increase of temperature for both the interactions. However, the Sussex interaction gives more binding. Lejeune *et al.* [5] have studied the properties of cold and hot nuclear matter in the framework of the Brueckner theory extended to finite temperature. They have used the Paris potential supplemented by the introduction of a three-body force. For reproducing the correct saturation effect at $T=0$, they have added a phenomenological term. They have calculated the binding energy per nucleon versus density at $T=10$ MeV. At $\rho=0.17$ fm^{-3}, they find the binding energy per nucleon to be around -12 MeV which agrees quite well with our calculation with density-dependent interaction. However, for large densities, the binding energy per nucleon increases rather sharply in our case.

In Fig. 2, we plot the single-particle potential versus momentum for nuclear matter at different temperatures and densities. We have varied density from $\rho=0.1$ to 0.4 fm^{-3} and temperature from $T=10$ to 30 MeV. We find that for a given density, single-particle potential curves corresponding to different temperatures almost meet at a point. Using the relation $\rho=2k_F^3/(3\pi^2)$, we find that this point almost corresponds to the Fermi momentum of the system. Below this point, the attractive mean field decreases as the temperature increases. However, at higher momentum, the reverse is the case. Again the depth of the potential increases with density for low momentum. Lejeune *et al.* [5] have calculated the single-particle potential at different densities and temperatures. At normal nuclear density and at $T=10$ MeV, they find the depth of the potential at low momentum to be around -75 MeV which agrees quite well with our result. They also find the depth of the potential increasing with density for low momentum. However, in our case, the increase in the depth of the potential with density is relatively smaller.

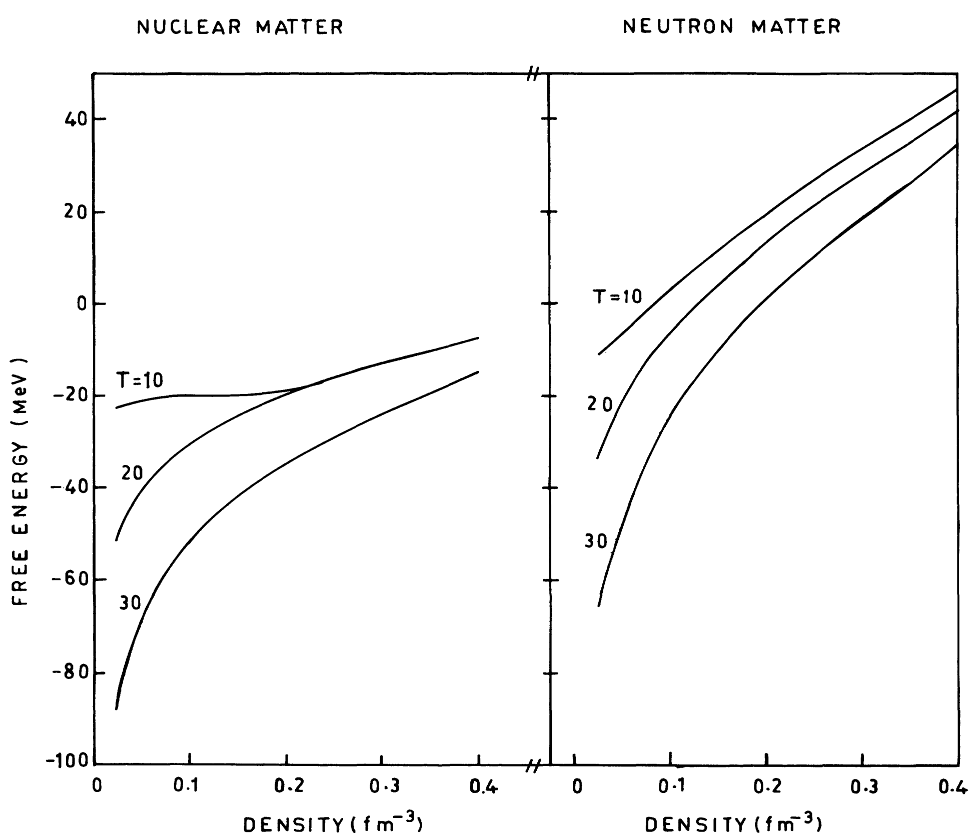

FIG. 4. Free energy versus density for nuclear and neutron matter at $T=10$, 20, and 30 MeV.

Again at high momentum, the single-particle potential is much more repulsive in our case. Baldo *et al.* [7] have studied the symmetrical nuclear matter at finite temperature in the framework of the Brueckner-Hartree-Fock approximation extended to include single-particle correlations. They have calculated the single-particle potential at $T=0$, 10, and 18 MeV at $\rho=0.2$ fm^{-3}. As in our case, they also find that the single-particle potential graphs for these three temperatures meet at a point which almost corresponds to the Fermi momentum. However, they find the depth of the potential to increase with temperature at low momentum.

In Fig. 3, we plot the single-particle potential versus momentum for neutron matter at $T=10$, 20, and 30 MeV and $\rho=0.1$, 0.2, and 0.3 fm^{-3}. Since in neutron matter, only $T=1$ part of the nuclear force operates, the single-particle potential is more repulsive than the nuclear matter case. Otherwise, the behavior of the curves is almost similar to the nuclear matter case.

The free energy for nuclear and neutron matter are calculated using Eq. (15). The results are plotted in Fig. 4 for $T=10$, 20, and 30 MeV. From these graphs, we find that at constant density, the free energy decreases with the increase of temperature. Again with the increase of density, the free energy shows an increasing trend for a given temperature. Friedman and Pandharipande [8] have calculated the free energy at different temperatures and densities and tabulated their values for both the nuclear matter and neutron matter. There is an overall agreement between our values and the values reported by them. Our results also agree more or less with those of Baldo *et al.* [7] and of Lejeune *et al.* [5] for nuclear matter.

The influence of temperature ($T=10$, 15, 20, and 30 MeV) on the energy per nucleon is shown in Fig. 5 for nuclear and neutron matter. The differences at different temperatures are largest at small densities for neutron matter. Our results agree quite well with those of Weber and Weigel [9] for neutron matter.

In Fig. 6, we have plotted entropy per nucleon as a function of density for different temperatures for nuclear matter. The experimental results of Ref. [10] are shown by the shaded areas. The negatively sloped, positively sloped, and vertical lines are for $T=18$, 25, and 35 MeV. Our results are in good agreement with experiment. In Ref. [1], we had calculated the entropy per nucleon taking only the Sussex interaction. Introduction of the density-dependent term in the present calculation in-

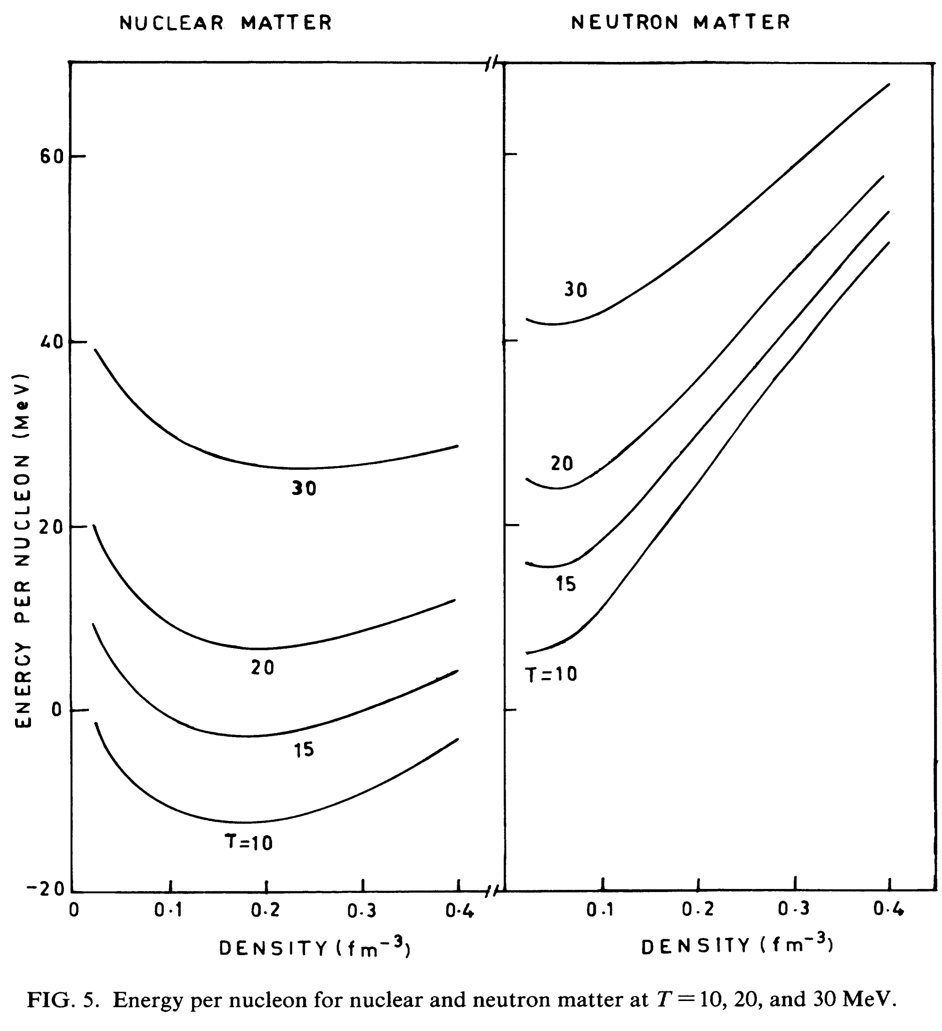

FIG. 5. Energy per nucleon for nuclear and neutron matter at $T=10$, 20, and 30 MeV.

creases the entropy by about 5%. The results are still in agreement with experiment within the experimental errors. Without assuming breakup into two phases, our model is thus able to reproduce correctly the experimental value of entropy for nuclear matter. Figure 7 gives the entropy per nucleon for neutron matter. Our results agree well with those of Friedman and Pandharipande [8] both for nuclear matter and neutron matter.

In Fig. 8, the pressure versus density is shown for $T=10$, 20, and 30 MeV for nuclear matter and neutron matter. We find that phase transitions are absent for neutron matter. However, for nuclear matter, we have a different story. As we have pointed out earlier, we calculate the chemical potential and the single-particle potential self-consistently and then proceed to calculate other thermodynamic quantities. Ours is a one phase model. In our calculation we find that for nuclear matter below $T=10$ MeV, we have a convergence problem within a given density range. For example, at $T=9$ MeV, we cannot get convergence within a density range around $\frac{1}{3}\rho_0$. However, we do not find any such convergence problem either at higher density or at lower density at this temperature. As we go to lower temperature, the range of density over which we have such a convergence problem, increases. Now the question is whether this lack of convergence points to the defect of the model at low temperature or gives insight to some new physics. To show that the model is correct, we have repeated the calculation for neutron matter for temperatures below $T=10$ MeV. We

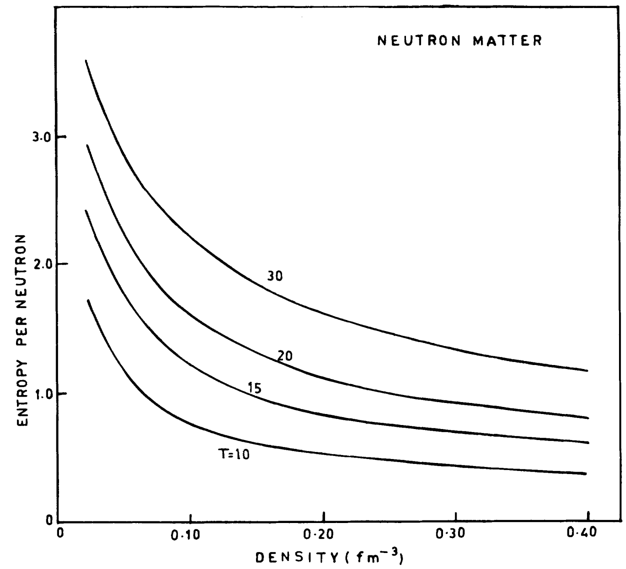

FIG. 7. Entropy per nucleon versus density at $T=10$, 15, 20, and 30 MeV for neutron matter.

do not face any convergence problem even up to $T=3$ MeV over the entire density range. As has been pointed out, we consider only one phase in our model. One possible explanation for the lack of convergence in nuclear matter is that below $T=10$ MeV and within the density

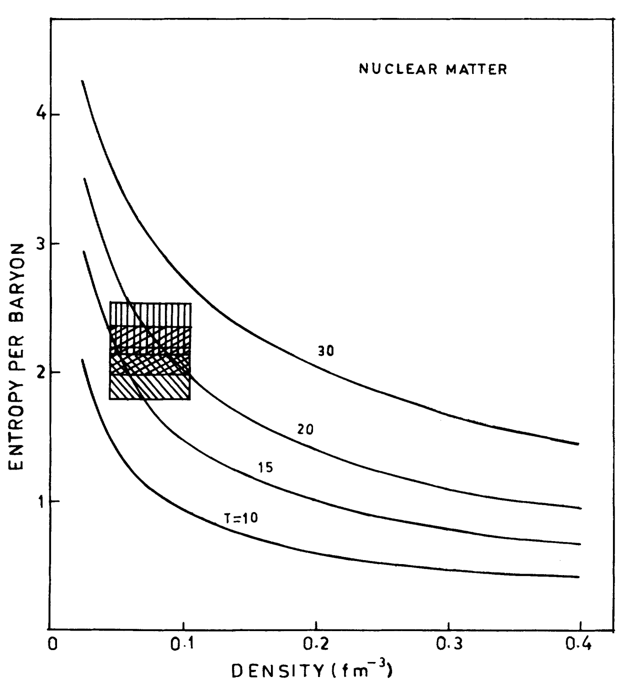

FIG. 6. Entropy per baryon versus density at $T=10$, 15, 20, and 30 MeV for nuclear matter. The experimental results of Ref. [10] are shown by shaded areas. The negatively sloped, positively sloped, and verticle lines represent the experimental values for $T=18$, 25, and 35 MeV.

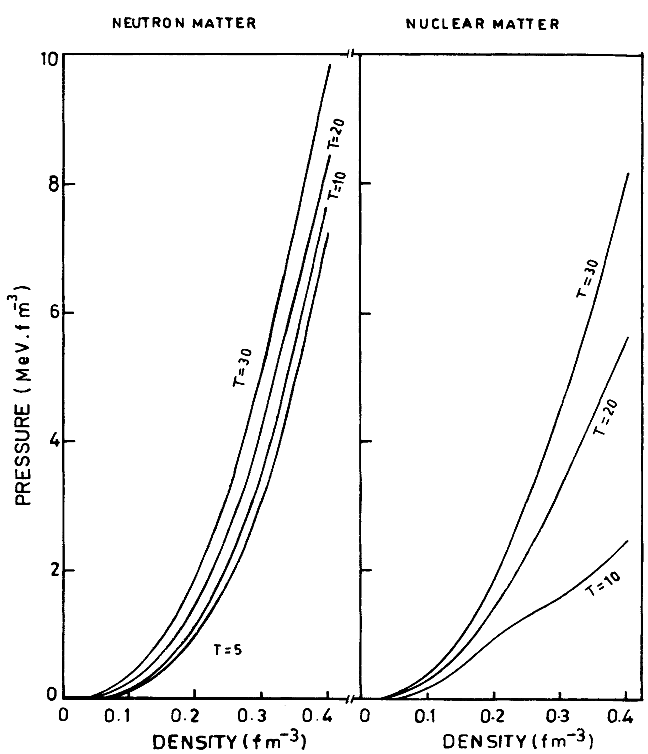

FIG. 8. Equation of state for nuclear and neutron matter. We do not observe liquid-vapor phase transition for neutron matter. But a phase transition is observed in nuclear matter with a critical temperature of $T=9$ MeV.

range, we have a mixture of two phases. The chemical potential and single-particle potential in the two phases may be different. Hence when we tried to calculate them self-consistently assuming that there exists only one phase, we failed. That our above interpretation is correct can be seen from the study of asymmetric nuclear matter [14]. As discussed above, for symmetric nuclear matter ($\gamma=1$), the highest temperature at which we start facing the convergence problem is $T=9$ MeV. However, we found that as the proton to neutron ratio γ decreases, this temperature goes on decreasing and finally for pure neutron matter ($\gamma=0$), this temperature is zero. Hence it seems that the lack of convergence in the calculation points to the occurrence of phase transition. The critical temperature decreases from $T=9$ MeV as the proton to neutron ratio γ decreases. Finally, for neutron matter this critical temperature is zero, implying the nonexistence of phase transition. If this interpretation is

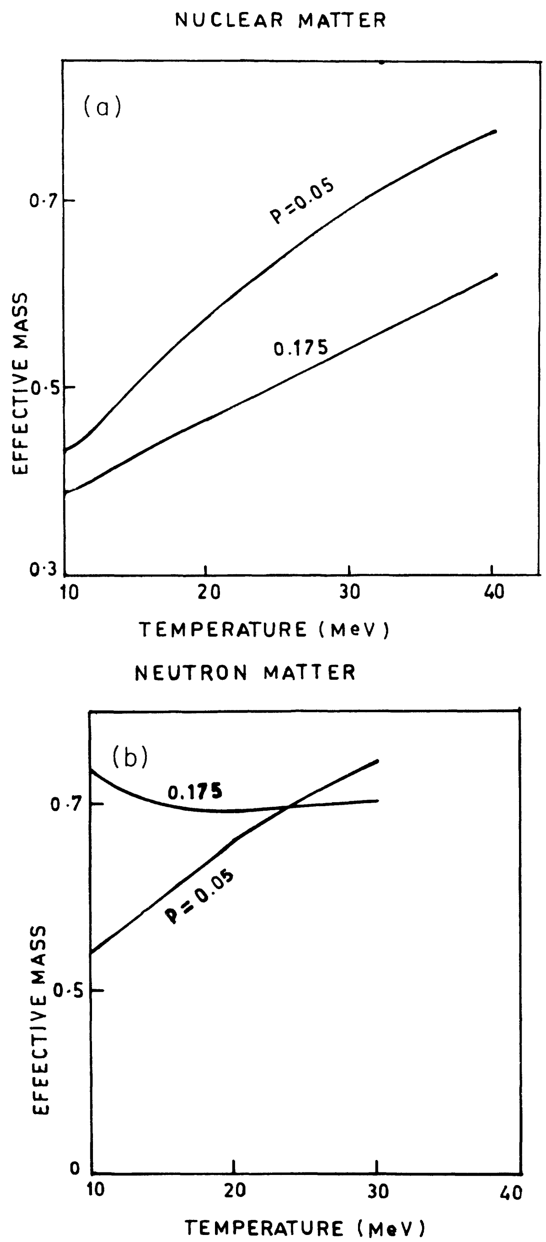

FIG. 9. Effective mass versus temperature for $\rho=0.05$ fm^{-3} and $\rho=0.175$ fm^{-3} in (a) nuclear matter and (b) neutron matter.

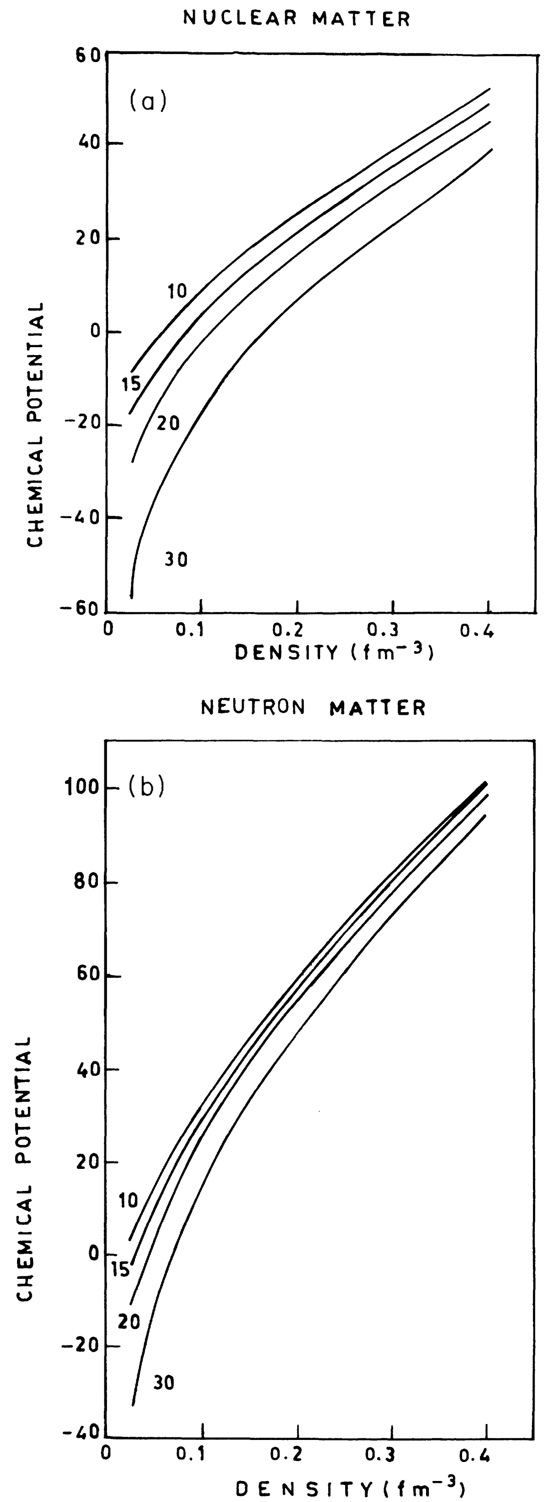

FIG. 10. Chemical potential versus density at $T=10$, 15, 20, and 30 MeV in (a) nuclear matter and (b) neutron matter.

correct, then we predict the existence of liquid-vapor two-phase equilibrium below $T=10$ MeV and around $\frac{1}{3}\rho_0$ in symmetric nuclear matter. Our value of critical temperature is far below the values reported by Friedman and Pandharipande [8], Satpathy *et al.* [11], Baldo *et al.* [7], etc. However, Haar and Malfliet [12,13] had performed a fully self-consistent Dirac-Brueckner calculation and found the critical temperature to be below 10 MeV. We had also obtained a similar value for critical temperature using only the Sussex interaction [1].

The temperature dependence of the effective mass is investigated for nuclear matter and neutron matter in Figs. 9(a) and 9(b). In calculating the effective mass we have made use of Eq. (14). For nuclear matter, our results are in fairly good agreement with the ones reported in Ref. [11].

In Fig. 10, we plot the chemical potential for nuclear matter and neutron matter.

IV. CONCLUSION

We have tried to calculate different thermodynamic properties of nuclear and neutron matter within a fully self-consistent model. The effective interaction consists of the original Sussex interaction plus a density-dependent term. The latter is added to correctly reproduce the saturation properties of nuclear matter. We have calculated the single-particle potential for both nuclear and neutron matter at different temperatures and densities. Our results at low momentum agree quite well with those of Lejeune *et al.* [5] for nuclear matter. The calculated free energy for nuclear and neutron matter agrees with the results reported by Friedman and Pandharipande [8]. Without assuming breakup into two phases, our model is able to reproduce correctly the experimental value of entropy. We do not observe any phase transition for neutron matter. However, for nuclear matter, we feel that a phase transition occurs at $T=9$ MeV. This value of critical temperature is substantially smaller than the values reported by other calculations [7,8,11]. However, Haar and Malfliet [12,13] found a phase transition for nuclear matter below $T=10$ MeV within their self-consistent Dirac-Brueckner calculation. We have also studied the temperature dependence of effective mass and chemical potential.

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